Document de travail
Discussion paper

N°2026-06

https://lem.univ-lille.fr

Is Inequality Inevitable ? Trajectories of Development in Early Agricultural Societies

Pablo Hernandez-Lagos
Sy Syms School of Business, Yeshiva University, New York, New York 10016| Pablo.hernandezlagos@yu.edu

Tanguy Le Fur
Univ. Lille, CNRS, IESEG School of Management, UMR 9221 – LEM, Lille Economie Management, F-59000, France | tanguy.le-fur@univ-lille.fr

Is Inequality Inevitable? Trajectories of Development in Early Agricultural Societies

Pablo Hernandez-Lagos* Tanguy Le Fur†

May 27, 2026

Abstract

We propose a Malthusian model of conflict in the Neolithic to study the dynamics of inequality from the onset of agriculture. Societies evolve in a region where the total supply of land is fixed and undergo three distinct stages of development: An initial foraging stage, an expansion stage when arable land is abundant, and an encroachment stage when land becomes scarce, and societies compete for its control. For each stage, we characterize the division of economic surplus between elites and workers. Our main result is that inequality is not destined to rise with the adoption of agriculture: depending on the output elasticity of labor—our summary measure of workers' effective influence over the surplus—societies may follow trajectories that rise, fall, or exhibit a hump. Using data from 1,173 archaeological sites across six independent centers of agricultural origin, we document distinct regional paths consistent with the model. Inequality increased monotonically in West Asia and Europe, but followed inverse U-shaped patterns in South and Mesoamerica, as well as in East Asia. We argue that these heterogeneous trajectories reflect variation in a small set of ecological and technological parameters that shape workers' marginal productivity and, therefore, their bargaining power.

1 Introduction

The Neolithic Revolution is arguably as important a civilizational event as the Industrial Revolution, yet it remains much less understood. Archaeologist Vere Gordon Childe, who coined the term in 1936, proposed that the transition from hunting and gathering to agriculture around 10,000 BCE was driven by changing constraints on resource access and use, especially under environmental pressure. The recent *economic prehistory* literature has made important progress in understanding how the adoption

*Sy Syms School of Business, Yeshiva University, New York, New York 10016. Email: pablo.hernandezlagos@yu.edu

†Univ. Lille, CNRS, IESEG School of Management, UMR 9221 – LEM – Lille Economie Management, F-59000 Lille, France. Email: tanguy.le-fur@univ-lille.fr

of agriculture has shaped those constraints and determined long-run economic growth (Weisdorf, 2005; Guzmán and Weisdorf, 2011; Bowles and Choi, 2019) and the origins of government (Dal Bó et al., 2022; Mayshar et al., 2022; Dow and Reed, 2023; Allen et al., 2023). A key theme in all this work is the evolution of income and wealth inequality: the Neolithic Revolution is associated with the rise of stratified societies and persistent differences in consumption and wealth (Dow and Reed, 2013; Bogaard et al., 2019; Bowles and Fochesato, 2024; Kohler et al., 2025). Is inequality inevitable? Did agriculture mechanically increase inequality, or did its distributive consequences depend on the conditions under which early agricultural societies developed? Answering these questions is the focus of this paper.

We propose a theory of early development across three distinct stages, motivated by the history of the origins of civilization in the Fertile Crescent (Allen, 2024). In an initial *foraging stage*, agricultural technology is not yet sufficiently productive to justify adoption and hunting-gathering thus prevails. As technology advances, societies enter an *expansion stage* in which agriculture becomes attractive while land remains abundant. In this stage, societies invest in agricultural infrastructure, such as traction animals and irrigation canals, and expand onto the unclaimed land around them. Eventually, population growth pushes expanding societies into an *encroachment stage*, characterized by land scarcity. As expanding agricultural societies abut one another, competition for land becomes central, and military capabilities determine their ability to secure a share of the region's fixed land supply. These three stages—initial foraging, agricultural expansion, and encroachment—provide a natural framework for examining the long-run evolution of inequality and living standards.

Using this framework, we document substantial heterogeneity in the trajectories of inequality across stages among early agricultural societies in the Fertile Crescent and beyond, in regions of the globe that independently adopted agriculture. We use a dataset of 1,173 geolocated archaeological sites compiled by Kohler et al. (2025), which provides estimates of wealth inequality based on variation in residential house sizes, expressed as Gini coefficients. To identify the stage of development associated with each site, we combine information on the mode of production from Kohler et al. (2025) with spatial data from Bird et al. (2022), a global database of 180,070 radiocarbon-dated archaeological sites extending back 55,000 years. By measuring the spatial proximity of contemporaneous settlements, we infer whether a given site operated in a foraging, agricultural expansion, or encroachment context. While the average Gini coefficient increases from foraging to expansion and again to encroachment at the global level, this pattern is driven in part by West Asia and Europe, the region most closely associated with the canonical narrative. North America, however, exhibits an inverse-U shape. East and South Asia display declining inequality in the encroachment stage. Mesoamerica shows particularly pronounced variation, with some of the lowest observed inequality during foraging and some of the highest during expansion.

To explain these heterogeneous trajectories, we develop a Malthusian growth model with three stages of development, in which technological and ecological parameters shape how elites allocate the economic surplus in a region with multiple competing societies. In each society, an elite—whether an elderly council, a tribal leader, or a chief and associates—governs the mode of production and allocates output among their own consumption, redistribution to workers, investment in agricultural infrastructure, and military spending. Redistribution to workers matters because it raises workers' fertility and thus fuels population growth, which in turn affects future labor supply, territorial expansion, and military capabilities. We characterize how the elite's allocation of the surplus evolves as societies transition across stages of development and how these allocations shape inequality and per capita income over the long run.

Our main result is that inequality is a negative function of workers' marginal contribution to production, which determines the value of redistribution for the elite. We interpret this as workers' bargaining power. What drives the trajectory of inequality during early development is that the determinants of this bargaining power change across stages. In the foraging stage, it simply corresponds to the returns to labor in foraging activities. In the expansion stage, the returns to labor in foraging are replaced by the returns to labor in agriculture. Moreover, it also includes the returns to territorial expansion, i.e., the additional acres of land a worker can occupy and use for agriculture. In the expansion stage, returns to expansion are replaced by returns to population size in military capabilities, as land has become scarce and competition between societies prevails.

Because the values of the parameters governing these elasticities vary across regions, the model generates a range of inequality trajectories, driven by a small set of technological and ecological parameters, consistent with the heterogeneity observed in the archaeological record. Importantly, development need not be monotonic: as workers' bargaining power evolves across stages, inequality can rise, fall, or follow distinct non-linear paths. In some regions, inequality may have increased sharply with agriculture; in others, only when land became scarce. Some societies may have experienced egalitarian expansion phases in which elites redistributed resources aggressively to fuel agricultural growth, whereas others may have seen inequality decrease only when competition for land intensified. We then interpret trajectories of inequality across stages through the lens of our model and sketch tentative analytical narratives of early development in regions that adopted agriculture independently, drawing on archaeological and anthropological research.

Our model also sheds light on the evolution of standards of living following the adoption of agriculture. Although humans should have benefited from the introduction of new, more productive technologies, archaeological evidence indicates that agriculturalists may have been worse off than hunter-gatherers. In our Malthusian model, because redistribution to workers fuels population growth, it mechanically reduces steady-state per capita income at each stage. The same parameters governing inequality, therefore, drive the evolution of standards of living during the course of early development, and agriculturalists might indeed have been worse off in the long run, but only if the competition for land in the encroachment stage induced the elite to redistribute substantially in order to raise fertility. As such, we argue that it is not the adoption of agriculture *per se* that led to a decline in standards of living, or a rise in inequality for that matter, but technological and ecological conditions determining the allocation of the agricultural surplus between its various uses that might have been different in various regions of the globe.

This paper contributes to the literature on economic prehistory, which studies the origins of growth, the evolution of inequality, and also the role of conflict in early development (Dal Bó and Dal Bó, 2011; Mayshar et al., 2022; Dal Bó et al., 2022; Dow and Reed, 2023). By integrating fertility decisions, public-good investment, and competition for land into a tractable growth model, we offer a unified framework for understanding the long transition from foraging to early agricultural civilization (Weisdorf, 2005). Our focus on inequality and living standards connects to a rich body of work documenting the evolution of inequality in the Neolithic (Bogaard et al., 2019; Dow and Reed, 2013; Angelopoulos et al., 2023; Bowles and Fochesato, 2024; Kohler et al., 2025; Bogaard et al., 2025). We show that while land-based constraints can explain the experiences of West Asia, Europe, and East Asia, cross-regional variation in parameters such as ecological productivity and the returns to military competition helps account for the different trajectories observed in the Americas. Our framework also complements the literature on the potential decline in living standards following the transition to sedentism and agriculture (Guzmán and Weisdorf, 2011; Allen, 2024), by linking redistribution and population growth in a Malthusian fashion. Finally, it contributes to research on the origins and consequences of warfare (Keeley, 1997; Ferguson, 2013; Dow et al., 2017; Chu et al., 2024; Le Fur and Wasmer, 2025) by showing how competition over scarce resources interacts with fertility, investment, and technology to generate conflict. By distinguishing between endogenous choices—fertility, redistribution, agricultural investment, and military spending—and exogenous constraints such as geography and climate, we provide a set of predictions linking resource scarcity to inequality and violence. These predictions help rationalize empirical patterns documented in archaeological and anthropological studies (Kohler et al., 2014; Allen et al., 2016; Kennett et al., 2022; McCool et al., 2022; Fibiger et al., 2023).

The remainder of the paper proceeds as follows. Section 2 outlines our view of the stages of early development and empirically documents heterogeneous trajectories of inequality across the globe. Section 3 sets up a theory of development for early agricultural societies. Section 4 studies the evolution of standards of living and trajectories of inequality across the three stages of early development. Section 5 compares the model's predictions to the global archaeological record and offers a qualitative account of the evolution of inequality in different regions. Section 6 concludes.

2 Historical Background and Empirical Motivation

We draw on extensive research on the origins of civilization in the Fertile Crescent to develop a guiding principle for our paper. We explain briefly the evolution of societies in three macro stages: foraging, expansion, and encroachment, and show how this framework helps us highlight the evolution of inequality in other regions. Using archaeological data, we then document that, while inequality did increase in Mesopotamia, this might not have been the case in other regions that evolved through the same stages.

2.1 Defining three stages of development

Allen (2024) proposes that civilization was preceded by societies that evolved through a series of modes of production, starting from foraging, then planting wild seed, cultivating with hoes, farming with ploughs, and irrigated agriculture. Modes of production have both technological and organizational components. Because technology and organization depend on each other, we first summarize the evidence for their co-evolution and then present an analytical framework to examine it.

Foraging Foraging existed because it was more efficient than domestication. For most of the approximately 2.8 million years of human evolution, plants have been a more costly source of calories than large animals. Geographic conditions (Ice Age Retreating) created a forager's paradise around the area highlighted in the top-left panel in Figure 1. Foraging is a labor-intensive activity that requires basic tools or weapons only occasionally. Once large animals were extinct because of human hunting, plants became an efficient source of calories, comparable to hunting small animals. (Mason, 1992; Kelly, 2013; Rosenberg, 2008; Ben-Dor and Barkai, 2021).

Foragers were initially mobile, but when land became abundant, they could survive by settling. Examples of plentiful locations include the slopes of Galilee, the Golan Heights, Anatolia (the Upper Euphrates Valley and Karacadag), and places in the Zagros Mountains (Garrod, 1957). The Natufians lived in their "Garden of Eden" south of the Levant some 15,000 years ago. Cereals, however, were not an essential part of their diet, although their importance increased with experimentation with agriculture (Allen, 2024, p.14-15). Agriculture was not adopted from the beginning of human settlement because it was costly in terms of labor requirements.

Although the origins of agriculture remain debated, for our purposes, it has two main elements that drive consensus. The first is the need to move to wetter areas following the Younger Dryas (c. 10,800 to c. 9,600 BCE). A drier climate forced people to experiment with agriculture to cope with a less fertile environment (Bar-Yosef and Meadow, 1995). The other element is that once irrigation was needed, small

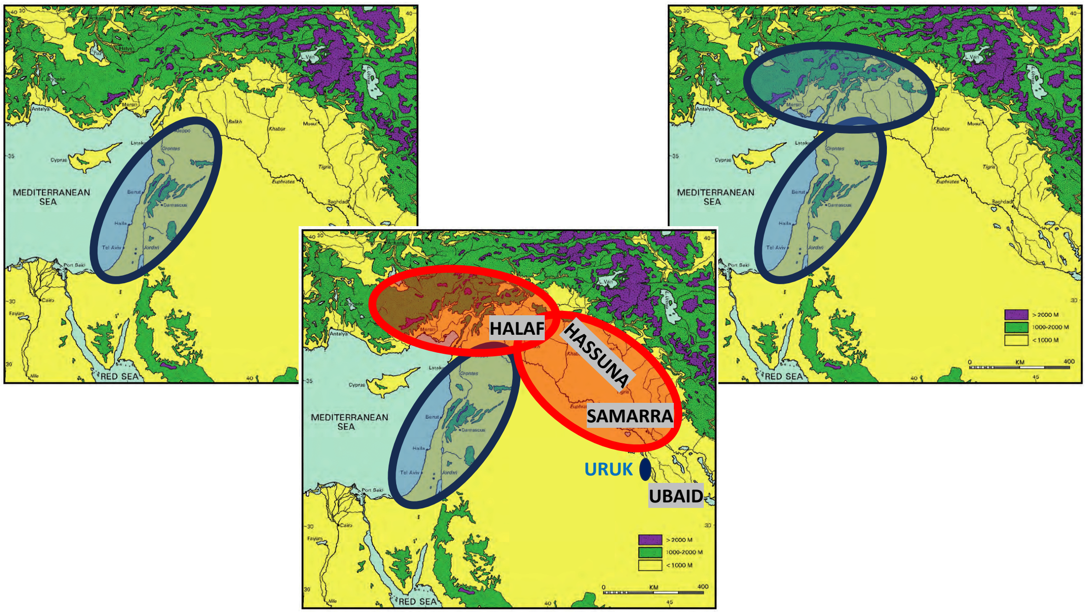

Figure 1: Black: No evidence of mass conflict; Red: Evidence of mass conflict

bands or inchoate societies needed investment in public infrastructure (irrigation canals) and a way to appropriate the harvest (Bowles and Choi, 2019). Under these conditions, small communities could experiment with agriculture on small plots, relatively far apart. The population grew, and as agricultural modes of production became the dominant technologies, it spread in all directions. Anatolia became part of the most fertile regions for cereals, as illustrated in the top-right panel of Figure 1. At the end of the stage, marked differences in consumption across population segments emerged.

Expansion The group adopting the most effective combination of technology and organization, the one with the highest labor productivity, increased its population, becoming the dominant society until it was encroached upon by another successful group emerging somewhere else.

The land beyond the Fertile Crescent, not naturally plentiful, remained unoccupied. Starting about 10,000 BCE, the expansion was slow toward northern Mesopotamia, particularly the Jazira (the "island" between the Tigris and the Euphrates) region (Fuller et al., 2012). Cultivators could now get calories from the land they settled on and produce beyond what was needed to subsist. Investments in infrastructure and the plough increased yields. Surpluses eventually led to systematic inequality across societies. The use of land demanded traction animals and eventually reduced labor's importance in production and increased capital's importance (Fochesato et al., 2021). The owners of capital consumed more, and inequality increased further (North and Thomas, 1977; Bowles and Choi, 2019).

However, labor's reduced importance in food production did not necessarily imply a reduced role for human work. Expansion became a consequence of population growth and required humans to occupy new territory, prepare the land, and scout new regions. Labor intensity may have been reduced, but labor extensity increased: food production required less labor and more capital per plot of land, while the number of plots cultivated increased. The domestication of seed and animals increased the area where grain could be profitably grown c. 9,000 to 6,000 BCE (Maeda et al., 2016). Expansion to and within Anatolia and the Jazira presaged organized competition for land (Skourtanioti et al., 2020). The top-right panel of Figure 1 roughly encompasses the sites that show domestic grain expansion between c. 9,000 and 6,000 BCE in Figures 6 and 7 in Allen (2024).

Encroachment Although the debate is not entirely settled, scholars agree that violent competition for resources before c. 6,500 BCE occurred only sporadically and in very small numbers. Ferguson (2013), for example, documents that there is no skeletal evidence of large-scale conflict or fortifications from 13,000 BCE to 3,200 BCE. However, in Anatolia and the Northern Tigris, there is evidence of warfare dating to around the Pottery Neolithic (6,400 BCE) and perhaps earlier, during the pre-Pottery Neolithic (9,500-6,000 BCE), when forager communities were first experimenting with agriculture. Due to climate and population pressures, agricultural experiments and successes (seed domestication) expanded into the Jazira region after 6,000-5,400BCE, and with them, the first evidence of encroachment and organized conflict (Wilkinson et al., 2024). Eventually, there was no more free land in the Jazira, and warfare emerged over land control — for example, control of the obsidian trade routes — depicted by the red ovals in the central panel of Figure 1.

Initially, there was free land in the Jazira, but evidence suggests that it was later privatized and traded (Bogaard et al., 2019). When land became scarce, evidence of conflict abounded. Examples are Tell Maghzalia IV (c. 6,500) BCE, the first defensive wall, Hacilar II (c. 5,600), evidence of war, Domuztepe (5,700-5,600) pit with 400 victims of violence, Ras Shamra (5,234) destroyed by fire, then a defensive wall was built. The timeline in Table 1 based on Allen (2024) provides approximate dates for these stages.

2.2 Inequality across stages of development

A dataset compiled by Kohler et al. (2025) summarizes the remarkable effort by scientists across disciplines to measure inequality using differences in housing size across 1,173 archaeological sites. For each site, the data provide a Gini coefficient, the date it was populated, its longitude and latitude, and a variety of other information, including, crucially, the prevailing mode of production. Each site can thus be assigned to one of eight regions that independently developed agriculture.

We classify each archaeological site in Kohler et al. (2025) into one of three stages of development (foraging, expansion, or encroachment) in the first column of Table 1 to show the evolution of inequality

Stage	Archaeological period	Years BCE	Mode of production	Approx. location
Foraging	Natufian and Younger Dryas	13,000 - 9,600	Settled and mobile foragers	Palestine, Israel, Jordanian Hills, upper Euphrates
Expansion	PPNA and PPNB	9,600 - 6,000	Wadis, natural fields using wild seeds, cultivate domestic seed and animals	Jordanian rift valley, upper Euphrates, southern Anatolia
Encroachment	Pottery Neolithic, Halaf, Hassuna, and Samarra periods	6,000 - 5,400	Rain-fed farming	Jordanian rift valley, upper Euphrates, southern Anatolia, Jazira

Table 1: Food production stages. Adapted from Allen (2024) Table 2.

across regions. The evolution from foraging to expansion can be determined solely from the characteristics of each site available in Kohler et al. (2025). Encroachment, however, needs data about societies inhabiting nearby areas. To assign sites to the encroachment stage, we complement the data in Kohler et al. (2025) with Bird et al. (2022), which collects, dates, and geolocates 180,070 sites globally. Although no estimate of the Gini coefficient is available there, this much larger number of archaeological sites allows us to compute the number of societies in a given area at a given date.[1] We also use information on the mode of production of each site, in particular the use of plant cultivation, suggestive of the adoption of agriculture. The three stages of development present in the model are defined and computed as follows:

- **Foraging** is equal to one if plant cultivation *was not* commonly used before the site was inhabited. It is zero otherwise
- **Expansion** is equal to one if plant cultivation *was* commonly used before the site was inhabited and there are no neighbors within a 50 km radius. It is zero otherwise.[2]
- **Encroachment** is equal to one if plant cultivation *was* commonly used before the site was inhabited and there is at least one neighbor within a 50 km radius. It is zero otherwise.

[1] The site's date combines data on the earliest and latest occupation and residences. As such, we estimate the site date as an interval in calendar years, with negative values indicating dates before the common era (BCE). The dataset used in this study is detailed explained in Ortman et al. (2025). The site date interval is defined as the calendar years between $\max\{beginsite, beginlocation\}$ and the $\min\{endsite, endlocation\}$. These variables are defined in page 4 of the Supporting Online Material accompanying Ortman et al. (2025): "*beginsite*, *endsite*, *beginlocation*, *endlocation* (BCE/CE)— beginning and ending date of the associated site and its other attributes and buildings; and the beginning and ending dates representing the period of continuous occupation at that location. The second set of dates can be longer than the first. BCE dates are negative."

[2] The 50 km radius is arbitrary, but is roughly the distance between two warring inchoate city states, Lagash and Umma. Longer radii do not change the qualitative results.

The next variable we compute is the region that developed agriculture independently. This variable is crucial because it identifies potentially distinct paths of development, ranging from foraging to encroachment. The eight regions identified in the data are listed in Table 2.

Independent Agriculture	**Regions in Kohler et al. (2025)**
W Asia & Europe	C Europe, E Europe, Great Britain, SE Europe, W Asia and Cyprus, W Europe
East Asia	E Asia
South Asia	S Asia
Mesoamerica	Central Mexico, Maya, Southern Mexico
South America	Central Andes, Southern Andes
North America	Great Plains, Northeast NA, Northwest NA, Southeast NA, Southwest NA
Africa	E Africa, Horn of Africa, North Africa, S Africa
Oceania	Melanesia, Polynesia

Table 2: Regions that developed agriculture independently computed from grouping the variable `region` in Kohler et al. (2025).

We then assign each archaeological site to one of the eight regions, and Table 3 presents summary statistics. The regions with the most sites (observations) are West Asia & Europe (475), North America (278), and East Asia (160). The ones with the fewest are Oceania (24), South Asia (15), and Africa (15). In West Asia & Europe, the most common stage is Expansion (292), and the least common is Foraging (26). The same pattern holds for North America (111 and 74, respectively). The majority of sites in East Asia are in the Encroachment stage (95), whereas the fewest are in the Expansion stage (26). Meso and South America have very few observations in the Foraging stage (less than 1% and 9%, respectively), but 155 and 54 sites, respectively. Likewise, there is no South Asia site in the Encroachment stage. For this reason, we group relatively close regions together, East and South Asia, as well as South and Meso America, when computing the Gini. Note that Kohler et al. (2025) also provides an estimate of income based on the average house size in each archaeological site. Mean Gini is highest in Meso America (0.42) and lowest in North America and East Asia (both at 0.23). Mean income is also highest in Meso America (mean log of household area 5.76, roughly 317 square meters, or 3,412 square feet) and lowest in Africa (2.75, approximately 15 square meters, or 161 square feet). The numbers for West Asia & Europe and North America are 560 and 344 square feet, respectively.

Figure 2 top left panel shows that the average inequality of the 475 sites located in West Asia and Europe increases from foraging to expansion to encroachment. This is the story of increasing inequality as societies evolve, leading to the dawn of the first city-states. However, Kohler et al. (2025) data also allows us to plot other regions that adopted agriculture independently. The evolution of inequality in East Asia and Europe does not repeat in any other region. Across the 278 sites in North America, inequality remains largely flat across regions. When combining the sites of Meso (155) and South (54)

Variable		1 W Asia & Europe	2 North America	3 Meso America	4 South America	5 East Asia	6 South Asia	7 Oceania	8 Africa
	Sites:	475	278	155	54	160	15	24	15
Foraging	N	26	74	1	5	39	8	8	1
Expansion	N	292	111	26	27	26	7	6	7
Encroachment	N	157	93	128	22	95	0	10	7
Gini	N	475	278	155	54	160	15	24	15
	Mean	0.31	0.23	0.42	0.33	0.23	0.32	0.41	0.30
Income	N	475	278	155	54	160	15	24	15
	Mean	3.97	3.47	5.76	4.36	2.80	3.87	3.76	2.75

Table 3: Summary Statistics. N: Non-missing observations.

America, inequality presents an inverse U-shape: it is lowest during foraging and highest during expansion. Something similar occurs in the East (160) and South (15) Asian sites combined, although in this case, the lowest inequality occurs during encroachment. Oceania (25) and Africa (15) present even more striking differences, but the number of observations is much smaller.

The evolution from foraging to cultivation, and then from cultivation to encroachment, occurs worldwide. However, inequality, which is an outcome of pre-institutional variables (geographic, climatic, and biodiversity) and decisions, presents a different evolution. The model we lay out in the next section offers an explanation for such heterogeneity in inequality trajectories.

3 The Model

N societies indexed by $j = 1, 2, ..., N$ evolve in a region where the supply of land is fixed and equal to X. Each society is populated by a number $\mathcal{L}_t$ of adult individuals in period t, from which a fixed and exogenous fraction θ belongs to the elite: $L_t = \theta\mathcal{L}_t$. The word elite does not refer to any prior income or consumption differences, but to a subset of society that makes resource allocation decisions; think of a council of the elderly or a tribal chief. The rest of the population are workers, and their number is thus $l_t = (1-\theta)\mathcal{L}_t$. We assume that members of the elite do not work or spend resources to have children. The population grows according to the following:

$$\mathcal{L}_{t+1} = n_t(1-\theta)\mathcal{L}_t \tag{1}$$

where n_t is the rate of fertility of workers.

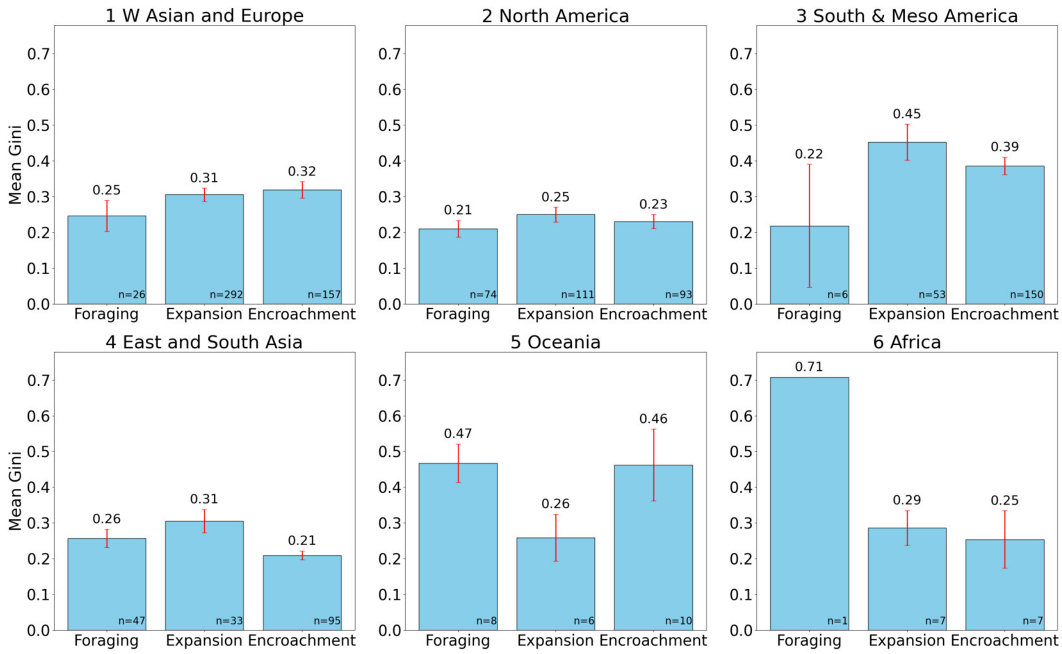

Figure 2: Gini by stage for each region.

3.1 Production technologies

Each society has access to two production technologies: mobile foraging and agriculture. Foraging requires only labor l_t, while agriculture requires labor and land, the use of which requires productive infrastructure investments G_t (e.g., irrigation canals). The crucial assumption of our model is that land must be occupied to be used for agriculture. The land holdings of a given society are denoted by x_t. There is progress in agricultural technology that raises the productivity of land A_t at an exogenous rate g. The production functions for foraging and agriculture are the following:

$$Y_t^F = l_t^\zeta \tag{2}$$

$$Y_t^A = l_t^\alpha (A_t G_t^\kappa x_t)^{1-\alpha} \tag{3}$$

with $\zeta \in (0,1)$, $\alpha \in (0,1)$, and $\kappa \in (0,1)$.

We assume that foraging is more labor-intensive than agriculture, such that $\zeta > \alpha$.

3.2 Occupying land

The key idea in our model is that land must be occupied to be used for agriculture. When free territories remain unoccupied, their occupation requires only workers. Land is thus initially abundant, and societies expand as their populations grow. At some point in the development of several societies, however, there is no more free land, and societies encroach. They compete for scarce land. Occupying land requires not only workers but territorial investments. Territorial investments are investments in human or physical capital that enable workers to expand or secure a portion of land. They could be thought of as an investment in military capability. Land is then distributed according to a Tullock contest between all societies, in which their relative *military capabilities* determine their respective share of the total supply of land. Thus, the amount of land that each society occupies and controls is given by:

$$x_t^i = \begin{cases} e(l_t^i) & \text{for} \quad \sum_{j=1}^N e(l_t^j) \leq X \\ \dfrac{\mathcal{M}_t^i}{\sum_{j=1}^N \mathcal{M}_t^j} \cdot X & \text{for} \quad \sum_{j=1}^N e(l_t^j) > X \end{cases} \tag{4}$$

The first row in Equation 4 states that when land is initially abundant, societies expand as their population grows. We assume the function $e(l_t^i)$ is increasing and concave, such that population density increases with expansion:

$$e(l_t^i) = (l_t^i)^\phi, \tag{5}$$

with $0 < \phi < 1$.

The second row in Equation 4 describes the competition for land, where $\mathcal{M}$ stands for *military capabilities*. We assume military capabilities to be a Cobb-Douglas function of both the worker population and military investment M_t, such that:

$$\mathcal{M}_t = M_t^\mu l_t^\lambda. \tag{6}$$

We do not restrict ourselves *a priori* to constant returns to scale. The fact that population size enters military capabilities implies that the elite will have additional incentives to influence workers' fertility decisions. In other words, the parameters μ and λ determine the trade-off between military investments and, indirectly, fertility decisions and consequently, economic growth.

We normalize the total supply of land $X = 1$ such that x_t^i characterizes the share of land in the region that society i controls.

3.3 Individual optimization

A society is divided into two social classes: the elite and the workers. The elite collects aggregate production and allocates it to its own consumption, wage payments to workers, investments in agricultural infrastructure, and military investments. Workers allocate their wages to consumption and child rearing.

Workers. Workers care about consumption and the number of children they have:

$$u_t = (1-\gamma)\log c_t + \gamma \log n_t,$$

where c_t is the consumption level of workers and n_t is the fertility rate. They take their revenue w_t as given and maximize their utility subject to their budget constraint $w_t = c_t + \tau n_t$, where τ is the fixed cost of child rearing. The optimization problem of workers is straightforward, and standard computations yield:

$$c_t = (1-\gamma)w_t \tag{7}$$

$$n_t = \frac{\gamma}{\tau} w_t. \tag{8}$$

Workers devote a constant fraction $1-\gamma$ of their revenue to their own consumption, and allocate the rest to child rearing. The key feature of workers' decisions is that the fertility rate increases with income in a standard Malthusian fashion. It is precisely this demographic feature that prompts the elite to redistribute a portion of economic output to workers. The wage the elite pays workers is the mechanism through which it can influence the future supply of labor, the share of land occupied as societies expand, and military capabilities when encroaching.

The elite. We assume that the elite cares about its own per capita consumption C_t and its *legacy*, characterized by aggregate output in the next period Y_{t+1}:

$$U_t = \log C_t + \beta \log Y_{t+1}.$$

The elite has access to unspecified coercion technology that allows it to collect production Y_t and allocate it between its own consumption C_t, workers' wages w_t, investment in infrastructure G_{t+1}, and military spending M_{t+1}. The budget constraint is:

$$Y_t = C_t L_t + w_t l_t + G_{t+1} + M_{t+1}. \tag{9}$$

Although the elite collects all produced resources, it will nonetheless allocate some of them to workers, infrastructure, and military investment because of their effects on future output. Redistributing income to workers influences their fertility rates. The effect of redistribution to workers initially comes solely from the extra labor force when foraging is the prevailing mode of production. When agriculture is adopted, the elite have further incentives to raise fertility to secure a greater share of land. And when societies encroach, the elite have additional incentives to increase fertility, as more people mean more soldiers.

The elite's decisions are more sophisticated than those of workers, as they involve choosing a production technology in addition to allocating resources. By choosing either $G_{t+1} = 0$ or $G_{t+1} > 0$, the elite decides whether or not to adopt agriculture. By choosing either $M_{t+1} = 0$ or $M_{t+1} > 0$, the elite decides on military investments. Military investments are not required when land is still abundant in the region ($M_{t+1} = 0$), but are necessary when societies encroach ($M_{t+1} > 0$). The combination of such corner solutions defines three technological stages:

1. The foraging (or pre-agricultural) stage where $G_{t+1} = 0$ and $M_{t+1} = 0$,
2. the expansion stage where $G_{t+1} > 0$ and $M_{t+1} = 0$, and
3. the encroachment stage where $G_{t+1} > 0$ and $M_{t+1} > 0$.

The distribution of aggregate output. The decisions of the elite determine redistribution, infrastructure investments, and military investments on future aggregate output. Solving the elite's maximization problem gives the shares of output that are allocated to each use–elite's consumption, redistribution, infrastructure investments, and military investments–at each stage of development. We show below how the distribution of the surplus depends on the three following elasticities:

$$\Omega_l = \frac{l_{t+1}}{Y_{t+1}} \frac{\partial Y_{t+1}}{\partial l_{t+1}} \quad ; \quad \Omega_G = \frac{G_{t+1}}{Y_{t+1}} \frac{\partial Y_{t+1}}{\partial G_{t+1}} \quad ; \quad \Omega_M = \frac{M_{t+1}}{Y_{t+1}} \frac{\partial Y_{t+1}}{\partial M_{t+1}} \tag{10}$$

Crucially, those elasticities will take different values in each of the three stages of development that we index by $k = 0, 1, 2$, as reported in Table 4. As the values of those elasticities change across stages, so does the optimal allocation of aggregate output from the elite's point of view. Those three elasticities quantify the value to the elite of various types of spending: redistribution to workers, infrastructure investment, and military investment. In the foraging stage, for example, the returns to investments in the agricultural infrastructure and military spending are zero, as only labor, not land, is used in production. The returns to redistribution come simply from the additional labor force that redistribution, by fueling workers' fertility, ensures. In the expansion stage, the returns to redistribution include an additional component beyond the returns to agricultural labor: the returns to land expansion, i.e., the additional land that additional workers can occupy. The returns to agricultural infrastructure become positive, whereas the returns to military investments remain zero, as land remains in abundant supply in the region and competition for its control has not yet begun. Finally, in the encroachment stage, the returns to redistribution no longer include returns to expansion, as land has become scarce, but instead include re-

Output elasticity of	Labor	Infrastructure	Military investment
	$\boldsymbol{\Omega_l^k}$	$\boldsymbol{\Omega_G^k}$	$\boldsymbol{\Omega_M^k}$
Foraging, $k=0$	ζ	0	0
Expansion, $k=1$	$\alpha+(1-\alpha)\phi$	$(1-\alpha)\kappa$	0
Encroachment, $k=2$	$\alpha+(1-\alpha)\lambda x^*_{t+1}$	$(1-\alpha)\kappa$	$(1-\alpha)\mu x^*_{t+1}$

Table 4: Output elasticities for each stage of development.

turns to population size in military capabilities. The returns to military investment have finally turned positive, as the share of land each society controls now depends on its military capabilities rather than simply on the size of its population. The returns to infrastructure investments are unchanged.

Note that $x^*_{t+1}=\sum_{j\neq i}x^j_{t+1}$ is the share of the land in the region controlled by all rival societies. x^*_{t+1} is an endogenous object, the outcome of the Tullock contest between all societies. While it is not possible to find a general closed-form solution to the model in the encroachment stage, we can nonetheless focus on a symmetric equilibrium in which all N societies are identical. In the symmetric equilibrium, each society controls a constant fraction $1/N$ of the land in the region and $x^*_{t+1}=x^*(N)=1-1/N$. The three elasticities thus remain constant and, importantly, depend on the number of societies in the region, or the intensity of competition N.

Solving the model for each stage distinctively is straightforward and gives us the allocation of aggregate output as the shares devoted to its own consumption S, redistribution to workers s, infrastructure investments $\mathcal{S}_G$, and military investments $\mathcal{S}_M$, for each stage:

$$S^k = \left(\frac{C_tL_t}{Y_t}\right)^k = \frac{1}{1+\beta\Omega^k} \tag{11}$$

$$s^k = \left(\frac{w_tl_t}{Y_t}\right)^k = \frac{\beta\Omega_l^k}{1+\beta\Omega^k} \tag{12}$$

$$\mathcal{S}_G^k = \left(\frac{G_{t+1}}{Y_t}\right)^k = \frac{\beta\Omega_G^k}{1+\beta\Omega^k} \tag{13}$$

$$\mathcal{S}_M^k = \left(\frac{M_{t+1}}{Y_t}\right)^k = \frac{\beta\Omega_M^k}{1+\beta\Omega^k} \tag{14}$$

where $\Omega^k=\Omega_l^k+\Omega_G^k+\Omega_M^k$.

In addition to β, the degree at which the elite favors future output relative to their own consumption, the elasticities defined above fully determine the allocation of the economic surplus. As one would expect, an increase in the returns to redistribution, infrastructure investment, or military investment will raise the associated share of output devoted to that investment and decrease the share of output appropriated by the elite for its own consumption.

Inequality. Using the solution to the workers' optimization problem, we can also introduce a measure of inequality based on consumption: $\mathcal{I}^k \equiv C_t^k / c_t^k$. It follows that:

Proposition 1 *Inequality is decreasing in workers' bargaining power. Let k indicate the stage and Ω_l^k the output elasticity of labor, then*

$$\mathcal{I}^k = \frac{1-\theta}{\beta\theta(1-\gamma)} \frac{1}{\Omega_l^k}, \tag{15}$$

where θ, β, and γ are fixed between zero and one.

The crucial parameter that will shape inequality is therefore the elasticity of aggregate output with respect to labor. Greater returns to labor, whether through additional labor input, a larger share of land occupied, or greater military capabilities, induce the elite to redistribute more to workers. Note that the relationship between workers' power and inequality does not depend on the stage: the proportionality constant $\dfrac{(1-\theta)}{\beta\theta(1-\gamma)}$ is the same across stages.

In the next section, we study how the transition from one stage of development to the next affects the distribution of aggregate output and inequality between the elite and workers.

4 Long-run dynamics and inequality trajectories

In this section, we study the evolution of N identical societies within a region. Because the initial level of agricultural technology A_0 is very low, all societies start by foraging in the first stage of development. We show that it is only when A_t reaches a critical threshold that the elite finds it beneficial to allocate resources to public infrastructure that supports agricultural production, and that $G_{t+1} > 0$. In this second stage of development, the fraction of total land each society controls and uses is sufficiently small that land remains an abundant factor in the region. As societies expand, however, they eventually encroach. Land becomes scarce, and the competition for its control requires military investment, so that $M_{t+1} > 0$ in the third and final stage of development. We first characterize the dynamics of income per capita across stages of development, and then describe how transitions between stages affect the elite's allocation of aggregate output and inequality between the elite and workers.

4.1 The dynamics of income per capita across stages of development

We begin by analyzing the dynamics of income per capita $y_t = Y_t/\mathcal{L}_t$ for a given society across the three stages, as well as the transitions between stages. It is straightforward to show that, due to diminishing returns to labor, there exists a unique steady-state level of income per capita in the initial foraging stage. Because foraging was prevalent for so long before the adoption of agriculture, we assume that societies reached this steady-state level. We then derive the condition on the level of agricultural productivity that triggers the endogenous transition from the foraging stage to the expansion stage, when societies adopt agriculture and land is abundant.

Whether a steady-state level of income per capita exists in the expansion stage depends on the returns to scale in the production function. A steady state exists only under decreasing returns, which may or may not be the case, depending on the returns to expansion ϕ and infrastructure investments κ. More specifically, a steady state exists if and only if $\phi + \kappa < 1$. If it does not, societies experience sustained growth in standards of living as their populations expand and their land holdings increase.

Population growth, however, eventually ends the expansion stage, and societies transition to the encroachment stage when there are no more free territories to expand into. In the symmetric equilibrium we analyze, the share of land controlled by each society is constant at $1/N$. Societies converge to a steady state level of income per capita. We summarize the dynamics in the following proposition:

Proposition 2 *Societies in a region undergo three stages of development: foraging, expansion, and encroachment. They transition from foraging to expansion when agricultural productivity is sufficiently high, and from expansion to encroachment when land becomes scarce. In particular:*

- *In the foraging stage, income per capita converges to a unique steady state.*
- *In the expansion stage, income per capita converges to a unique steady state if and only if $\phi + \kappa < 1$.*
- *In the encroachment stage, income per capita converges to a unique steady state.*

Whenever a steady state exists, steady-state income per capita is decreasing in workers' bargaining power.

Proof: see Appendix A.2

Intuitively, the steady state level of income per capita in each stage is a negative function of the share of the surplus redistributed to workers s^k. This is due to the familiar Malthusian population pressure: more redistribution leads to a higher rate of population growth, which, under diminishing returns to labor, lowers income per capita. Our framework is therefore particularly useful for addressing questions raised by the emerging literature on the economics of the Neolithic. While some scholars have argued that the adoption of agriculture made societies worse off in the long-run, we can derive the conditions under which this indeed occurs: if redistribution increases in the encroachment stage because the returns to population size in military capabilities are substantial, it is possible that the resulting population growth reduces steady state income per capita despite technological progress relative to the foraging steady state. Using the closed-form expression of steady state income in both the foraging stage and the encroachment stage derived in Appendix A.2, standards of living are permanently lower following the adoption of agriculture relative to foraging if and only if:

$$\frac{s^2}{s^0} \equiv \frac{1+\beta\Omega^0}{1+\beta\Omega^2}\frac{\Omega_l^2}{\Omega_l^0} > (1+g)^{\frac{1}{1-\kappa}}$$

Furthermore, the model can generate a substantial decline in standards of living during the transition from expansion to encroachment. When $\phi+\kappa > 1$, societies may experience sustained growth in income per capita during the expansion stage, but this growth depends on the availability of free land. Once societies encroach, land becomes fixed, the Malthusian constraint starts to bind and returns to scale are decreasing. The steady-state level of income per capita to which societies eventually converge may therefore be much lower than the level reached during expansion. As such, our theory suggests that it is not agriculture *per se* that might have led to a worsening of standards of living, but rather the end of land abundance and the Malthusian population pressure, fueled by redistribution, when societies competed for control of scarce land.

4.2 The dynamics of inequality across stages of development

Now that we have characterized the dynamics of the identical societies evolving in the region, we can turn to studying how transitions across stages affect inequality between the elite and workers. Proposition 1 shows that, within a given stage, inequality is decreasing in workers' bargaining power. A direct corollary is therefore:

Corollary 1 *Consider two consecutive stages of development, k and $k+1$, with $k \in \{0,1\}$. Inequality declines from stage k to stage $k+1$ if and only if workers' bargaining power is higher in the later stage:*

$$\mathcal{I}^{k+1} < \mathcal{I}^k \quad \Longleftrightarrow \quad \Omega_l^{k+1} > \Omega_l^k.$$

Equivalently, inequality increases from one stage to the next if and only if workers' bargaining power decreases.

Transitions across stages modify the economic value of the population and, therefore, workers' bargaining power. Recall that, in the three stages:

$$\Omega_l^0 = \zeta, \qquad \Omega_l^1 = \alpha + (1-\alpha)\phi, \qquad \Omega_l^2 = \alpha + (1-\alpha)\lambda x^*(N).$$

The dynamics of inequality are therefore driven by a small set of technological and ecological parameters: the returns to labor in foraging ζ, the returns to labor in agriculture α, the returns to population size in expansion ϕ, the returns to population size in military capabilities λ, and the degree of political fragmentation (or the number of competing societies in the region) N.

The transition from foraging to expansion: the adoption of agriculture. Although the adoption of agriculture is often associated with increasing inequality, our model shows that inequality is contingent on whether workers' bargaining power, and thus the returns to redistribution for the elite, rise or fall with agriculture when land is still abundant. The returns to redistribution in the foraging stage come solely from the additional labor force brought by a larger population. Since foraging is more labor-intensive than agriculture ($\zeta > \alpha$), increasing the number of workers should be more beneficial to the elite under foraging than under agriculture. However, during the expansion stage, a larger population also enables societies to control a greater share of the land. The returns to redistribution feature the returns to the additional land an additional worker can occupy $(1-\alpha)\phi$. Inequality might therefore decrease with the adoption of agriculture when land is free if the returns to population growth in territorial expansion ϕ are high enough. Formally, this is the case when:

$$\mathcal{I}^1 < \mathcal{I}^0 \quad \Longleftrightarrow \quad \Omega_l^1 > \Omega_l^0 \quad \Longleftrightarrow \quad \phi > \frac{\zeta-\alpha}{1-\alpha}.$$

The potential decline in inequality stems from the elite's incentives to reduce the share of the economic surplus that it appropriates for its own consumption, in order to invest in the infrastructure required for agriculture. This does not necessarily mean that the share devoted to redistribution increases, because the elite faces a trade-off between investing in the infrastructure or fueling population growth to expand and use a larger share of available land. For it to increase, the returns to redistribution must be sufficiently high relative to the returns to infrastructure investment. Proposition 3 in Appendix A.3 shows that redistribution does increase when $\phi > \frac{\zeta-\alpha}{1-\alpha} + \beta\zeta\kappa$. When that is the case, inequality decreases because the elite reduces the share they appropriate for their own consumption to invest in infrastructure and fuel population growth. If κ is high relative to ϕ, however, inequality can also decrease because the elite reduces both their own consumption and redistribution to invest in the infrastructure, but the former more than proportionally. Conversely, inequality can increase if the elite reduces the share redistributed to workers more than their own share. Finally, we can show that the elite increase their own share of the surplus when $\phi < \frac{\zeta-\alpha}{1-\alpha} - \kappa$, in which case the elite reduces redistribution to both invest in the infrastructure while raising their consumption.

The transition from expansion to encroachment: the competition for land. As both agricultural technology and population grow, societies gradually expand by using a larger share of the available land in the region. While land is abundant and requires only people to occupy it during the expansion stage, it becomes scarce once there are no free territories and the total supply of land is exhausted. Societies then encroach and compete for control of land, necessitating investments in military capabilities. Again, inequality can either rise or fall depending on whether encroachment raises workers' bargaining power and thus the returns to redistribution. The returns to redistribution still feature the returns to labor in agricultural production, but the returns to expansion are replaced by the returns to population size in military capabilities λ, weighted by the share of land controlled by all other societies in the region $x^*(N)$. Formally, inequality decreases when land becomes scarce, and societies encroach if and only if:

$$\mathcal{I}^2 < \mathcal{I}^1 \iff \Omega_l^2 > \Omega_l^1 \iff \lambda x^*(N) > \phi.$$

The reduction in inequality can occur because the elite reduces its own consumption beyond what goes to workers to finance both infrastructure and military investments, or because redistribution to workers increases significantly. Proposition 4 in Appendix A.3 states that for redistribution to increase with encroachment, the associated returns must not only be greater than in the expansion stage but also high relative to the returns to military capabilities. Conversely, encroachment can lead to rising inequality if a larger population is more valuable during expansion, when land is free, than during encroachment, when land becomes scarce. Inequality might increase because the elite reduces redistribution more than their own consumption or because they decide to raise the share of output they appropriate. It can be shown that this occurs when $\phi > (\lambda + \mu)x^*(N)$ such that the returns to expansion were so high that they surpass the overall returns to military capabilities. It is worth noting that when this is the case, the share of the surplus devoted to infrastructure investment increases with encroachment. This is because an extremely high ϕ means that expanding is more attractive than investing in the infrastructure when land is free. The sudden scarcity of land caused by encroachment thus raises the attractiveness of investments in the infrastructure if the returns to military capabilities are not high enough.

Political fragmentation in the encroachment stage. The role of political fragmentation, defined as the number N of societies in the region, in the encroachment stage is worth noting. In the symmetric equilibrium we study, an increase in political fragmentation reduces each society's share of land, thereby increasing competition. Because of diminishing returns to land, this raises the returns to military capabilities, and thus to both population growth and military investments. Noting that $x^*(N) = 1 - \frac{1}{N}$, it is straightforward to see that political fragmentation raises the returns to redistribution Ω_l^2 and therefore reduces inequality in the encroachment stage:

$$\frac{\partial \mathcal{I}^2}{\partial N} < 0$$

In Proposition 5 in Appendix A.3, we show that while military investments naturally increase with competition, redistribution increases only when the returns to population size in military capabilities are high enough relative to the returns to military investments. Interestingly, as the number of societies increases, the share of land each society controls is mechanically smaller. In addition to raising the returns to military capabilities, political fragmentation reduces the returns to investments in agricultural infrastructure and the share of resources devoted to it. This illustrates the trade-off between competing for land and investing in infrastructure. In a region with more but smaller societies, the intensity of conflict reduces inequality at the expense of productive investments. Large societies in less fragmented regions, on the other hand, are likely to be more unequal but to devote more resources to productive investment.

From foraging to encroachment: the long-run perspective. It is also useful to compare the final encroachment stage directly with the initial foraging stage. This long-run comparison determines whether inequality is ultimately higher or lower after societies adopt agriculture, expand, and eventually compete over scarce land. The same logic applies: inequality is lower in the encroachment stage only if the returns to redistribution are higher than in the foraging stage. Formally:

$$\mathcal{I}^2 < \mathcal{I}^0 \iff \Omega_l^2 > \Omega_l^0 \iff \lambda x^*(N) > \frac{\zeta - \alpha}{1 - \alpha}.$$

For inequality to remain permanently lower when land becomes scarce, the returns to population size in military capabilities must be large enough to induce greater redistribution to workers. This condition is naturally independent of the returns to expansion ϕ: returns to expansion instead determine what happens between the initial foraging stage and the final encroachment stage. As such, long-run comparisons of inequality levels between foraging and mature agricultural societies might conceal subtler trajectories of development, as we show in the next section.

4.3 A taxonomy of inequality trajectories

The previous section outlined the conditions under which inequality increases or decreases as societies transition from one stage to the next, and showed that there is a wide range of possible scenarios for the long-run evolution of inequality in early agricultural societies. Did inequality increase when societies adopted agriculture? Did it decline instead? What happened when land eventually became scarce?

Interestingly, the dynamics of inequality need not be linear: inequality might have decreased in the expansion stage relative to the foraging stage, only to rise again with encroachment, or conversely, it might have increased with the adoption of agriculture but decreased when land became scarce. In the long run, the level of inequality in societies that encroach into a region can be lower or higher than the initial level before the adoption of agriculture. Our framework allows us to characterize the conditions under which each scenario prevails, and small differences in the values of a small number of technological and ecological parameters can yield heterogeneous inequality trajectories. More specifically, we outline the parameter values that lead to each of the six possible trajectories:

$$\mathcal{I}^2 > \mathcal{I}^0 > \mathcal{I}^1, \qquad \mathcal{I}^2 > \mathcal{I}^1 > \mathcal{I}^0, \qquad \mathcal{I}^1 > \mathcal{I}^2 > \mathcal{I}^0,$$

$$\mathcal{I}^0 > \mathcal{I}^2 > \mathcal{I}^1, \qquad \mathcal{I}^0 > \mathcal{I}^1 > \mathcal{I}^2, \qquad \mathcal{I}^1 > \mathcal{I}^0 > \mathcal{I}^2.$$

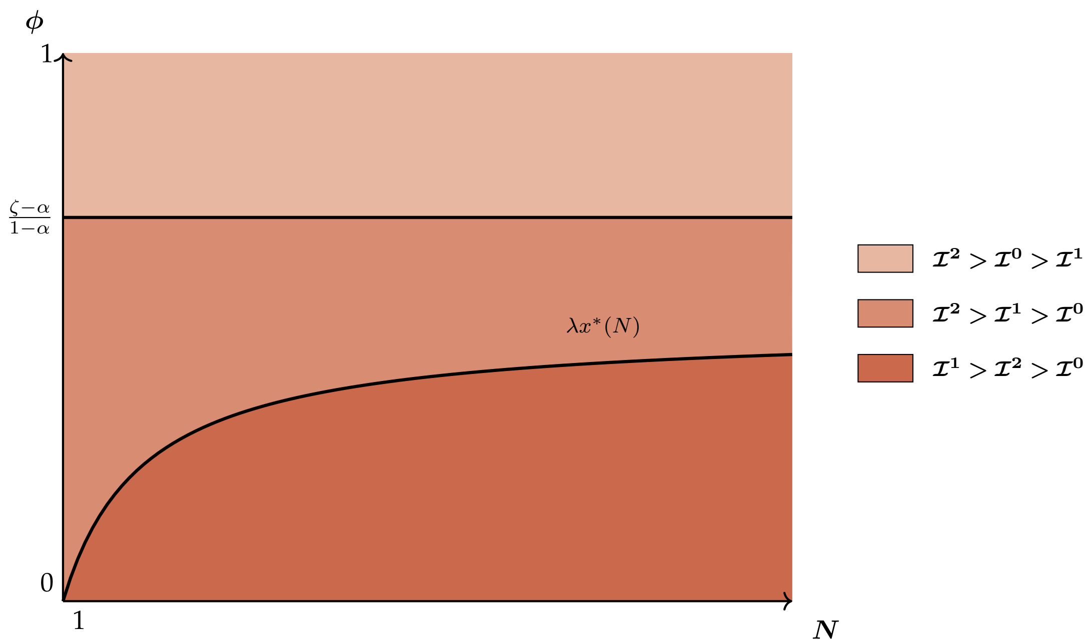

Figure 3: Inequality in the Neolithic when $\lambda < \dfrac{\zeta - \alpha}{1 - \alpha}$

When inequality is greater in the long-run. A commonly held view is that inequality was permanently higher in mature agricultural societies relative to their foraging predecessors. In our framework, this corresponds to the case in which inequality is greater in the final encroachment stage than in the initial foraging stage, i.e., $\mathcal{I}^2 > \mathcal{I}^0$. This is always the case when $\lambda < \dfrac{\zeta - \alpha}{1 - \alpha}$, such that the returns to population size in military capabilities are low, even as $x^*(N)$ tends to one with extreme political fragmentation. This situation is depicted in Figure 3. Although inequality is greater in the long run, the trajectory is not necessarily linear, and the three regions depict what occurs during the intermediate expansion stage.

When $\phi < \dfrac{\zeta - \alpha}{1 - \alpha}$, the transition from foraging to agriculture leads to an increase in inequality because the returns to expansion are not high enough for the elite to redistribute a large share of the surplus when land is free. Now, if the returns to expansion are nonetheless above the returns to military capabilities, inequality follows a linear upward path as encroachment further reduces the incentives of the elite to redistribute. However, if the returns to expansion are even lower than the returns to population size in military capabilities, as in the darker brown region, encroachment actually reduces inequality relative to the expansion stage, even if it remains greater than initially. This describes a case in which the higher level of inequality in the long run obscures an even more unequal period in which the elite had little incentive to improve workers' standards of living, because faster population growth yielded little benefit in terms of expansion.

The most interesting case, however, might be when $\phi > \frac{\zeta - \alpha}{1 - \alpha}$, as in the lighter brown region. In this scenario, inequality initially decreases with the transition from foraging to agriculture before increasing again with encroachment, to a greater extent than initially. This possibility suggests that comparing the levels of inequality in established agricultural societies with those of foraging societies to argue that agriculture caused a rise in inequality might be misleading, precisely because of the non-linear trajectory of inequality. Although agriculture does lead to greater inequality in the long run, it masks a more egalitarian intermediate period during which the elite redistributes a substantial share of the surplus to workers, fueling population growth and expanding into free territories. It is therefore not the adoption of agriculture that causes inequality to rise, but the scarcity of land when societies encroach and the returns to population size in military capabilities are not high enough for the elite to keep pursuing a relatively egalitarian policy of redistribution.

When inequality is lower in the long-run. As shown by the archaeological data in Section 2, it is not necessarily the case that inequality ends up greater in the encroachment stage than in the foraging stage. This occurs when the returns to population size in military capabilities λ are high enough such that, as the number N of societies in the region grows, they eventually surpass the returns to labor in foraging. This is the case when $\lambda > \dfrac{\zeta - \alpha}{1 - \alpha}$ that is depicted in Figure 4, which allows for $\mathcal{I}^2 < \mathcal{I}^0$.

It is straightforward to see that there exists a threshold number of societies $\tilde{N} = \dfrac{(1 - \alpha)\lambda}{\alpha - \zeta + (1 - \alpha)\lambda}$ above which inequality is actually lower in the long run, even when land is scarce. The degree of political fragmentation in a region, therefore, matters, as the intensity of competition for resources influences how the elite allocates the surplus among its various uses. When N is lower than the threshold, inequality is greater in the encroachment stage than it was before the adoption of agriculture. A smaller number

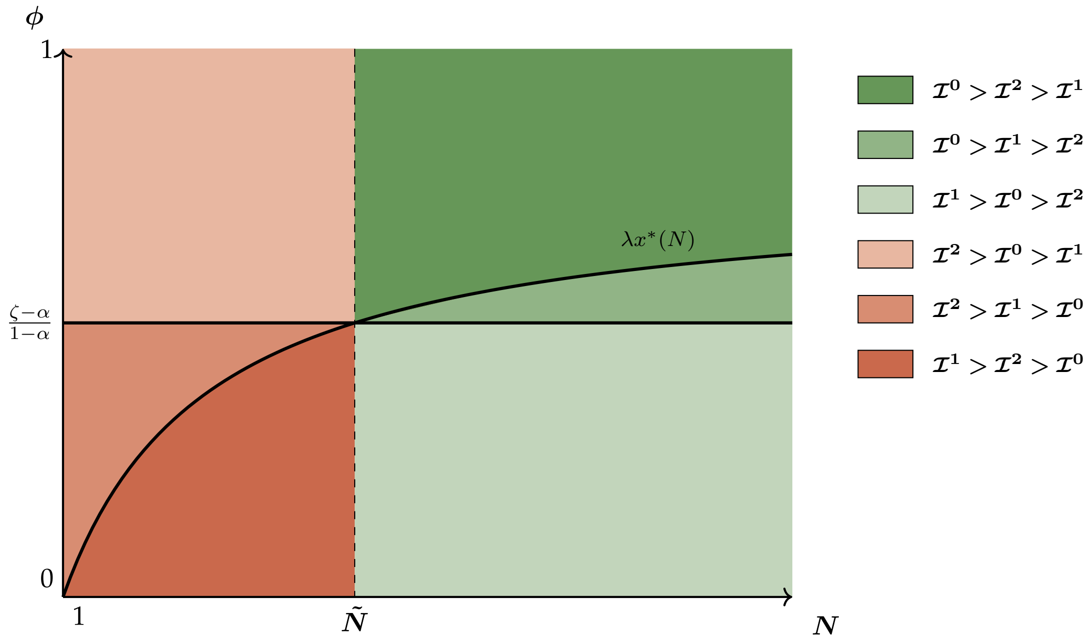

Figure 4: Inequality in the Neolithic when $\lambda > \dfrac{\zeta - \alpha}{1 - \alpha}$

of societies controlling a larger share of the total land available in the region have fewer incentives to redistribute in order to increase population size when they encroach, and we uncover the three cases described previously. In a fragmented region, however, each society controls a smaller share of the total land supply, implying that the returns to competition (through military investments or redistribution to increase the population) are substantial.

Again, if inequality is lower in the long run in a fragmented region, its trajectory is linear for intermediate values of ϕ, but this need not be the case. If the returns to population size in expansion are very low, as in the light green region, redistribution initially decreases with the adoption of agriculture, then increases as societies encroach. As such, the lower long-run level of inequality hides a much more unequal intermediate period of expansion. Conversely, if the returns to expansion are substantially high, as in the darker green region, inequality decreases with the adoption of agriculture but increases with encroachment, even though it remains lower than initially. Finally, for intermediate values of ϕ in a fragmented region, inequality follows a downward trajectory.

A wealth of heterogeneous inequality trajectories. All six scenarios are depicted in Figure 4, which shows that a subtle interplay between the returns to population size in expansion, the returns to population size in military capabilities, and the degree of competition for scarce resources shapes the trajectory of inequality. Our framework accounts for all possible trajectories of inequality over the long run, and we believe that, although there is convincing evidence that inequality was greater in some developed agricultural societies than in earlier hunting-gathering societies, the full picture may be more nuanced.

An interesting case is the one depicted in which inequality is indeed greater in the latter stage than initially, but potentially lower in between, as depicted in light brown in the top left quadrant of Figure 4, when $N < \tilde{N}$ and $\phi > \frac{\zeta - \alpha}{1 - \alpha}$. It is possible that the adoption of agriculture per se did not lead to an increase in inequality, but rather the scarcity of land as societies encroached. The adoption of agriculture in this case initially reduced inequality, as the elite were willing to redistribute more of the surplus to workers to fuel expansion through population growth. The small number of societies in the region allowed each to expand substantially, so that when they finally encroached, they nonetheless controlled a fairly large fraction of the land. With the emergence of competition, societies began investing in military capabilities, thereby diverting some resources from redistribution. Furthermore, due to diminishing returns on both land and the overall resources devoted to conflict, larger societies reduced redistribution even further as they found population growth less valuable. Finally, the larger share of land each society controlled increased the returns to investment in infrastructure. All those forces eventually drove inequality up, but land scarcity is the culprit, not the agricultural technology itself, and the late rise in inequality masks a period of egalitarian redistribution by the elite.

5 The evolution of inequality in the Neolithic

The main conclusion of our theoretical analysis is that inequality can follow very different trajectories across our three stages of development, depending on factors that govern workers' bargaining power. Figure 2 shows that inequality did indeed follow very different trajectories across different regions of the world. While West Asia and Europe experienced a sustained increase in inequality over the long run following the adoption of inequality that aligns with the standard narrative, the evolution of inequality was non-linear in other regions of the world. In South and Mesoamerica, inequality increased substantially with the adoption of agriculture, but remained higher than that of foraging societies when competing societies encroached. North America follows a similar but much flatter pattern. In East and South Asia, inequality also rose with agriculture, but remained substantially lower than in the initial encroachment stage. Looking at Oceania, one might think that inequality remained relatively constant, but the expansion stage was actually a period of dramatically lower inequality. Africa exhibits a sustained decline in inequality across stages of development, but given the limited number of observations, these results should be interpreted with caution.

Because our theoretical framework allows for such different trajectories of inequality in the early stages of development and highlights the role of a small number of exogenous parameters, we can leverage it to provide a tentative account of what happened in each region. Based on the results in

Figure 2, we can first locate in Figure 5 our six broad regions on the graph characterizing the paths of inequality according to the parameters of the model.[3]

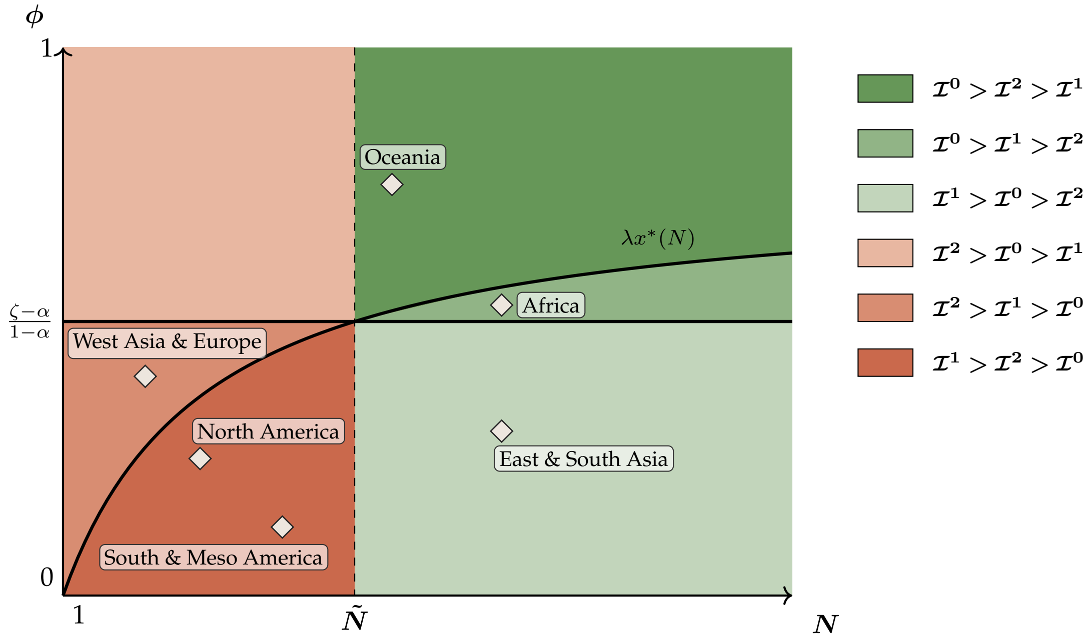

Figure 5: Inequality in the Neolithic

West Asia and Europe. In West Asia and Europe, inequality rose monotonically through the three stages of development, consistent with the canonical view that the adoption of agriculture fostered hierarchical social structures. In our framework, the productivity of labor in agriculture α was relatively low compared to foraging ζ, so that expansion capacity ϕ did not provide sufficient incentives for elites to redistribute in order to stimulate population growth. Archaeological evidence indeed suggests that early farming in Western Eurasia was increasingly land-limited as population grew, consistent with a relatively low marginal product of labor α (Bogaard et al., 2019; Bowles and Fochesato, 2024).

From the Late Neolithic onward, Western Asia and parts of Europe saw the emergence of large territorial states and empires (Uruk, Akkad, later Assyria, Mycenae, etc.), in which a small number of societies monopolized large agricultural basins. The encroachment stage was therefore characterized by limited political fragmentation N, with each society controlling a substantial agricultural area, which reduced the marginal returns to conflict. Elites therefore allocated a larger share of the surplus to infrastructure — potentially including military spending if μ was high — rather than to redistribution, and inequality increased further.

[3]Naturally, those six regions might differ in terms of parameters others than ϕ and N, the areas of the graph characterized by the various lines and curves might thus be different.

North America. North America exhibits a mild inverted-U pattern: inequality rose modestly with the adoption of agriculture and declined slightly when societies encroached, ending slightly above the initial level. This suggests that expansion ϕ was not particularly valuable, while returns to population in military capabilities were sufficiently high to generate some redistribution once a larger number of smaller societies N encroached. Archaeological studies of North American agricultural societies, particularly in the Southwest (Pueblo regions), show that maize agriculture expanded within ecological constraints that produced neither binding land scarcity nor simple expansion opportunities. Variability in maize yields and water access meant that returns to expansion were only limited (Kohler et al., 2014).

Ethnographic and archaeological evidence indicate that warfare in the Pueblo Southwest and other regions was episodic and related to resource scarcity, with evidence of increased competition as settlement densities rose, raising the returns to population in military capabilities (Allen et al., 2016). Yet, as social integration increased, violence declined, suggesting incentives for elites to support a modest level of redistribution to secure population and defensive capabilities (Kohler et al., 2014).

South and Mesoamerica. South and Mesoamerica display the most pronounced inverted-U trajectory. Inequality was initially low in the foraging stage, consistent with a relatively high productivity of foragers ζ compared to agricultural labor. The persistence of tropical foraging in the region suggests this might have been the case (Capriles et al., 2019).

After the adoption of agriculture, expansion incentives remained limited: a low ϕ is consistent with production being concentrated in specific areas, reflecting the geographic constraints of these regions. Mesoamerican and Andean agricultural landscapes are indeed among the world's most spatially constrained, and productive land was concentrated in valley bottoms, lake basins, volcanic highlands, or terraced mountain slopes. In the Basin of Mexico, agricultural intensification (e.g., chinampas) developed precisely because horizontal expansion was limited (Nichols, 2015). In the Maya lowlands, terracing systems expanded as populations grew, indicating investment in agricultural infrastructures rather than expansion. In the Andes, agricultural production depended on carefully engineered terraces and irrigation systems in steep valleys and high-altitude environments (Denevan, 2001), again pointing towards large returns to ambitious public works κ relative to expansion.

The encroachment stage was characterized by competition among multiple rival societies, thereby raising the incentives for elites to redistribute to increase military capabilities. Archaeological evidence on the prevalence of warfare in the region (Chase and Chase, 1998; Arkush and Tung, 2013) do seem to indicate relatively high returns to population in military capabilities λ, which might have contributed to the decline in inequality in this final stage. Yet, although inequality fell relative to the expansion stage, it remained above the initial foraging level.

East and South Asia. In East and South Asia, inequality increased with the adoption of agriculture but ultimately fell below its foraging level once societies encroached. Although rice-based systems of agriculture in the region are labor-intensive with a high productivity of labor α, increases in output mostly came from the intensive rather than the extensive margin, suggesting potentially low returns to expansion ϕ (Fuller, 2020; Fochesato et al., 2021). Elites were thus incentivized to divert surplus toward themselves and/or to agricultural infrastructures whenever κ was sufficiently high. The large α might nonetheless explain why the increase in inequality was less pronounced than in South and Mesoamerica.

In the encroachment stage, the substantial reduction in inequality indicates that returns to population in military capabilities λ were high relative to returns to military investments μ, and/or that political fragmentation N was large enough to increase the marginal value of land for many small societies. Historically, both East and South Asia have exhibited long periods of political fragmentation and competition for scarce resources: the Warring States period in China before unification, and the many early Indian Mahajanapadas are prime examples.

Oceania. In Oceania, inequality appears stable when comparing the first and last stages, but the expansion stage is markedly more egalitarian. This pattern suggests that returns to agricultural expansion, ϕ, exceeded returns to foraging, ζ, thereby giving elites strong incentives to redistribute to foster population growth. Evidence of substantial territorial expansion at the emergence of the first Hawaiian proto-states (Field et al., 2011) and of settlement in East Polynesia (Sear et al., 2020) points to substantial returns to expansion.

The subsequent increase in inequality once societies encroached implies that the returns to population in military capabilities, λ, were relatively limited. Polynesian societies famously developed elaborate chiefly hierarchies, but much of the violence and coercion appears to operate through status competition, ritual, and control of land and labor, rather than dense, manpower-maximizing state warfare of the Eurasian kind. Kirch (1989) studied Polynesian chiefdoms and emphasized hierarchical control over land and labor, but on islands with limited absolute population and often without large standing armies. Nevertheless, the presence of warfare due to local rivalries in multi-societies islands with a relatively large N might have kept inequality close to its foraging-stage level rather than substantially above it.

Africa. Africa exhibits a substantial decline in inequality across stages, though this pattern should be interpreted cautiously given the limited data. The decline suggests that workers' bargaining power was high once agriculture spread, probably due to a high expansion capacity ϕ, consistent with the argument of Herbst (2015) that, unlike Europe, Africa was never a continent on which land was scarce. In regions of the continent where societies nonetheless encroached, the historically high degree of political fragmentation in Africa was probably sufficient to raise the returns to military capabilities, generating additional redistributive incentives from elites to workers and further reducing inequality.

6 Conclusion

The growth-theoretic framework we propose integrates various key elements crucial to the dynamics of both standards of living and inequality during the Neolithic Revolution. Workers' fertility decisions induce elites to consent to redistribution to fuel population growth. Population growth imposes Malthusian pressure on per capita income, but also allows societies to expand freely when land is abundant in the region, or to compete for its control when it becomes scarce as societies encroach. Infrastructure investments are required for agriculture, but the elite face a trade-off between raising the productivity of the land they control through such investments and increasing the share of land available for production, whether through expansion or conflict. All these ingredients lead to a wide range of possible development scenarios, as they influence the elite's incentives to allocate the surplus across various uses, which in turn affect both standards of living and inequality.

Our main message is that the evolution of both standards of living and inequality in the Neolithic is likely to have been non-monotonic. As such, when comparing levels of inequality in agricultural societies centuries after the adoption of agriculture with those in earlier hunting-gathering societies, one must be cautious before claiming that agriculture raised inequality. In our framework, it is possible that, rather than agriculture itself, the scarcity and competition for land as societies encroach may have caused inequality to increase. Such crude comparisons across two distant points in time might obscure a more egalitarian age in which the elite redistributed to workers to fuel expansion through population growth. The same caution should apply when comparing standards of living: while it is possible that agriculturalists eventually became worse off than their hunter-gatherer ancestors, this might not be due to the new mode of production, agriculture, but rather to the scarcity of land once enough agricultural societies had settled in a region.

References

Allen, M. W., R. L. Bettinger, B. F. Codding, T. L. Jones, and A. W. Schwitalla (2016). Resource scarcity drives lethal aggression among prehistoric hunter-gatherers in central california. *Proceedings of the National Academy of Sciences 113*(43), 12120–12125.

Allen, R. C. (2024). The neolithic revolution in the middle east. *The Economic History Review 77*(4), 1154–1196.

Allen, R. C., M. C. Bertazzini, and L. Heldring (2023). The economic origins of government. *American Economic Review 113*(10), 2507–2545.

Angelopoulos, K., S. Lazarakis, R. Mancy, D. Agol, and E. Papyrakis (2023). Resource risk and the origins of inequality: Evidence from a pastoralist economy.

Arkush, E. and T. A. Tung (2013). Patterns of war in the andes from the archaic to the late horizon: insights from settlement patterns and cranial trauma. *Journal of Archaeological Research 21*(4), 307–369.

Bar-Yosef, O. and R. H. Meadow (1995). The origins of agriculture in the near east. *Last hunters, first farmers*.

Ben-Dor, M. and R. Barkai (2021). Prey size decline as a unifying ecological selecting agent in pleistocene human evolution. *Quaternary 4*(1), 7.

Bird, D., L. Miranda, M. Vander Linden, E. Robinson, R. K. Bocinsky, C. Nicholson, J. M. Capriles, J. B. Finley, E. M. Gayo, A. Gil, et al. (2022). p3k14c, a synthetic global database of archaeological radiocarbon dates. *Scientific Data 9*(1), 27.

Bogaard, A., P. Cruz, M. Fochesato, J. Birch, G. Cervantes Quequezana, S. Chirikure, E. R. Crema, G. M. Feinman, A. S. Green, H. Hamerow, et al. (2025). Labor, land, and the global dynamics of economic inequality. *Proceedings of the National Academy of Sciences 122*(16), e2400694122.

Bogaard, A., M. Fochesato, and S. Bowles (2019). The farming-inequality nexus: new insights from ancient western eurasia. *Antiquity 93*(371), 1129–1143.

Bowles, S. and J.-K. Choi (2019). The neolithic agricultural revolution and the origins of private property. *Journal of Political Economy 127*(5), 2186–2228.

Bowles, S. and M. Fochesato (2024). The origins of enduring economic inequality. *Journal of Economic Literature 62*(4), 1475–1537.

Capriles, J. M., U. Lombardo, B. Maley, C. Zuna, H. Veit, and D. J. Kennett (2019). Persistent early to middle holocene tropical foraging in southwestern amazonia. *Science advances 5*(4), eaav5449.

Chase, A. F. and D. Z. Chase (1998). Late classic maya political structure, polity size and warfare arenas. In *Anatomía de una civilización: aproximaciones interdisciplinarias a la Cultura Maya*, pp. 11–30. Sociedad Española de Estudios Mayas.

Chu, A. C., P. F. Peretto, and Y. Furukawa (2024). Political fragmentation versus a unified empire in a malthusian economy. *Journal of Economic Behavior & Organization 222*, 284–293.

Dal Bó, E. and P. Dal Bó (2011). Workers, warriors, and criminals: social conflict in general equilibrium. *Journal of the European Economic Association 9*(4), 646–677.

Dal Bó, E., P. Hernández-Lagos, and S. Mazzuca (2022). The paradox of civilization: Preinstitutional sources of security and prosperity. *American Political Science Review 116*(1), 213–230.

Denevan, W. M. (2001). *Cultivated landscapes of native Amazonia and the Andes*. Oxford University Press, USA.

Dow, G. K., L. Mitchell, and C. G. Reed (2017). The economics of early warfare over land. *Journal of Development Economics 127*, 297–305.

Dow, G. K. and C. G. Reed (2013). The origins of inequality: Insiders, outsiders, elites, and commoners. *Journal of Political Economy 121*(3), 609–641.

Dow, G. K. and C. G. Reed (2023). *Economic prehistory: six transitions that shaped the world*. Cambridge University Press.

Ferguson, R. B. (2013). The prehistory of war and peace in europe and the near east. *War, peace, and human nature: the convergence of evolutionary and cultural views*, 191–240.

Fibiger, L., T. Ahlström, C. Meyer, and M. Smith (2023). Conflict, violence, and warfare among early farmers in northwestern europe. *Proceedings of the National Academy of Sciences 120*(4), e2209481119.

Field, J. S., T. N. Ladefoged, and P. V. Kirch (2011). Household expansion linked to agricultural intensification during emergence of hawaiian archaic states. *Proceedings of the National Academy of Sciences 108*(18), 7327–7332.

Fochesato, M., C. Higham, A. Bogaard, and C. C. Castillo (2021). Changing social inequality from first farmers to early states in southeast asia. *Proceedings of the National Academy of Sciences 118*(47), e2113598118.

Fuller, D. Q. (2020). Transitions in productivity: Rice intensification from domestication to urbanisation. *Archaeology International 23*(1), 88–103.

Fuller, D. Q., G. Willcox, and R. G. Allaby (2012). Early agricultural pathways: moving outside the 'core area'hypothesis in southwest asia. *Journal of experimental botany 63*(2), 617–633.

Garrod, D. A. (1957). The natufian culture: the life and economy of a mesolithic people in the near east. *Albert Reckitt archaeological lecture*.

Guzmán, R. A. and J. Weisdorf (2011). The neolithic revolution from a price-theoretic perspective. *Journal of Development Economics 96*(2), 209–219.

Herbst, J. (2015). *States and power in Africa: Comparative lessons in authority and control: Comparative lessons in authority and control*. Princeton University Press.

Keeley, L. H. (1997). *War before civilization*. Oxford University Press.

Kelly, R. L. (2013). *The lifeways of hunter-gatherers: the foraging spectrum*. Cambridge University Press.

Kennett, D. J., M. Masson, C. P. Lope, S. Serafin, R. J. George, T. C. Spencer, J. A. Hoggarth, B. J. Culleton, T. K. Harper, K. M. Prufer, et al. (2022). Drought-induced civil conflict among the ancient maya. *Nature communications 13*(1), 3911.

Kirch, P. V. (1989). *The evolution of the Polynesian chiefdoms*. Cambridge University Press.

Kohler, T. A., A. Bogaard, S. G. Ortman, E. R. Crema, S. Chirikure, P. Cruz, A. Green, T. Kerig, M. D. McCoy, J. Munson, et al. (2025). Economic inequality is fueled by population scale, land-limited production, and settlement hierarchies across the archaeological record. *Proceedings of the National Academy of Sciences 122*(16), e2400691122.

Kohler, T. A., S. G. Ortman, K. E. Grundtisch, C. M. Fitzpatrick, and S. M. Cole (2014). The better angels of their nature: Declining violence through time among prehispanic farmers of the pueblo southwest. *American Antiquity 79*(3), 444–464.

Le Fur, T. and E. Wasmer (2025). Fighting for resources: a unified growth model of the great divergence. *Journal of Economic Growth*, 1–56.

Maeda, O., L. Lucas, F. Silva, K.-I. Tanno, and D. Q. Fuller (2016). Narrowing the harvest: Increasing sickle investment and the rise of domesticated cereal agriculture in the fertile crescent. *Quaternary Science Reviews 145*, 226–237.

Mason, S. L. R. (1992). *Acorns in human subsistence*. University of London, University College London (United Kingdom).

Mayshar, J., O. Moav, and L. Pascali (2022). The origin of the state: Land productivity or appropriability? *Journal of Political Economy 130*(4), 1091–1144.

McCool, W. C., B. F. Codding, K. B. Vernon, K. M. Wilson, P. M. Yaworsky, N. Marwan, and D. J. Kennett (2022). Climate change–induced population pressure drives high rates of lethal violence in the prehispanic central andes. *Proceedings of the National Academy of Sciences 119*(17), e2117556119.

Nichols, D. L. (2015). Intensive agriculture and early complex societies of the basin of mexico: The formative period. *Ancient Mesoamerica 26*(2), 407–421.

North, D. C. and R. P. Thomas (1977). The first economic revolution. *The Economic History Review 30*(2), 229–241.

Ortman, S. G., A. Bogaard, J. Munson, D. Lawrence, A. S. Green, G. M. Feinman, S. Chirikure, J. H. Uhl, and S. Leyk (2025). Changes in agglomeration and productivity are poor predictors of inequality across the archaeological record. *Proceedings of the National Academy of Sciences 122*(16), e2400693122.

Rosenberg, D. (2008). The possible use of acorns in past economies of the southern levant: a staple food or a negligible food source? *Levant 40*(2), 167–175.

Sear, D. A., M. S. Allen, J. D. Hassall, A. E. Maloney, P. G. Langdon, A. E. Morrison, A. C. Henderson, H. Mackay, I. W. Croudace, C. Clarke, et al. (2020). Human settlement of east polynesia earlier, incremental, and coincident with prolonged south pacific drought. *Proceedings of the National Academy of Sciences 117*(16), 8813–8819.

Skourtanioti, E., Y. S. Erdal, M. Frangipane, F. B. Restelli, K. A. Yener, F. Pinnock, P. Matthiae, R. Özbal, U.-D. Schoop, F. Guliyev, et al. (2020). Genomic history of neolithic to bronze age anatolia, northern levant, and southern caucasus. *Cell 181*(5), 1158–1175.

Weisdorf, J. L. (2005). From foraging to farming: explaining the neolithic revolution. *Journal of Economic surveys 19*(4), 561–586.

Wilkinson, T. J. et al. (2024). Settlement development in the north jazira, iraq: a study of the archaeological landscape.

A Appendix

A.1 The elite's optimization program

The Lagrangian associated to the elite's optimization problem is:

$$\max_{C_t, w_t, G_{t+1}, M_{t+1}} \mathscr{L}_t = \log C_t + \beta \log Y_{t+1} - \lambda_t [Y_t - C_t L_t - w_t l_t - G_{t+1} - M_{t+1}] \tag{16}$$

The first order conditions of the elite's optimization problem are:

$$\frac{\partial \mathscr{L}_t}{\partial C_t} = 0 \quad \Leftrightarrow \quad \frac{1}{C_t} = \lambda_t L_t \tag{17}$$

$$\frac{\partial \mathscr{L}_t}{\partial w_t} = 0 \quad \Leftrightarrow \quad \beta \frac{n'(w_t)}{n(w_t)} \Omega_l^k = \lambda_t l_t \tag{18}$$

$$\frac{\partial \mathscr{L}_t}{\partial G_{t+1}} = 0 \quad \Leftrightarrow \quad \beta \frac{\Omega_G^k}{G_{t+1}} = \lambda_t \tag{19}$$

$$\frac{\partial \mathscr{L}_t}{\partial M_{t+1}} = 0 \quad \Leftrightarrow \quad \beta \frac{\Omega_M^k}{M_{t+1}} = \lambda_t \tag{20}$$

Solving this system of equations gives the optimal allocation of aggregate output by the elite.

A.2 Proof of proposition 2

The proof of proposition 1 is divided into distinct parts: the proof of existence of a steady state level of income per capita in each of the three stages, then the derivation of the conditions that characterize the transition between the stages.

The foraging stage. While the shares devoted to both elite consumption and redistribution to workers are constant, the the foraging stage is characterized by population dynamics. Using the fact that $\mathcal{L}_{t+1} = n_t(1-\theta)\mathcal{L}_t$ and the optimal rate of fertility of workers, we find the law of motion of population:

$$\mathcal{L}_{t+1} = \frac{\gamma}{\tau} \frac{\beta\zeta}{1+\beta\zeta} [(1-\theta)\mathcal{L}_t]^{\zeta} \tag{21}$$

It is straightforward to see that, with decreasing returns to labor in the foraging technology $\zeta < 1$, population converges to a constant level and so does output per capita:

$$\mathcal{L}^0 = \left\{ \frac{\gamma}{\tau} \frac{\beta\Omega_l^0}{1+\beta\Omega^0} (1-\theta)^{\zeta} \right\}^{\frac{1}{1-\zeta}} \tag{22}$$

$$y^0 = \frac{\tau}{\gamma} \frac{1+\beta\Omega^0}{\beta\Omega_l^0} \tag{23}$$

The expansion stage. Income per capita in $t+1$ in the expansion stage can be written as:

$$y_{t+1} = \frac{Y_{t+1}}{\mathcal{L}_{t+1}} = (1-\theta) \left\{ \frac{A_{t+1} G_{t+1}^{\kappa} X}{l_{t+1}^{1-\phi}} \right\}^{1-\alpha} \tag{24}$$

Therefore:

$$\frac{y_{t+1}}{y_t} = \left\{ \frac{A_{t+1}}{A_{t+1}} \left(\frac{G_{t+1}}{G_t} \right)^{\kappa} \left(\frac{l_t}{l_{t+1}} \right)^{1-\phi} \right\}^{1-\alpha} \tag{25}$$

Using the fact that $A_{t+1} = (1+g)A_t$ and $l_{t+1} = (1-\theta)n_t l_t$, and also expressing workers' fertility as a function of income per capita, we obtain:

$$y_{t+1} = \left\{ (1+g) \left(\frac{G_{t+1}}{G_t} \right)^{\kappa} \left(\frac{\tau}{\gamma} \frac{1+\beta\Omega^1}{\beta\Omega_l^1} \right)^{1-\phi} \right\}^{1-\alpha} y_t^{\phi+(1-\phi)\alpha} \tag{26}$$

Using the shares of aggregate output devoted to infrastructure investments, we also know that:

$$\frac{G_{t+1}}{G_t} = \frac{Y_t}{Y_{t-1}} = 1 + g_{Yt} \tag{27}$$

such that infrastructure investments grow with aggregate output. For a steady level of income per capita to exists, the rate of growth of aggregate output must converge to a constant. Again, using the solution to the elite's optimization program, we can write:

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{A_{t+1}}{A_t} \right)^{1-\alpha} \left(\frac{Y_t}{Y_{t-1}} \right)^{\Omega^1} \tag{28}$$

$$\Leftrightarrow 1 + g_{Yt+1} = (1+g)^{1-\alpha} (1 + g_{Yt})^{\Omega^1} \tag{29}$$

It is straightforward to see that the rate of growth of aggregate output converges to a constant if and only if returns to scale are decreasing $\Omega^1 = \alpha + (1-\alpha)(\phi+\kappa) < 1$, which is the case if and only if $\phi + \kappa < 1$. When that is the case, the stationary rate of growth of aggregate output is $1 + g_Y = (1+g)^{\frac{1}{1-(\phi+\kappa)}}$. Plugging that back into equation 26, we can easily solve for the steady state level of income per capita.

$$y^1 = \frac{\tau}{\gamma} \frac{1+\beta\Omega^1}{\beta\Omega_l^1} (1+g)^{\frac{1}{1-(\phi+\kappa)}} \tag{30}$$

The encroachment stage. We follow the same strategy for the encroachment stage when the share of land controlled by each identical society is constant and equal to $1/N$. As such, income per capita can be written as:

$$y_{t+1} = (1-\theta) \left\{ \frac{A_{t+1} G_{t+1}^{\kappa}}{l_{t+1}} \frac{X}{N} \right\}^{1-\alpha} \tag{31}$$

Again, using the fact that $A_{t+1} = (1+g)A_t$ and $l_{t+1} = (1-\theta)n_t l_t$, expressing workers' fertility as a function of income per capita, and recognizing that infrastructure investments grow with aggregate output, we can write:

$$y_{t+1} = \left\{ (1+g)(1+g_{Yt})^{\kappa} \left(\frac{\tau}{\gamma} \frac{1+\beta\Omega^2}{\beta\Omega_l^2} \right) \right\}^{1-\alpha} y_t^{\alpha} \tag{32}$$

Looking at the long-run growth rate of aggregate output in the encroachment stage gives:

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{A_{t+1}}{A_t}\right)^{1-\alpha}\left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha+(1-\alpha)\kappa} \tag{33}$$

$$\Leftrightarrow 1+g_{Y\,t+1} = (1+g)^{1-\alpha}(1+g_{Y\,t})^{\alpha+(1-\alpha)\kappa} \tag{34}$$

Naturally, as long as there are decreasing returns in infrastructure investments ($\kappa < 1$) the growth rate of aggregate output converges to a constant $1 + g_Y = (1+g)^{\frac{1}{1-\kappa}}$. Plugging this back into equation 32 gives:

$$y^2 = \frac{\tau}{\gamma}\frac{1+\beta\Omega^2}{\beta\Omega^2_l}(1+g)^{\frac{1}{1-\kappa}} \tag{35}$$

The transition from foraging to expansion. The elite finds it beneficial to allocate some resources to the infrastructure necessary for agriculture if and only if it raises their utility relative to foraging; that is, when $U^1 > U^0$. In order to study the transition from foraging to agriculture undertaken by the elite, we can simply plug optimal solutions in both cases into the utility function of the elite and compare. After some substitutions, the elite's utility can be show to be greater with agriculture when the following condition holds:

$$\frac{Y^A_{t+1}}{Y^*} > \left(\frac{S^0}{S^1}\right)^{\frac{1}{\beta}} \equiv \left(\frac{1+\beta\Omega^1}{1+\beta\Omega^0}\right)^{\frac{1}{\beta}} \tag{36}$$

Intuitively, the elite decides to adopt agriculture once the level of aggregate output that can be achieved using the agricultural technology is sufficiently higher than the steady state level under foraging. After some algebra, we can show that this corresponds to:

$$A_t > \frac{1}{1+g}\left(\frac{(S^0/S^1)^{\frac{1}{\beta}}}{\left[(1-\theta)\frac{\gamma}{\tau}s^1\right]^{\Omega^1_l}\mathcal{S}_{G^1}^{\Omega^1_G}}Y^{0^{1-\Omega^1}}\right)^{\frac{1}{1-\alpha}} \equiv \mathcal{A} \tag{37}$$

The transition from expansion to encroachment. societies encroach when there are no more free territories in the region, that is when the sum of the share of land they each control is greater than one, or:

$$l_{t+1} > \left(\frac{X}{N}\right)^{\frac{1}{\phi}} \tag{38}$$

Using the fact that $l_{t+1} = (1-\theta)n_t l_t$, the optimal fertility rate of workers, and the shares of aggregate output redistributed by the elite, we get the following condition:

$$Y_t > \frac{\tau}{\gamma(1-\theta)}\frac{1+\beta\Omega^1}{\beta\Omega^1_l}\left(\frac{X}{N}\right)^{\frac{1}{\phi}} \tag{39}$$

which concludes the proof. To summarize:

- In the foraging stage, there exists a unique steady state level of income per capita: $y^0 = \frac{\tau}{\gamma}\frac{1+\beta\Omega^0}{\beta\Omega_l^0}$.

$\hookrightarrow$ The transition from the foraging stage to the expansion stage occurs when the elite allocates resources to infrastructure investments and adopts agriculture, as $A_t > \mathcal{A}$ where $\mathcal{A}$ is a constant.

- In the expansion stage, there exists a unique steady state level of income per capita if and only if $\phi + \kappa < 1$: $y^1 = \frac{\tau}{\gamma}\frac{1+\beta\Omega^1}{\beta\Omega_l^1}(1+g)^{\frac{1}{1-(\phi+\kappa)}}$.

$\hookrightarrow$ The transition from the expansion stage to the encroachment stage occurs when there are no more free territories in the region, as $Y_t > \frac{\tau}{\gamma(1-\theta)}\frac{1+\beta\Omega^1}{\beta\Omega_l^1}\left(\frac{1}{N}\right)^{\frac{1}{\phi}}$.

- In the encroachment stage, there exists a unique steady state level of income per capita if and only if $\kappa < 1$: $y^2 = \frac{\tau}{\gamma}\frac{1+\beta\Omega^2}{\beta\Omega_l^2}(1+g)^{\frac{1}{1-\kappa}}$.

QED.

A.3 Additional Theoretical Results

The transition from foraging to expansion

Proposition 3 *The transition from foraging to expansion modifies the allocation of the surplus as follows:*

- *The share of aggregate output appropriated by the elite decreases with the adoption of agriculture ($S^1 < S^0$) if and only if $\Omega_l^1 + \Omega_G^1 > \Omega_l^0$.*

- *The share of aggregate output redistributed to workers increases with the adoption of agriculture ($s^1 > s^0$) if and only if $\frac{\Omega_l^1}{1+\beta\Omega_G^1} > \Omega_l^0$.*

- *The share of aggregated output devoted to infrastructure investments turns positive ($\mathcal{S}_G^1 > S_G^0 = 0$).*

Figure 6 depicts the four potential scenarios following the adoption of agriculture, depending on the relative returns to expansion through population growth ϕ and infrastructure investments κ. The green areas feature a reduction in inequality when ϕ is sufficiently high. In the upper left green region, inequality declines as the elite both reduce the share they appropriates and increases redistribution. When the returns to infrastructures investments κ are high relative to ϕ in the lighter green region, inequality might also be lower despite less redistribution because the elite reduces its own share more than it reduces redistribution. In the brown regions, inequality increases with the adoption of agriculture because the returns to expansion through population growth are not high enough to induce redistribution. In the lighter brown region, the elite reduces the share of the surplus redistributed to

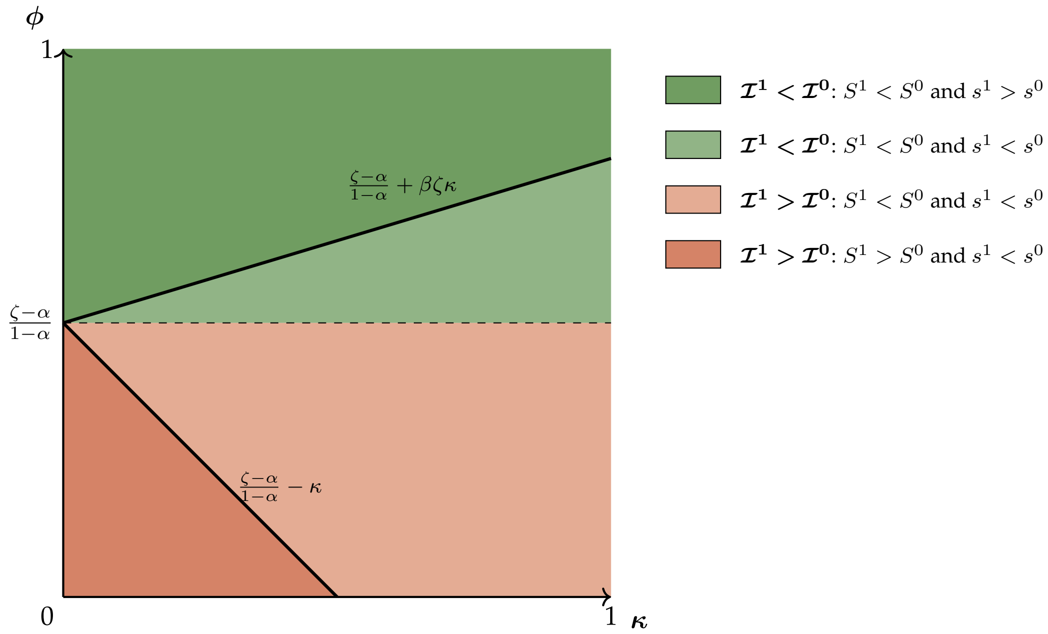

Figure 6: The evolution of inequality during the adoption of agriculture

workers more than the share they appropriates for their consumption. In the darker brown region, both ϕ and κ are so low such that the elite increases their own consumption.

The transition from expansion to encroachment

Proposition 4 *The transition from expansion to encroachment modifies the allocation of the surplus as follows:*

- *The share of aggregate output appropriated by the elite decreases when societies encroach ($S^2 < S^1$) if and only if $\Omega_l^2 + \Omega_M^2 > \Omega_l^1$.*
- *The share of aggregate output redistributed to workers increases when societies encroach ($s^2 > s^1$) if and only if $\Omega_l^2 \dfrac{1+\beta\Omega_G^1}{1+\beta(\Omega_G^2+\Omega_M^2)} > \Omega_l^1$.*
- *The share of aggregate output devoted to infrastructure investments increases when societies encroach ($\mathcal{S}_G^2 > \mathcal{S}_G^1$) if and only if $\Omega_l^2 + \Omega_M^2 < \Omega_l^1$.*
- *The share of aggregate output devoted to military investments turns positive ($\mathcal{S}_M^2 > \mathcal{S}_M^1 = 0$).*

Figure 7 depicts the potential change in redistribution and inequality following the transition from the expansion to the encroachment stage. From Proposition 4, we can derive a threshold $\tilde{\phi}(\kappa) = \dfrac{([1+\beta(1-\alpha)\kappa]\lambda - \beta\alpha\mu)\, x^*(N)}{1+\beta(1-\alpha)[\kappa+\mu x^*(N)]}$ such that redistribution increases when $\phi < \tilde{\phi}(\kappa)$. This threshold can be shown to be an increasing and concave function of κ: because of the trade-off between investment in the infrastructure and expansion, a greater κ implies a lower redistribution in in the expansion stage, thus raising the probability that it might increase with encroachment.

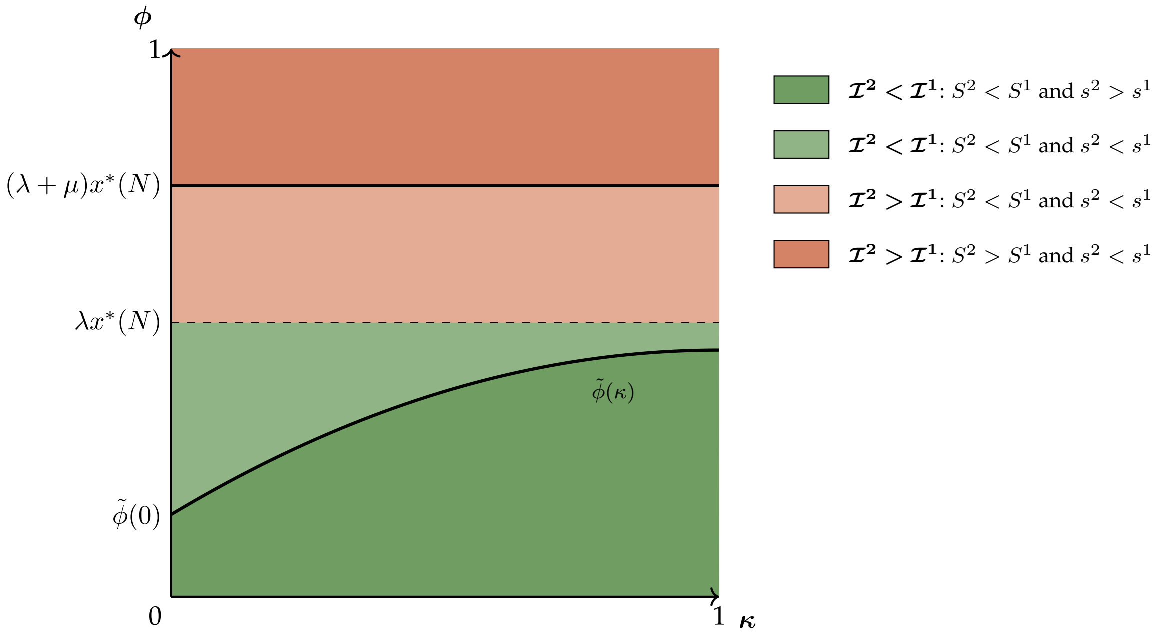

Figure 7: The evolution of inequality when land becomes scarce

The effects of political fragmentation

Proposition 5 *Political fragmentation modifies the allocation of the surplus in the encroachment stage as follows:*

- $\frac{\partial S^2}{\partial N} < 0$: *political fragmentation reduces the share of aggregate output appropriated by the elite*
- $\frac{\partial s^2}{\partial N} > 0$ *if and only if* $\frac{\lambda}{\mu} > \frac{\beta\alpha}{1+\beta(1-\alpha)\kappa}$: *political fragmentation increases the share of aggregate output redistributed to workers when the relative returns to population size in military capabilities are high*
- $\frac{\partial S^2_M}{\partial N} > 0$: *political fragmentation increases the share of aggregate output devoted to military investments*
- $\frac{\partial S^2_G}{\partial N} < 0$: *political fragmentation increases the share of aggregate output devoted to infrastructure investments*