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Do’s and dont’s

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Identification of spatial spillovers: Do's and dont's

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Abstract

The notion of spatial spillovers has been widely used in applied spatial econometrics. In this paper, we consider how they can be identified in both structural and causal reduced-form models. First, discussing the various threats to identification in structural models, we point out that the typical estimation framework proposed in the applied spatial econometric literature boils down to considering spatial spillovers as a side-effect of a data-driven chosen specification. We also discuss the limits of blindly relying on interaction matrices purely based on geography to identify the source and content of spillovers. Then, we present reduced forms impact evaluation models for spatial data and show that the current spatial versions of usual impact evaluation models are not fully satisfactory when considering the identification issue. Finally, a set of recommendations for applied articles aimed at identifying spatial spillovers is proposed.

JEL Classification: *C18, C21*
Keywords: spatial spillovers; spatial interactions; causal inference; interference; structural and reduced form identification.

1 Introduction

Spatial spillovers, which we define here as referring to both substantive geographical interactions and/or spatial externalities between neighboring observations, are at the heart of applied spatial econometrics, as typically used in regional/political science, among others. Building upon Tobler's first law in geography, countless empirical papers have estimated spatial spillover effects by specifying a geographically-based connectivity matrix[1] and using one of the classical spatial econometric specifications, such as the spatial Durbin model, the spatial lag model, etc. In essence, a typical paper in applied spatial econometrics aims to analyze the "impact" (quotation marks for purpose) of some explanatory variable(s) on an explained variable based on a model estimated on georeferenced data and *accounting* for cross-sectional dependence in various forms.[2] Following a long-standing tradition in spatial econometrics, the analysis is often based on specific-to-general or general-to-specific specification search strategies, and is most of the time followed by a discussion of interpretations in terms of *implied* spatial spillovers of this data-driven specification search.[3]

However, this practice has been largely criticized for various reasons. The most heavy critique probably arose from Gibbons & Overman (2012) who pointed out major identification problems, when spatial econometric specifications are used for an explanatory purpose, with the more or less explicit aim of being able to draw causal interpretations of the estimated parameters. Their conclusion best illustrates their arguments: "Identification problems bedevil applied spatial economic research. Spatial econometrics usually solves these problems by deriving estimators assuming that functional forms are known and by using model comparison techniques to let the data choose between competing specifications. We argue that in many situations of interest this achieves, at best, only a very weak identification. Worse, in many cases, such an approach will be uninformative about the causal economic processes at work, rendering much applied spatial econometric research "pointless," unless the main aim is a description of the data." Partridge et al. (2012) also point out that standard spatial econometrics is not capable of "differentiating when outcomes in nearby areas are spatially correlated [...] versus spatial causality" while McMillen (2012) argues that standard spatial econometric models are falsely used as a quick fix for any model misspecification. As mentioned by Mur (2013), this lack of attention of spatial econometrics to identification and causality is probably related to its history, as its evolution initially mimicked that of the time series literature.

As a result, we can only observe that while the standard spatial econometric toolbox is widely used in empirical articles in regional science and related disciplines, the papers in top field or top 5 economic journals that make use of "spatial econometric models" are now almost exclusively anchored in the literature on network econometrics. In the latter, the connections between individuals are modeled along a social network and the main focus of interest is to derive identification strategies to disentangle the endogenous effects from the exogenous and correlated effects, following the terminology of Manski (1993), with the

[1]This matrix is also called a spatial weights matrix or interaction matrix. In the literature on social networks, it is referred to as the adjacency matrix. However, we prefer the term connectivity or interaction to spatial weights for reasons that will become apparent later.

[2]Recent reviews in spatial econometrics can be found in Kelejian & Piras (2017), Elhorst et al. (2021), Arbia (2024), Otto et al. (2024), and Yang et al. (2025).

[3]Figure 1 in Elhorst (2010) illustrates these nested relationships between spatial models in the cross-sectional case.

final aim of deriving a causal interpretation of these effects. The empirical literature dealing with interactions in local policy choices is representative of this trend. Standard spatial lag or spatial Durbin models were widely used in the early 2000s to assess the existence and extent of fiscal competition among neighboring local jurisdictions. These models were labeled first-generation models by Agrawal et al. (2022). Yet, they were widely criticized on various grounds related to identification: the exclusion restrictions are never discussed, the instruments are weak, and these models are unable to discriminate between underlying mechanisms (fiscal competition, yardstick competition, Tiebout sorting, etc.). All these pitfalls led the recent literature on fiscal federalism to heavily rely on quasi-experiments to obtain more credible instruments (see Lyytikäinen 2012, Parchet 2019).

In the twentieth century, identification was mainly discussed within the framework of simultaneous equation models (demand and supply), in the spirit of the Cowles Foundation. This approach, called structural econometrics, where explicit economic theories are combined with statistical models, has been at the center of the economics literature for a long period and is based on explicitly stating modeling and behavioral assumptions.[4] However, following the credibility revolution of Angrist & Pischke (2010), much effort has been made to design identification strategies aimed to be as close as possible to a random experiment: the experimentalist paradigm. This second approach, also called reduced form, treatment effect, impact evaluation of public policy, or causal inference, aims at abstracting from behavioral assumptions typically made in the structural approach to assess the impact of a policy on an outcome, but at the cost of eluding the question of transmission channels. Although in practice, the separation between structural and reduced forms might not be so clear-cut, we use this distinction as the starting point for clarity purpose.

Our first aim in this paper is to reconsider the notion of spatial spillovers in light of this fundamental econometric question of identification. Although the econometric literature has developed many different concepts of identification, our paper focuses only on its most widely used notion, namely point identification. According to Lewbel (2019, p.842) the parameter θ is point identified (often just called identified) if there exist no pairs of possible values θ and $\widetilde{\theta}$ that are different but observationally equivalent. In this paper, we discuss the (point) identification of spatial spillovers in both structural and causal reduced-form models. In the first case, i.e. structural models, spatial spillovers should be the main parameters of interest, as endogenous effects are the main parameters of interest in network econometrics. Yet, we point out that the typical statistical strategies (specific-to-general or general-to-specific) proposed in the spatial econometrics literature boil down to considering spatial spillovers as a side-effect of the data-driven chosen specification, which is not an appropriate identification strategy. In addition, we discuss other threats to identification traditionally considered in the network econometrics literature, such as unobserved heterogeneity and endogenous sorting, typically not considered. In particular, in line with Neumayer & Plumper (2016), we argue that the use of interaction matrices based solely on geographical criteria, as a proxy of numerous transmission mechanisms, prevents the identification of associated spillovers.

In causal reduced models, interest typically relies on assessing the causal impact of a treatment on an outcome. Among other assumptions, most papers since Rosenbaum & Rubin (1983) impose a noninterference cross unit assumption: the stable unit value assumption (SUTVA). However, for the past 20 years, there has been an explosion of work relaxing this

[4]See Reiss & Wolak (2007) for a good overview of the logic of structural econometric models.

assumption.[5] Initially, the articles aimed to assess the bias of the causal effects estimated under SUTVA and to propose designs of experiments that avoid interference. Then, the focus has progressively switched to a substantive identification of these spillovers, both in terms of estimands of interests and design of experiments, as they are a crucial component in understanding the full impact of an intervention. In particular, if beneficial spillovers exist, a lower percentage of the population might need treatment. Furthermore, information on the nature and extent of spillover effects may allow targeting specific individuals or groups of individuals to increase the overall impact of the intervention. The notion of spillovers and related concepts has consequently become central in program evaluation (see Angelucci & Di Maro 2016). Most discussions in this literature focus on spillovers along networks, yet a limited statistical literature exists with respect to spatial spillovers. We discuss this emerging literature and also show that the current spatial versions of public policy evaluation models, such as difference-in-differences models including a spatial lag of the endogenous variable, are not satisfactory when considered from the identification perspective.

These attempts illustrate how the identification of spatial spillovers has recently witnessed an upsurge of interest. Our second aim is to review the current state of affairs. Some reviews of this emerging literature are already available on some specific issues. On the one hand, Gibbons et al. (2015) focus on the structural approach and outline the threats to the identification and specification of the connectivity matrix. On the other hand, Kolak & Anselin (2020), Akbari et al. (2023) and Gao et al. (2022) focus more on the causal inference framework. For example, Kolak & Anselin (2020) review how spatial effects (namely spatial dependence and spatial heterogeneity) lead to the violation of the SUTVA assumption and provide an overview of existing attempts, within the spatial econometrics/regional science literature, to extend impact the common evaluation identification strategies and models (difference-in-differences, propensity score matching, regression discontinuity, and instrumental variables) in the presence of spatial effects. They focus on the counterfactual framework and discuss how spatial effects affect the assignment to treatment, the potential sources of variation in the treatment variables, and the estimands.

Compared to the existing literature, we go one step further in that we consider the general notion of identification. First, we discuss it in both structural and reduced forms frameworks so that we are able to provide bridges between them. Second, and complementing the previous reviews, we provide a comprehensive view of identification in the presence of spatial heterogeneity that we apprehend both in terms of (spatial) varying treatment effect and spatial confounding. Third, we include a systematic discussion of the other identification threats, while previous reviews focus on spillovers only. Lastly, we mobilize the biostatistical literature, which has long focused on the definition of causal estimands in the presence of interference (spillovers). Conversely, we do not document the concept of Granger causality in a spatial context, nor do we discuss the empirical dynamic modeling or convergent cross-mapping (CCM) frameworks, as they are more rooted in the time series approach to causality. Finally, we do not consider graphical models or path analysis models as we only focus on the counterfactual and potential outcomes approach.[6] In a spatial context, these three approaches are discussed in Akbari et al. (2023) and Gao et al. (2022).[7]

[5]See among others VanderWeele (2015), Hong (2015), Angelucci & Di Maro (2016), Reich et al. (2021), Aronow, Eckles, Samii & Zonszein (2021).

[6]See Ogburn & VanderWeele (2014) for some insight in specifying causal diagrams with interference.

[7]See also Mur (2013), Herrera et al. (2016) for an extension of the Granger concept of causality for spatial

The paper is structured as follows. Section 2 presents a motivating example. Section 3 presents the structural approaches that have been developed to identify spatial spillovers, as well as the different threats to identification that are often overlooked in the applied spatial econometrics literature. Section 4 describes the causal inference methods that account for interference (spillovers) and discusses how they have been extended so far in the spatial context. Section 5 develops recommendations for applied papers aiming to identify spatial spillovers: definition of parameters of interest, structural versus reduced-form approaches, unit of analysis, choice of interaction matrix, spatial heterogeneity and structure of error terms. Section 6 concludes.

2 Motivating example: Determinants of local expenditure

Our motivating example refers to the determinants of local government expenditure. These determinants are of significant interest to economists and policymakers, as they play a crucial role in shaping the distribution and size of public goods, infrastructure, and services that directly affect economic welfare and local development. Local jurisdictions, responsible for providing essential services such as education, healthcare, transportation, and public safety, must carefully allocate resources to meet the needs of their populations while maintaining budget balance.

A key factor in understanding local government expenditure is the presence of strategic interactions between neighboring local jurisdictions. These interactions arise when the spending decisions of one jurisdiction affect neighboring jurisdictions. Their existence is grounded in the theoretical framework of strategic complementarity or substituability (see for instance Brueckner 2003). For example, if a local government increases its expenditures on public services, nearby jurisdictions may feel compelled to follow to avoid losing residents or businesses to neighboring areas with better amenities. This spatial competition can lead to an upward pressure on public spending, often referred to as a “race to the top”. Neighboring jurisdictions engaging in strategic competition over public services may overextend their budgets, leading to fiscal imbalances or cuts in other essential areas. Conversely, some jurisdictions may reduce expenditure if they can benefit from the spending of neighboring areas, a form of free-riding behavior, leading, in the contrary, to a “race to the bottom”. Consequently, these spatial interactions have profound implications for local policy choices as they may lead to inefficient resource allocation or policy competition. Empirical evidence of such strategic interactions in local public expenditure is well documented, although their estimation raises major identification issues to disentangle these substantive interactions from similar behaviors implied by similar characteristics and/or common shocks (Agrawal et al. 2022).

Although strategic interactions are central to understanding local expenditure, several other factors also play an important role. Larger populations tend to demand more public services, which increases total expenditure. However, economies of scale in the provision of services may imply that per capita expenditure decreases as population size increases, up to the point where diseconomies of scale arise, leading to a U-curved relationship between

data.

population size/density and expenditure (Ladd 1992). In addition, the demographic composition of a jurisdiction, such as the proportion of children, elderly, or working-age adults, can influence the types of services demanded. Local economic conditions also significantly affect local government expenditure. High employment and income levels expand the local tax base, allowing governments to generate more revenue and thus increase spending on public goods and services. In addition to demographic and economic variables, institutional and political factors also shape local government expenditure. Political color may influence the prioritization of spending on certain services, while institutional configurations, such as the degree of fiscal decentralization or the presence of intergovernmental transfers, can play a critical role. More decentralized systems often give local governments greater discretion in spending decisions, potentially leading to higher levels of local spending (Oates 1972). However, obviously, most of these factors might be endogenous due to simultaneity or omitted variables.

In studying these issues, various identification strategies might be adopted depending on the parameters of interest. On the one hand, one can rely on a structural approach rooted in the fiscal federalism theory to identify the substantive strategic interactions between local jurisdictions (see Agrawal et al. 2022). The identification strategy should therefore be able to identify these interactions in a context where, as mentioned above, neighboring jurisdictions might share similar sociodemographic characteristics or shared infrastructure, leading to similar expenditure, or might experience common spatial institutional or environmental shocks. In addition, the spatial range of interactions should be defined in order to operationalize the notion of neighbors. Finally, when local jurisdictions belong to intermunicipal groups, an additional issue of self-selection arises. We explore all these topics in Section 3.

On the other hand, when the interest is on the impact of one particular variable, say population size, on the level of local expenditure, but staying agnostic as to underlying mechanisms, one can adopt a reduced-form approach and try to leverage shocks, such as changes in the institutional design of intermunicipal groups, or discontinuities to causally identify this effect. The identification strategy should then deal with mitigating self-selection and omitted variables while avoiding the presence of bad controls. In addition, in the context of local expenditure, local jurisdictions might interact with their neighbors in various ways, by mimicking their neighbors' behavior or because they share similar infrastructure or characteristics, leading to what is called *interference*, a violation of the SUTVA assumption. The identification strategy must be adapted to account for this interference, as discussed in Section 4.

3 Structural approaches to (spatial) interactions

3.1 Related literature

The structural modeling approach is based on the development of a structural model, where identification focuses on the associated (deep) parameters, giving them a *ceteris paribus* interpretation (Clarke & Windmeijer 2012). The literature in structural spatial and network economics has developed several models to identify spillovers between observations, which can be microeconomic units individuals or firms, for instance) or more aggregate units (countries, regions).

Starting with the case of structural models involving aggregate units, Lopez-Bazo et al.

(2004) model externalities of production between regions. Ertur & Koch (2007) develop a spatially augmented Solow model where interactions between economies are motivated by technological interdependencies. Egger & Pfaffermayr (2006) follow their approach to study the consequences of spatial dependence on convergence, while Pfaffermayr (2009) contrasts the spatial Solow model with the Verdoon model and demonstrates that the speed of convergence in both models depends on the remoteness and the income gaps of all regions. In a subsequent paper, Ertur & Koch (2011) integrate technological interdependencies in a Schumpeterian growth model, allowing one to cast both models in an integrated theoretical and methodological framework. When confronting their model to data, the four first papers rely on a geographically-based interaction scheme. In contrast, Ertur & Koch (2011) further consider an interaction scheme based on bilateral trade. However, they point out that the interaction matrix should ideally be theory-based, as the implementation of spatial methods requires accurate identification of the relevant interacting space (Ertur & Koch 2011, p.236). A step towards this direction is due to Behrens et al. (2012) who derive a quantity-based structural gravity equation in which both trade flows and error terms are cross-sectionally correlated. To the best of our knowledge, this is the only paper in which the channel through which interactions occur and the precise functional form of the interaction scheme, constructed from the population shares, are completely derived from economic theory.

At the microeconomic level, Schone et al. (2013) develop a theoretical model which shows that cities' growth control decisions are mainly driven by political struggle between different groups of voters and lobbies. Furthermore, by accounting for the residential mobility of renters in the city, their model implies spatially interdependent local growth control policies so that cities engage in strategic interactions.

In the context of social networks, the literature has also developed models to theoretically justify the presence of peer effects. These models, derived from game theory with strategic interactions, mainly explain spillovers by conformity or strategic complementarities between individuals.[8]

3.2 Applied spatial econometrics

Gibbons et al. (2015) propose a general framework that captures almost all channels through which spatial effects can be included in a regression model. For the cross-sectional case, this model is shown in (1):

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + \lambda\sum_{j=1}^{n} w_{ij}^{y} y_j + \sum_{j=1}^{n} w_{ij}^{x}\mathbf{x}_j\boldsymbol{\gamma} + \sum_{j=1}^{n} w_{ij}^{z}\mathbf{z}_j\boldsymbol{\theta} + \sum_{j=1}^{n} w_{ij}^{v}\mathbf{v}_j\boldsymbol{\kappa} + \varepsilon_i, \quad i = 1, \cdots, n \tag{1}$$

where y_i is the outcome for individual i, $\sum_{j=1}^{n} w_{ij}^{y} y_j$ is some aggregate value of the neighboring outcomes of i, $\mathbf{x}_i$ is a vector of individual characteristics and $\sum_{j=1}^{n} w_{ij}^{x}\mathbf{x}_j$ represents an aggregate value of neighborhood's characteristics, $\sum_{j=1}^{n} w_{ij}^{z}\mathbf{z}_j$ is a vector of observed common factors and $\sum_{j=1}^{n} w_{ij}^{v}\mathbf{v}_j$ represents an aggregate of unobservables across observations. This last term may represent interactions between observations in unobserved dimensions or spatially autocorrelated errors. Finally, ε_i is the idiosyncratic error term. In the literature

[8] Topa & Zenou (2015) and Bramoullé et al. (2020) provide excellent overviews of the literature on these models.

on social interactions, $\sum_{j=1}^{n} w_{ij}^{y} y_j$ refers to *endogenous effects*, $\sum_{j=1}^{n} w_{ij}^{x}\mathbf{x}_j$ is named *contextual effects* while $\sum_{j=1}^{n} w_{ij}^{v} v_j$ represents *correlated effects* (Manski 1993). Model (1) is also the starting point for the identification of policy interactions between local jurisdictions as described in Section 2. In this case, the model corresponds to the policy reaction functions (Brueckner 2003, Agrawal et al. 2022) with $\lambda \neq 0$ representing strategic interactions (race to the top / bottom mechanism), while $\gamma \neq 0$ indicates spillovers from neighboring jurisdictions, such as inhabitants of neighboring jurisdictions using public good and services of a jurisdiction.

Model (1) allows for different connectivity schemes for y, $\mathbf{x}$, $\mathbf{z}$ and v. However, in the vast majority of works, these matrices are identical, notably due to the difficulty of justifying different interaction schemes for different variables. When individuals interact in separate groups of the same size (for example, farmers who interact with all other farmers in the same villages, but not with farmers in other villages, with villages showing the same number of farmers), Manski (1993) shows that it is impossible to distinguish between endogenous and contextual effects even in the absence of correlated effects. This is known as the reflection problem. However, as soon as we depart from this specific interaction scheme, either by assuming different group sizes (Lee 2007, Davezies et al. 2009) or considering interactions along networks (Bramoullé et al. 2009), these two different types of spillovers can be disentangled in the absence of correlated effects. Therefore, considering the possibility of correlated effects is of the greatest importance for identifying spillover effects.

In applied spatial econometrics, the selection of the econometric specification is done using one of the two main methods developed now. The general-to-specific approach consists in estimating a Spatial Durbin model (SDM), shown in (2), which encompasses the spatial autoregressive model (SAR) when $\boldsymbol{\gamma} = 0$, the spatial X model, (SLX) when $\lambda = 0$ and the spatial error model (SEM) when $\boldsymbol{\gamma} = -\lambda\boldsymbol{\beta}$. However, this SDM assumes away correlated effects and observed common factors while both are prevalent in most applied work, threatening the identification of spillovers. We come back to this point in Section 3.3.3.

$$y_i = \mathbf{x}_i\boldsymbol{\beta} + \lambda \sum_{j=1}^{n} w_{ij} y_j + \sum_{j=1}^{n} w_{ij}\mathbf{x}_j\boldsymbol{\gamma} + \varepsilon_i, \quad i = 1, \cdots, n, \tag{2}$$

A second issue is that this SDM specification is most of the time not driven by economic arguments but by statistical considerations. LeSage & Pace (2009) argue that the SDM generalizes the spatial autoregressive model to account for possible omitted variables that are spatially autocorrelated (i.e neighborhood characteristics).[9] Although this problem is indeed pervasive in applied economics, other threats to identification, presented in the next section, also need to be considered to plausibly identify spillovers.

In a large majority of papers, once this SDM is estimated, using quasi-maximum likelihood, Bayesian methods, two-stage least squares[10], or a generalized method of moments, the econometric reduced form is computed and marginal effects are calculated. These marginal effects are interpreted as the direct, indirect, or total effect of a change in each determinant on the outcome of interest, accounting for the presence of spillovers between observations.

[9] The inclusion of neighborhood characteristics also implies more heterogeneity in the total impacts (see LeSage & Pace 2009, p. 40).

[10] In the SDM model, internal instruments for the 2SLS may be weak since only higher order contextual effects can be used while being potentially highly correlated with the contextual effects.

The marginal effect for determinant k is shown in equation (3) when the reduced form of model (2) is written in matrix form.

$$\frac{\partial \mathbb{E}(\mathbf{y} \mid \mathbf{X}, \mathbf{W})}{\partial \mathbf{X}_k} = (\mathbf{I}_n - \lambda \mathbf{W})^{-1}(\mathbf{I}_n \beta_k + \mathbf{W}\gamma_k), \tag{3}$$

with $\mathbf{I}_n$ stands for the identity matrix of dimension n.

The second strategy used to select the correct specification is the specific-to-general approach. It consists of starting with a linear model (with or without neighborhood's characteristics) and relies on specification tests (typically of the Lagrange multiplier or Wald type) to find the most relevant spatial model (namely the one that best fits the data). The objective of this approach is to assess whether cross-sectional dependence should be modeled as endogenous effects (SAR model), contextual effects (SLX model), and/or in the error term (spatial lag in the errors or spatial moving average errors). Once the "best" specification is selected, the marginal effects (average direct, indirect, and total effects) are reported and interpreted as the impact change in a given explanatory variable on the outcome. This model selection procedure borrows from the Box-Jenkins approach developed for time-series data. As already pointed out in Gibbons & Overman (2012), this specific-to-general model selection procedure, initially motivated by computational reasons[11], is strongly hampered by the fact that it is not related to any consideration of the economic reasons for the presence of spillovers and their transmission channels.

An implication of both approaches is that spillover effects, under the form of endogenous and/or contextual effects, are relegated as side-products of the specification. In fact, interpretations are based on marginal effects, which compute the effect of a change in the (exogenous) determinants on the outcome, accounting for spillovers. However, the spillovers *per se* are not of interest. Yet, given the structural constraints inherent to these specifications, for instance, the motivation for spillovers, the construction of the interaction scheme (selection of the relevant interaction space, the functional form of the assumed links), and its properties (possible evolution over time, endogeneity, mismeasurements, etc.), the proper identification of these spillovers should be at the center of interest and not considered as a data-driven consequence of the specification.

3.3 Spatial threats to identification

As the previous section has made clear, the presence of a spatial lag, the *endogenous effect*, should be closely related to the behavioral assumptions. The empirical modeling of interactions thus requires questioning the unit of decision and its possible discrepancy with the unit of observation, something that has been mostly overlooked in applied spatial econometrics. From the economic perspective, units of decisions might be individuals, firms, local jurisdictions, governments, etc. that typically optimize objective functions under constraints. On the contrary, the unit of observation is the unit on which the empirical analysis is performed (typically depending on data availability). Discrepancies between the two types of unit raise classical caveats largely documented in the spatial statistical literature: the ecological fallacy and the change of support problem. The ecological fallacy arises when conclusions obtained for aggregated data do not reflect the reality of individuals belonging to this aggregation.

[11] Spatial models were traditionally estimated by maximum likelihood method, which required the computation of the Jacobian of the transformation, computationally costly.

Also called aggregation bias, it has received lots of attention in regional and political sciences as it is common to have data at aggregated spatial levels while the meaningful mechanisms are at a lower spatial level. The change of support problem refers to issues related to the combination of spatial data observed at various scales and support.[12] The consequences of the discrepancy between units of decision and observations are further amplified in studies of spatial spillovers, as the threats to their identification are even more acute. These threats can be gathered into three main categories: i) Nature and construction of the interaction matrix; ii) Use of proxy variables for interaction; iii) Presence of spatial heterogeneity, and these are the topics of the next sections.

3.3.1 Role of the interaction matrix

The interaction matrix plays a crucial role in spatial econometric models from the identification, estimation, or interpretation perspectives. In the context of model (2) and assuming exogenous explanatory variables, Bramoullé et al. (2009) show that the interaction scheme should include intransitive triads, i.e. triads such as "peers of my peers are not my peers" to identify endogenous effects.[13] Developed for social networks, this paper spells out the conditions originally developed by Kelejian & Prucha (1998, 1999) in the context of SAR models. Turning to estimation, the (quasi-) maximum likelihood approach requires a correct definition of the functional form and of the interaction matrix to provide unbiased estimators. Finally, the interpretation of the model is mainly based on marginal effects, which explicitly depend on this matrix.

The literature has early realized the importance of the specification in the interaction scheme. As such, several works have studied the consequences of a misspecified interaction matrix (see, among others, Florax & Rey 1995, Paez et al. 2008, and Vande Kamp 2019). Besides, several statistical approaches have been developed to select the connectivity schemes. Kelejian (2008) has initiated a series of papers that use the J test of Davidson & MacKinnon (1981) to select the most relevant interaction scheme, among a finite number of candidates. In addition, Jin & Lee (2013) develop Cox-type tests to choose between two competing interaction matrices, while Bayesian model averaging procedures have also been derived (LeSage & Pace 2009, Zhang & Yu 2018). In addition, models that simultaneously introduce several interaction matrices in the SAR model have been developed (see among others Lee & Liu 2010, Badinger & Egger 2011, Han et al. 2017, Hazir et al. 2018, Debarsy & LeSage 2022).

Nevertheless, the large majority of papers in applied spatial econometrics specify the interaction scheme in terms of geographic proximity such as a function of the k nearest neighbors, contiguity, or other decreasing functions of distance (distance threshold, inverse distance, etc.). They also include a robustness section with respect to the choice of the interaction matrix, typically considering alternative functional forms of distance or different numbers of neighbors.[14]

[12] Gotway & Young (2002), Wakefield & Salway (2001) and Gelfand (2010), among others, provide a complete overview of the various statistical issues related to these topics. Also, Briant et al. (2010) assess the effect of the Modifiable Areal Unit Problem (MAUP) on economic geography estimates.

[13] We assume here that individuals do not interact in groups. For the latter case, Lee (2007) shows that as soon as the size of the group changes, the identification of endogenous effects is warranted.

[14] LeSage & Pace (2014) argue that this fine-tuning of the connectivity matrix is unnecessary and constitutes the biggest myth in spatial econometrics.

The first invoked reason to rely on a geographically-based connectivity matrix is its exogeneity (and non-stochasticity). This assumption allows to greatly simplify identification and estimation, but at a very high cost. Indeed, it generally impedes the modeling of relationships between aggregate units using socio-economic indicators, which generally lies at the heart of interactions. Furthermore, this precludes taking into account changes in the interaction matrix resulting from a (exogenous) change in some of the determinants.[15]

As a solution, Corrado & Fingleton (2012) advocate the use of economically-based interaction matrices, as was already done in Case et al. (1993). This might complicate the estimation strategy as the exogeneity is not guaranteed, but several methods have been developed to account for potentially endogenous interaction schemes. In the context of panel data models, Kelejian & Piras (2014) propose an IV procedure that directly instruments the elements of $\mathbf{W}$. This procedure has been applied by Agrawal et al. (2024) who use syndicate membership as instruments for endogenous intermunicipal cooperation between jurisdictions. Alternatively, Qu & Lee (2015) develop an estimator where the endogeneity of $\mathbf{W}$ is modeled as originating from the (economic) variables used to build the connectivity scheme. This methodology has been extended to dyadic data (Qu et al. 2021), to panel data models (Qu et al. 2017*a*, Shi & Lee 2018), and also to social networks (Johnsson & Moon 2021). In this context, Jochmans (2023) derives an IV estimator that takes advantage of the restrictions in $\mathbf{W}$ to construct instruments from leave-own-out networks. Besides, Kuersteiner & Prucha (2020) derive a GMM estimator for a panel data model which allows for endogenous time-varying networks, common factors and sequentially exogenous determinants. Finally, the peer effects literature has approached the endogeneity of the network by specifying a structural model of network formation in addition to the outcome model (see, for instance, Goldsmith-Pinkham & Imbens 2013, Hsieh & Lee 2016, and De Giorgi et al. 2022).

The second reason for the use of geographically based connectivity matrices is that they may act as a proxy for many economic phenomena (mobility of firms or consumers, transport costs, traded goods, capital movements, etc.). Following Neumayer & Plumper (2016), we argue that $\mathbf{W}$ must capture the *causal* mechanism of spillovers and thus reflect the connectivity between units. In other words, $\mathbf{W}$ should define the transmission channel through which interactions occur. Using geographic proximity as a proxy thus prevents drawing sound conclusions on spillovers for at least two reasons. Firstly, it remains silent about their underlying causal mechanism(s) and therefore cannot help distinguish alternative theories justifying these spillovers. Secondly, a-theoretical geographical proximity is at best a mismeasurement of connectivity, leading to misspecification problems discussed above, and at worst, completely unrelated to the true channels driving interactions, leading to unreliable conclusions.

In the context of microeconomic models, geographical space may play an important role in understanding spillovers between units. We already mentioned the work of Schone et al. (2013) on local growth control decisions, which shows that closer (in a spatial sense) cities interact more than cities located far away from each other. Glaeser et al. (1996) explain the high variance of the crime rate between cities using a model where the agents' propensity to engage in criminal activities partly depends on the propensity of other agents in the neighborhood. In addition, Kim et al. (2023) develop a new theory of social tie formation

[15] Boucher & Fortin (2016) develop and discuss a three-case categorization of connectivity matrices depending on their exogeneity and randomness.

in which people care about the geographical location of other individuals, to account for the cost of transport inherent in social interactions. In contrast, Del Bello et al. (2015) show that interactions in education occur through social proximity between students rather than through their spatial proximity. Finally, Janeba & Osterloh (2013) develop a model of tax competition between jurisdictions with locally mobile capital assuming different competition patterns depending on size: small jurisdictions compete locally with their geographically close neighbors while large jurisdictions (cities) compete both locally and with interregional cities. The geographical nature of interactions is motivated by recent empirical findings (notably Brett 2000, Buettner 2001, Brueckner & Saavedra 2001, Strauss-Kahn 2009) and results of their own survey of political decision-makers on the definition of the relevant neighbors in terms of competition for businesses.

The discussion above clearly shows the consequences of the discrepancy between units of decision and units of observation. By specifying a model in which the origin and nature of interactions are specified,the aggregation level at which the model should be estimated is known, as is the connectivity matrix that should best reflect the causal mechanism of spillovers. In contrast, estimating a model with spillovers on some (geographically) aggregated data not linked to decision units does not allow sound interpretations of the results.

The last point we make about the connectivity matrix relates to its normalization. In the vast majority of papers, the connectivity matrix is row-normalized to interpret the spatial lag as a weighted average of the neighbors. The problem with this normalization is that it is not neutral and creates misspecification if not derived from theory. As shown by Kelejian & Prucha (2010), there is no one-to-one correspondence between the row-normalized model and the original one due to a different normalizing factor for each row.[16] Additionally, row-normalization alters the informational content of the connectivity matrix by converting absolute distances (geographic or not) into relative ones. For example, if the connectivity between two observations should represent transport costs, then bilateral links should be expressed as absolute distances and not as relative ones. As such, without clear justification, row normalization should not be used.[17] An example of theoretical motivation is the paper of Patacchini & Zenou (2012), who develop a model showing that conformism to a social norm deters criminal activities and where conformity is modeled through a row-normalized connectivity matrix.

As wrap-up, we plead for giving up the blind use of a-theoretical geographically based connectivity matrices on the ground that it is exogenous, deterministic, and acts as a proxy for many channels. Geographically based proximity, unless theoretically grounded, such as in Schone et al. (2013) or Janeba & Osterloh (2013), jeopardizes identification of the causal mechanism of spillovers and further prevents to figure out the underlying economic reasons leading observations to interact, as should ideally do structural models, which underpin spatial econometrics models. Furthermore, the row normalization of the connectivity matrix should be avoided unless there are theoretical motivations, such as those in Liu et al. (2014).

[16]Kelejian & Prucha (2010) propose alternative matrix norms that do not alter the model, and which consist of dividing all elements of the connectivity matrix by the same factor (the spectral radius, or the minimum between the maximum of the row and column sums).

[17]Some estimators may require the assumption of row-normalized matrices. Within the framework of panel data models, Lee & Yu (2010) introduce a QML estimator that uses a transformation approach to eliminate time-fixed effects. Nevertheless, the existence of the (quasi-)likelihood function necessitates the row-normalization of the matrix.

As stated by Neumayer & Plumper (2016, p.177), "Specification choices [of $\mathbf{W}$] should follow theory rather than convention."

3.3.2 Proxy variables to model interactions

As seen above, the workhorse model for applied spatial econometrics is the SDM model. A constrained version of the model, shown in (4) is obtained by reexpressing a linear model with spatially correlated errors (SEM).

$$\begin{aligned} \mathbf{y} &= \mathbf{X}\boldsymbol{\beta} + (\mathbf{I}_n - \lambda\mathbf{W})^{-1}\mathbf{u}, \quad \mathbf{u} \sim iid(\mathbf{0}, \sigma^2\mathbf{I}_n) \\ &= \lambda\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\gamma} + \mathbf{u}, \end{aligned} \tag{4}$$

where $\boldsymbol{\gamma} = -\lambda\boldsymbol{\beta}$. It is fundamental to distinguish between a true SDM and a SEM as the *economic* implications are different. In the SDM model (2), a unit's behavior is affected by its characteristics and the behavior and characteristics of its neighbors, leading to endogenous and exogenous spillovers between observed variables. In contrast, in the SEM, interactions between units come from unobserved variables, leading to completely different economic interpretations. As noted in Boucher & Fortin (2016), the researcher does not always observe the "true" variable of interest and uses proxies. For example, in terms of obesity, papers use Body Mass Index (BMI) to proxy effort to reduce weight (see among others Christakis & Fowler 2007, O'Malley et al. 2014). Boucher & Fortin (2016) show that the apparent contagion effect of BMI may come from the unobserved effort the individual makes to reduce her weight and which depends on her peers' effort. Hence, the derived economic model is similar to an SEM model. If SDM is estimated instead, one might wrongly conclude to the presence of contagion effects of BMI.

In light of this discussion, the common factor statistic, derived by Burridge (1981), which tests whether the SDM model can be reduced to a SEM specification, should be a fundamental part of the empirical spatial toolbox.

3.3.3 Spatial heterogeneity

The spatial econometrics literature is well aware of the observational equivalence between spatial heterogeneity and spatial spillovers in cross-sectional settings (Anselin & Bera 1998). In other words, by looking at a map with some clustering, it is impossible to distinguish between a data-generating process that includes spatial regimes, common factors, spillovers, or a combination. For instance, when observing a spatial cluster of firms, we cannot know whether it is due to spatially varying characteristics of the locations in terms of access to population, amenities, etc. or because of direct interactions between firms. This issue is also known in epidemiology as "true" versus "apparent" contagion (Messner & Anselin 2004).

Nevertheless, spatial heterogeneity has often been presented as a secondary econometric issue with respect to spatial autocorrelation. For instance, Anselin (2001, p. 311) states that spatial heterogeneity is "simply structural instability, either in the form of nonconstant variances in a regression model (heteroskedasticity) or in the form of variable regression coefficients. Most methodological issues related to spatial heterogeneity can be tackled by means of the standard econometric toolbox". However, we argue that spatial heterogeneity is a major threat to identification that is generally overlooked, which strongly contrasts with the literature in network econometrics, where accounting for unobserved heterogeneity is an important part of the identification strategy.

The first source of spatial heterogeneity results from unmodeled spatially varying coefficients. For example, McMillen (2003) shows that spatial autocorrelation is also often the result of incorrect functional forms and spatially autocorrelated omitted variables in space. Typically, discrete spatial heterogeneity can be handled with dummy variables reflecting the spatial regimes or spatial switching regressions, while continuous spatial heterogeneity might be tackled with the inclusion of coordinates. Although this may have somewhat evolved with the advent of geographically weighted regression or semiparametric and nonparametric spatial approaches (see, among others, McMillen 2010, Basile et al. 2014, Osland et al. 2016, and Géniaux & Martinetti 2018), there is still a view that spatial heterogeneity does not involve as many conceptual complexities as spillovers. However, failing to properly handling spatially varying relations will result in the inability to identify spillovers, whether they result from endogenous or contextual effects.

The second source of spatial heterogeneity refers to the presence of common factors, which are frequent whenever spatial units are affected by unobserved shocks or events occurring at a higher spatial scale. While distinguishing between endogenous (direct spatial interactions) and exogenous (group-average) effects is already challenging, the possible occurrence of such unobserved common shocks adds additional complexity. If there are valid arguments that these unobservables factors are uncorrelated with the included control variables, robust inference (including clustered standard errors) could be applied. However, most of the time, these unobserved common factors are likely to be correlated with the determinants. To account for the effects of common factors, Bai & Li (2021), and Shi & Lee (2017, 2018) develop a spatial panel models where unobserved heterogeneity is modeled with interactive fixed effects rather than with additive fixed effects, to account for more flexible unobserved patterns.[18] In the context of tax competition, Chirinko & Wilson (2017) estimate the common correlated effects model of Pesaran (2006) and find that heterogeneous responses to common shocks, as well as delayed reaction, are crucial for understanding spillovers between US States. These authors show that States' tax rate follow a "ride on a seesaw" rather than a race to the bottom, as usually found in the literature. Alternatively, Mínguez et al. (2020) develop a semiparametric approach which consists of including smooth (via penalized splines functions) interactions between the time trend and the spatial coordinates of the observations, to account for heterogeneous effects common factors might have on each unit.

Finally, the impact of spatial sorting is not always considered. Spatial sorting refers to the situation in which heterogeneous individuals self-select themselves into various locations due to observed but also unobserved characteristics. It may act on the analysis through different channels. First, it may generate an endogenous interaction matrix as individuals choose their location. It might also act as an omitted variable bias and generate endogenous regressors.[19] If individual unobserved characteristics are correlated with both outcome and location, then spatial fixed effects (spatial differencing) are of little help in restoring identification.

[18]In the absence of spillovers, and for panels with "large T," popular (least-squares type) methods include those developed by Pesaran (2006) and Bai (2009), known in the literature as common correlated effects and principal components, respectively.

[19]For instance, when the control reflects characteristics of the population (structure by age, professional category, etc.) which are directly linked to the location decisions and hence to the resulting spatial sorting, including such variables in a policy reaction functions leads to identification issues, as they form bad controls.

The classical way to deal with endogenous variables consists in finding instruments. In fact, a large range of papers have considered the estimation of spatial models with endogeneity coming from both the endogenous spatial lag and additional endogenous variables. These papers are reviewed extensively in Le Gallo & Fingleton (2019) for both cross-sectional and panel data models and for single-equation and multi-equation models. They essentially consist in using moment conditions where, in addition to the traditional powers of spatial lags of the explanatory variables to instrument for the endogenous spatial lags, external instruments are added. Gibbons & Overman (2012) heavily criticize this identification strategy for the endogenous spatial lag as it is not clear whether these instruments satisfy the exclusion restriction. In the context of local policy choices, Agrawal et al. (2022) argue that special attention should be paid to the selection, ideally grounded in theory, of the spatial lags of explanatory variables that might be used as instruments. Alternatively, quasi-experiments might be used as instruments, such as exploiting reforms by higher-level governments in the context of the identification of tax interactions (Lyytikäinen 2012).

3.4 Structural identification strategies

From an identification perspective, understanding the causes of nonrandom spatial distribution of observations necessitates discriminating between all these various effects (endogenous effects, exogenous effects, spatially heterogeneous relations, common shocks, spatial sorting) while mitigating the impact of omitted variables. In the context of peer effects, Gibbons et al. (2015), Bramoullé et al. (2020) and An et al. (2022) discuss several identification strategies, based on exogenous sources of variation (random peers, random shocks), nature of data (panel), or structural models (network formation or modeling of structured omitted variables), which may constitute food for thought for applied spatial interactions. A substantive part of identification strategies developed in the peer effects literature have been derived to solve the complexities induced by endogenous modeling of the interaction scheme, constructed from socioeconomic indicators (interpersonal links, for instance).

However, some problems related to the definition of the interaction matrix remain. In the context of cross-sectional data, Boucher & Houndetoungan (2023) and Lewbel et al. (2023) develop identification strategies for spillover parameters when the adjacency matrix is unknown, but individuals are assumed to interact within many small (known) networks, such as classrooms or villages. The identification results of Boucher & Houndetoungan (2023) rely on the specification of a network formation model, while Lewbel et al. (2023) focus on the reduced form of the linear in-means model. Within a panel data model, de Paula et al. (2024) directly estimate the adjacency matrix from the reduced form of the SDM model and show the conditions for the identification of spillovers (endogenous and contextual effects parameters). They further apply their approach to reassess the magnitude of tax competition between US states. They show that the relevant neighborhood for each state is mainly based on economic similarity rather than geographic proximity (as typically assumed in the literature).

Finally, the analysis of time-varying networks are only at their early stages, a topic for which methods developed in the already extensive theoretical spatial econometric literature might be useful (see, among others Yu et al. 2008, Kelejian & Piras 2014, Qu et al. 2016, 2017*b*, Shi & Lee 2017, 2018).

A summary of issues discussed above is shown in Table 1.

4 Reduced form approach to spillovers

We now turn to reduced-form approaches, where the parameter of interest is a treatment effect. For instance, we might want to causally assess the impact of the political color or the belonging to an intermunicipal group on local expenditure without looking for the underlying causal mechanism. This experimental paradigm, with randomized controlled trials as the gold standard, is the core of most applied econometrics.[20] Although this causal inference literature avoids behavioral assumptions, one of the founding blocks of all its methods is, nevertheless, of behavioral nature: the Stable Unit Treatment Value Assumption (see Rubin 1974). This assumption prevents general equilibrium effects, social (spatial) interactions, spillovers, or dynamic behavior.

In this context, spillovers, named interference in the biostatistics and epidemiology literature, have been first considered a nuisance that confounds the identification of the parameter of interest. However, a rapidly growing literature has started to develop new causal estimands that capture interference, whose presence is viewed as a way to enrich the identification and interpretation of the parameter of interest.[21] In this section, we first review the canonical causal inference model and describe the concept of interference. We then focus on how spatial interference has been regarded so far in statistics. Finally, we also critically discuss the attempts made in the applied spatial econometric literature that have "spatialized" reduced-form models by including endogenous spatial lags and/or spatial error terms.

4.1 The canonical impact evaluation model

The canonical impact evaluation model of Rubin (1974) examines the effect of a binary treatment T, such as a public policy intervention or an environmental shock that only affects part of the sample.[22] Each observation has two potential outcomes: y_{0i}, the outcome that would be observed for unit i in the absence of intervention ($T_i = 0$), and y_{1i}, the outcome that would be observed in case of treatment ($T_i = 1$). The causal impact of the intervention for unit i is $\Delta_i = y_{1i} - y_{0i}, \forall i = 1, ...N$. As only one of these two outcomes is observed, the fundamental problem for estimating Δ_i is then a missing observation problem (Holland 1986).

The definition of a causal effect should precede the stages of identification and estimation (Heckman & Vytlacil 2007). A causal effect may be defined for an individual, a subpopulation, or the entire population and always involves one or more counterfactuals. At the population level, the literature focuses mainly on the Average Treatment Effect (ATE), $\mathbb{E}(y_{1i} - y_{0i})$, and the Average Treatment Effect on Treated (ATT), $\mathbb{E}(y_{1i} - y_{0i}|T_i = 1)$. At the subgroup level, defined for different values of the variables X, the Conditional Average Treatment Effect (CATE) is also defined as $\mathbb{E}(y_{1i}|X = x) - \mathbb{E}(y_{0i}|X = x)$. This estimand

[20] We do not provide an extensive literature review of the potential outcomes approach. The interested reader may consult, among others, Imbens & Rubin (2015), Abadie & Cattaneo (2018), Fougère & Jacquemet (2021), Heckman & Pinto (2022) or Imbens (2024) for excellent surveys or textbooks.

[21] See for instance VanderWeele, Ogburn & Tchetgen Tchetgen (2012), VanderWeele & An (2013), Ogburn et al. (2020), Reich et al. (2021).

[22] We only consider here the binary treatment framework.

Table 1: Identification threats in structural approaches

General threat	Specific problems	Possible solutions
Parameter of interest	Choice of the specification based on data-driven considerations	Motivate, ideally through an economic model, the presence of spillovers (endogenous or contextual effects).
Discrepancy between unit of decision and observation	Ecological fallacy	Investigate the spatial aggregation level at which the decision process occurs and try to collect data accordingly.
	Change of support	Use geostatistical model-based approaches, spatial disaggregation methods.
Interaction scheme	Selection of the connectivity matrix	Avoid using geographically-based connectivity matrices unless these spatial interactions can be grounded on economic arguments.
		Use alternative proximity measures that reflect causal mechanism and account for the econometric challenges, such that missing or misspecified links, endogeneity, time evolution, or response to changes in determinants.
		Use models that simultaneously integrate several interaction matrices, or statistical tests to choose between them when several causal channels may be justified.
	Unsupported row-normalization	Ground the row-normalization in the economic model.
		Use alternative matrix normalizations, typically based on the division of all elements by the same factor (spectral radius, etc.).
Unobserved variable at the source of interactions	Confusion between SDM and SEM	Model the source of interactions within the economic model.
		Test for the appropriateness of the SDM compare to the SEM using the common factor test.
Spatial heterogeneity	Common factors	Account for the unobserved heterogeneity originating in hierarchical models through higher level fixed effects, interactive
	Spatial variation of relationships	Include location specific effects, rely on heterogeneous coefficients models, or estimate nonparametric models.
	Spatial sorting	Assess the effect on network exogeneity, and rely on the literature on network endogeneity if needed.
		Look for instruments or quasi-experimental variations.

accounts for a varying effect along the different values of X. Importantly, the variables in X should be *predetermined* relative to the treatment, that is, they cannot be changed by active manipulation of the treatment T. We come back later to this important issue.

Identification of the causal effects of interest can only be achieved if the counterfactual quantities can be equated in some way with observable population data without introducing selection bias. As such, a fundamental identifying assumption in the Rubin causal model is the Stable Unit Treatment Value Assumption, presented below.

Assumption 1 (Stable Unit Treatment Value Assumption, SUTVA) *The potential outcomes for any unit do not vary with the treatment assigned to other units, and, for each unit, there are no different forms or versions of each treatment level, leading to different potential outcomes.*

SUTVA first assumes the absence of treatment spillovers between units.[23] It also states that a single version exists for each treatment. Spatial analysis is mainly concerned with the first point as it imposes the absence of spatial spillovers. The methods developed to relax this assumption to account for interference will be the subject of the following sections.[24]

Identification also depends on the assignment mechanism of the treatment. For example, in the random assignment case, the "prima facie causal effect" $\mathbb{E}(Y|T=1) - \mathbb{E}(Y|T=0)$ identifies the ATE because the treatment is ignorable by definition (Holland 1986). When dealing with observational data, this difference between average observed outcomes differs from a population average causal effect due to selection bias, and further identification assumptions need to be imposed. If selection in treatment is due to observed determinants, causal estimands can be recovered under the assumption of strong ignorability, also termed strong unconfoundedness.[25]

By contrast, in case of confounding, i.e., the absence of conditional independence between the treatment and the potential outcomes, other identification strategies such as difference-in-differences, regression discontinuity or instrumental variables needs to be used, each one subject to particular assumptions. Confoundedness can occur due to the self-selection of agents in treatment based on their potential outcomes or when information correlated with potential outcomes is used for treatment assignment, such as place-based policies targeting specific areas. In addition, spatial heterogeneity constitutes also major threats to identification in this framework, which, in the statistical literature, is referred to as spatial confounding.

4.2 Causal inference without SUTVA

In this section, we analyze how the literature has dealt with the presence of interference between individuals in the experimentalist paradigm. Interference, a term developed in the biostatistical literature, "is said to be present when exposure or treatment received by one individual may affect the outcomes of other individuals" (Tchetgen & VanderWeele 2012). Initially, interference has been considered as a threat to the identification of the

[23] Manski (2013) calls this first condition the individualistic treatment response assumption.

[24] To the best of our knowledge, the second condition hasn't yet been studied in spatial applications.

[25] This assumption supposes that the treatment assignment is independent of the potential outcome, conditional on the set of determinants (Ignorability) and that for any value of the determinant, the probability of being treated is bounded away from 0 and 1 (Overlap).

main causal estimands, with papers assessing the bias of causal effects estimated under SUTVA (Sobel 2006) and proposing designs of experiments that avoid interference or adjust inference (Rosenbaum 2007, Athey et al. 2018). Then, the focus has progressively switched to a substantive identification of interference, both in terms of the definition of the estimands of interest and design of experiments.

The literature has first remained agnostic on the source of interference and has proceeded by extending the standard estimands discussed above to four key estimands, in the context of a design-based approach at the unit level. By defining $\mathbf{a}_j = (a_{ji}, \mathbf{a}_{j(-i)})$ the treatment program for all individuals in the group j, composed of the treatment for the individual i and all other individuals $\mathbf{a}_{j(-i)}$, and $y_{ji}(a_i, \mathbf{a}_{j(-i)})$ as the potential outcome for individual i in group j, subject to treatment $a_{ji} \in \{0, 1\}$, and the treatment program for all other individuals established at $\mathbf{a}_{j(-i)}$, Hudgens & Halloran (2008) define:

1. **Direct effect**: $DE_i(\mathbf{a}_{j(-i)}) = y_{ji}(a_{ji} = 1, \mathbf{a}_{j(-i)}) - y_{ji}(a_{ji} = 0, \mathbf{a}_{j(-i)})$, i.e. the difference between the potential outcome of the individual i given treatment compared to the potential outcome for that individual without treatment, all other things being equal (including the treatment status of the other individuals).

2. **Indirect effect**: $IE_i(\mathbf{a}_{j(-i)}, \mathbf{a}'_{j(-i)}) = y_{ji}(a_{ji} = 0, \mathbf{a}_{j(-i)}) - y_{ji}(a'_{ji} = 0, \mathbf{a}'_{j(-i)})$. This estimand is also called the spillover effects, as it compares an untreated subject i under scenarios where other subjects receive $\mathbf{a}_{j(-i)}$ versus $\mathbf{a}'_{j(-i)}$. It equals 0 if there is no interference. The indirect effect may also be computed for the treated individuals ($a_{ji} = 1$).

3. **Total effect**: $TE_i(\mathbf{a}_{j(-i)}, \mathbf{a}'_{j(-i)}) = y_{ji}(a_{ji} = 1, \mathbf{a}_{j(-i)}) - y_{ji}(a'_{ji} = 0, \mathbf{a}'_{j(-i)})$. This estimand reflects the difference in responses that would be seen in i, between the scenarios in which she is treated and the others receive the treatment program $\mathbf{a}_{j(-i)}$ and i is not treated while the others receive another treatment program $\mathbf{a}'_{j(-i)}$. In general, total causal effects are not commutative. Lastly, we have $TE_i(\mathbf{a}_{j(-i)}, \mathbf{a}'_{j(-i)}) = DE_i(\mathbf{a}'_{j(-i)}) + IE_i(\mathbf{a}_{j(-i)}, \mathbf{a}'_{j(-i)})$.

4. **Overall effect**: $OE_i(\mathbf{a}_j, \mathbf{a}'_j) = y_{ji}(\mathbf{a}_j) - y_{ji}(\mathbf{a}'_j)$. This effect looks at the overall difference in potential outcomes for unit i between two alternative population treatment programs $\mathbf{a}_j$ and $\mathbf{a}'_j$. This is similar to TE_i, but OE_i allows individual treatment to be determined by T (whereas TE_i imposes $a_{ji} = 1$, $a'_{ji} = 0$).

Finally, these effects are averaged (Hudgens & Halloran 2008). Allowing for interference makes causal inference challenging, so that in practice researchers usually impose an underlying structure limiting its scope. The first and still most common relaxation of the no interference assumption is that of *partial interference* or *clustered interference* (Sobel 2006). In this case, it is assumed that individuals can be partitioned into distinct groups and interference can occur within groups only. This assumption should approximately hold if individuals are clustered in space, time, or some other dimension. Secondly, there is an expanding literature that addresses cases where observations influence each other through connecting edges. This implies that the assumptions on interference are driven by the *network*'s structure. However, similar to the general practice in applied econometrics, this literature assume that the network is known and fixed *a priori*. Finally, *general interference* has been considered (see Aronow & Samii 2017) in which no explicit assumption on interference is made.

A popular approach to formalize causal inference under interference has been introduced by Halloran & Struchiner (1995) and Aronow et al. (2017) with the concept of *exposure mapping* that summarizes the impacts of other individuals' treatments in sufficient statistics. A mapping is specified that relates the vector of treatment assignments for the experimental units to a finite set of exposures. The most frequently used forms of exposure mapping are calculating the proportion of treated neighbors. Causal effects, called "exposure effects", can then be defined in terms of comparisons of outcomes under different exposures. This framework is flexible because one can use any form of mapping to characterize the interference structure, similar to the role of the connectivity matrix in the econometrics of interactions framework. In addition, for the spillover effects obtained to be valid, the exposure mapping must be correctly defined. If the specification is inappropriate, the resulting causal inference may be misleading (e.g., failure to detect treatment spillovers) with imprecise variance estimation. Sävje et al. (2021), Leung (2022), Sävje (2024) are the first to study these questions and to provide the assumptions under which these problems can be mitigated.

4.3 Causal inference with spatial interference and spatial confounding

When the validity of SUTVA might be questioned, alternative strategies have been suggested. The first one consists of constructing a control group such that spillovers across groups are prevented. For instance, buffer zones between the treated and the control observations might be used. In the context of our motivating example, if the aim is to causally assess the impact of being part of an intermunicipal group on local expenditure, the treated observations are all jurisdictions belonging to some intermunicipality group while the control group could consists of municipalities outside of any group and sufficiently far away to avoid spillovers arising from shared infrastructure or mobile fiscal base. A second strategy involves implementing forms of falsification tests, by focusing only on the nearly located treated and control observations and switching their role. ATE should not be significant if SUTVA holds.[26]

The next section focuses on identification strategies built to deal with violation of the SUTVA assumption and the presence of spatial confounding in observational spatial studies.[27]

4.3.1 Regression adjustment and propensity score methods

When the assumption of strong ignorability can be maintained, regression adjustment and/or propensity score matching methods are used to identify causal effects.

[26]See for instance Earnhart & Hendricks (2023) for an example in environmental economics.

[27]There is an extensive literature on causal inference under interference in experiments with design strategies incorporating network information and controlling treatment assignments to mitigate interference (see among many others Hudgens & Halloran 2008, Liu & Hudgens 2014, Basse & Airoldi 2018, Aronow, Eckles, Samii & Zonszein 2021, for literature reviews). The specific case of spatial experiments with interference is reviewed in Aronow, Samii & Wang (2021), Samii et al. (2023). In particular, they define a quantity called an average marginalized response, which measures how on average outcomes that are a given distance from an intervention site are affected by activating treatment at that site, taking into account ambient effects emanating from other intervention sites. Alternatively, Pollmann (2023) proposes an estimator based on a comparison between individuals near realized treatment locations with individuals near counterfactual (unrealized) candidate locations.

In the spatial context, Cerulli (2017) derives a regression adjustment estimator where the specification of the potential outcome y_{0i} is a parametric function of the potential outcomes y_{1j} of the neighboring observations. His model is as follows:

$$\begin{aligned} y_{0i} &= \mu_0 + \mathbf{x}_i\boldsymbol{\beta}_0 + \gamma\sum_{j=1}^{n_1} w_{ij}y_{1j} + \varepsilon_{0i} \\ y_{1i} &= \mu_1 + \mathbf{x}_i\boldsymbol{\beta}_1 + \varepsilon_{1i} \end{aligned} \tag{5}$$

where μ_0 and μ_1 are scalars, $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}_1$ are two unknown vectors defining the responses of unit i to the vector of controls, ε_0 and ε_1 are idiosyncratic error terms, w_{ij} are elements of a row-standardized interaction matrix and n_1 is the number of treated units. Cerulli (2017) then shows that substituting Eq.(5) into $y_i = y_{0i} + T_i(y_{1i} - y_{0i})$ and assuming that the expectation is conditionally unconfounded ($\mathbb{E}(y_{gi}|T_i, x_i) = \mathbb{E}(y_{gi}|x_i)$ with $g = 0, 1$), yields a consistent OLS estimator of the ATE when y_i is regressed on $(1, T_i, \mathbf{x}_i, T_i(\mathbf{x}_i - \bar{\mathbf{x}}), \mathbf{z}_i)$ with $\mathbf{z}_i = \mathbf{v}_i + w_i(\bar{\mathbf{v}} - \mathbf{v}_i)$ and $\mathbf{v}_i = \sum_{j=1}^{n_1} w_{ij}\mathbf{x}_j$. However, this parametric approach is not the most commonly used in the literature, which prefers to rely on the propensity score approach.

The propensity score, $e(x) = Pr(T = 1|X = x)$ is the conditional probability of exposure, given X, and acts as a balancing score when the strong ignorability condition holds (Rosenbaum & Rubin 1983). Propensity scores are typically estimated using a logistic or a probit regression model. Once the propensity score for each individual has been computed, different strategies can be used to estimate the ATE, such as inverse-probability weighted estimator (IPW), doubly robust estimators, or matching procedures.

Let us now consider the case where the ignorability assumption fails for variables specifically related to spatial heterogeneity. Then, as for the structural approach, the problem of unobserved spatial heterogeneity hampers identification. Several methods have been proposed to alleviate this issue, such as adding nonparametric functions of coordinates as additional explanatory variables or including spatial random effects. Gilbert et al. (2024) provide a recent review of this extensive literature. In the context of causal inference, Davis et al. (2019) includes a conditional autoregressive prior for the random terms of the propensity score model and the outcome models. They show that the inclusion of spatial random effects gives a lower bias and a lower RMSE. Papadogeorgou et al. (2019) extend propensity score-based methods to incorporate spatial distance between observations in addition to covariates. As a consequence, only units that are close in space can be matched.

The literature has also developed some rather *ad hoc* proposals to account for spatial spillovers. For example, to estimate propensity scores in the first stage, Chagas et al. (2012) use a probit model including both a spatial lag term and a spatial error term that they estimate using Monte Carlo Markov chains. This raises an important issue as the spatial lag term $\mathbf{Wy}$ cannot be considered as a predetermined variable. As noted in Gibbons et al. (2015, p.133), "the endogenous spatial lag, which is an aggregation of the dependent variable, cannot be directly, exogenously manipulated within the population of sample to which the model related. Further, it cannot be changed holding other factors (determinants) constants." Alternatively, Zigler et al. (2012), considering the impact of environmental regulations on air quality measures observed at spatial locations throughout the US, include the spatial information by modeling the two potential outcomes as a function of treatments and a spatially-varying Gaussian process. Giffin et al. (2023) include spatial spillovers in treatment using a Bayesian-spline-based regression model in the first step.

In the (general aspatial) statistical/biostatistical literature dealing with interference, the first identification and estimation methods under ignorability were derived under the assumption of partial interference. Hong & Raudenbush (2006) and Verbitsky-Savitz & Raudenbush (2012) mimic a two-stage experiment and base their identification on the generalization of the standard conditional randomization assumption made at the individual level (called a spatial ignorability assumption) and then rely on a multilevel propensity score stratification to account for partial interference. Also, Ferracci et al. (2014) develop a two-step identification method, which focuses first on the distribution of treatment between individuals within groups and then on the distribution of treatment across groups, with the assumption of ignorability maintained at both stages. Finally, Tchetgen & VanderWeele (2012) derive several inverse probability weighted estimators as well a new causal effects in this partial interference structure. A second strand of literature has then extended identification and estimation methods under ignorability to the case where interference occurs on a known network. These papers typically start by defining an exposure mapping function (or equivalent), which specifies how the treatment is propagated to immediate neighbors, and then develop either generalized propensity-weighted estimates (Liu et al. 2016), or generalized propensity score matching (Forastiere et al. 2021, 2024).

4.3.2 Difference-in-differences methods

Different strategies have been developed to account for confoundedness, relying on particular identification assumptions and/or specific design of the data.

We start with the difference-in-differences (DiD) model, which relies on the availability of data on at least two periods (before and after treatment) for both control and treated groups and the parallel trend assumption. Formally, suppose observations $i = 1, ..., n$ that are available for two time periods and a binary treatment that takes place in the second time period only.[28] The standard DiD equation reads as:

$$y_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 D_i + \alpha_3 \widetilde{T}_t + \alpha_4 D_i \widetilde{T}_t + \varepsilon_{it} \tag{6}$$

where D_i is a dummy indicating the treatment status of unit i, and $\widetilde{T}_t$ is a dummy taking the value of 1 in the second period. As is well known, if ε_{it} is an *iid* error term, uncorrelated with D_i and $\widetilde{T}_t$, t α_4 represents the ATT, so evidence of a causal effect is present when $H_0 : \alpha_4 = 0$ is rejected.

Several papers have considered a "spatial" version of the DiD model by including a spatial lag term in this equation and/or spatial autoregressive errors[29] The use of an endogenous spatial lag $\mathbf{Wy}$ in difference-in-differences models raises serious issues. First, similar to the case of the propensity score matching procedure, endogenous effects cannot be considered as a pretreatment variable. In addition, the presence of a spatial lag implies that the remaining coefficients cannot be interpreted as marginal effects anymore, preventing the interpretation of α_4 as ATT. In the best case, the average direct / indirect / total effects can be calculated (see LeSage & Pace 2009), but it remains to be demonstrated that the average direct effects (in spatial econometrics meaning) of the treatment could be causally interpreted.

[28] Repeated cross-sections or panel data fit this design.

[29] See among others Sunak & Madlener (2016), Diao et al. (2017), Dubé et al. (2017), Kaneko et al. (2019), Xu & Liu (2021), Zeng & Bao (2021).

Delgado & Florax (2015) examine the scenario in which an individual's potential outcomes are influenced both by their own treatment and by the treatment status of their neighbors, akin to a linear exposure mapping method.[30] This method has been extended to the multivariate case by Bardaka et al. (2019). Butts (2021*a*) instead develop an estimator that accounts for both local spillovers onto control units and onto treated units. This estimator explicitly relies on the exposure mapping, hence avoiding the linearity assumption of spillovers. The case of spatially targeted treatment is investigated in Butts (2023) with the estimation of a treatment effect curve. Also, Huber & Steinmayr (2019) derive an estimator in the case where SUTVA is satisfied at the aggregate level. This framework allows them to identify an individual effect and a within-aggregate spillover effect, driven by the treatment of other individuals in the group. As Roth et al. (2022), we expect in the near future significant advances in difference-in-differences models, particularly those incorporating spillover effects.

4.3.3 Regression discontinuity

Regression discontinuity designs are based on the existence of a known cut-off point in a known forcing variable and are therefore useful in the analysis of public policy characterized by discrete policy-led interventions above or below certain levels. The usual scenario is that all individuals or units are treated if their value of the forcing variable is above the cut-off point. If the unit's characteristics do not change abruptly at the threshold, then the causal effect of the treatment can be uncovered by the change in treatment status. Of particular interest in a spatial context are the geographic RDDs, where geographic borders serve as thresholds with only one side of the border being treated (see, among others Keele & Titiunik 2015, 2018, Butts 2021*b*). With regard to our illustrative example, Agrawal (2015) uses state borders to identify tax competition, without relying on the slope of the policy reaction function.

An obvious issue in these designs is spatial spillovers occurring within units on either side of the border, as well as those extending across the border, which preclude us from interpreting conventional RDD estimates as causal estimands. Aronow et al. (2017) show that in the context of an arbitrary and unknown interference structure and a local randomization assumption, the difference in means between units near the boundary can be interpreted as the average direct effects of Hudgens & Halloran (2008) for this subpopulation. Recently, Dal Torrione et al. (2024) extend the RDD to encompass scenarios in which units interact within a network, leading to a multiscore RDD with multidimensional boundaries. Finally, Auerbach et al. (2024) characterize the estimand of a RDD in which the outcome for a particular unit is linearly influenced by both the average treatment status and the average outcome of units with comparable running variable values. When these running variables are geographic coordinates, it accounts for spillover effects among spatially proximate units.

In the context of voter turnout during a presidential campaign, Keele & Titiunik (2018) assume, similarly to Verbitsky-Savitz & Raudenbush (2012), that interference occurs only when treated individuals are in close geographic proximity to a sufficiently high number of control individuals. This assumption let them define and non-parametrically identify

[30] Empirical applications of this approach in empirical spatial econometric papers can be found in Chagas et al. (2016), Han et al. (2018), Feng et al. (2021), Kosfeld et al. (2021), Madeira Triaca et al. (2021), Yu (2021).

estimands for the direct and indirect effects. Finally, Cornwall & Sauley (2021) consider the question of spatial spillovers in regression discontinuity design. Their approach consists in residualizing the outcome by applying a spatial Durbin model and then proceeding with the RDD usual estimates. The causal estimands at hand are unfortunately not defined clearly and, as we mentioned above, using parametric specifications with a spatial lag in a reduced-form context raises important identification issues.

4.3.4 Instrumental variables

Finally, the literature has also started to address the problem of non-compliance with treatment in the presence of interference, which calls for an instrumental variables framework. The idea of the first contributions in this field has been to extend the local average treatment effect framework of Imbens & Angrist (1994) to the presence of spillovers (see, among others, Kang & Imbens 2016, Kang & Keele 2018, and Imai et al. 2021). Vazquez-Bare (2023) goes one step further by developing a model allowing for spillovers on the outcome of interest, on treatment intake and multiple types of compliers under partial interference.[31] Still in the context of binary treatment (and binary instrument), Hoshino & Yanagi (2024) investigate the case of noncompliance into treatment for an unknown network and propose the concept of *instrument exposure mapping*, which extends the exposure mapping to the reduced form equation of the treatment and summarizes potentially complicated spillover effects into a fixed dimensional statistic of instrumental variables. Finally, Hoshino (2024) explore the framework of a continuous intervention framework incorporating spillover effects where people self-select into treatment (noncompliance). They develop a control function approach estimator based on some separability assumption in the reduced form equation, and instrument peers' treatment by the weighted sum of the value of the instruments for the peers (similarly to Kelejian & Prucha 1998 and Bramoullé et al. 2009). At last, they apply their model to determine the causal impact of regional unemployment on crime using data from Japan's city level. In this application, unemployment rate in neighboring cities is also accounted for, where neighborhood is defined as sharing a city-border, while unemployment rate is instrumented with childcare facilities.

A summary of issues discussed in this section can be found in Table 2.

4.4 From the sources of interferences to a convergence of approaches

An emerging literature in biostatistics focuses on sources of interference. According to Ogburn & VanderWeele (2014), there are three specific mechanisms, not mutually exclusive, that could cause interference. In the case of *direct interference*, the treatment T_i of individual i directly impacts the outcome Y_j of individual j, regardless of the value of Y_i. *Contagion interference* occurs when there is a dependence between outcomes Y_i and Y_j, and therefore treatment T_i can impact Y_j through its relationship to Y_i. Peer effects might be an example. Finally, *allocation interference* refers to the assignment of subjects to groups and Y_i can be impacted by the characteristics of the other subjects in that group.

[31] DiTraglia et al. (2023) consider a constrained model with one-sided compliance and spillovers in outcome only.

Table 2: Identification threats in causal inference models

Threat	**Possible solutions**
Parameter of interest	Define relevant causal treatment effects that distinguish between direct and spillovers effects (similar to contextual effects in the structural approach)
	Avoid introducing Wy as it cannot be treated as a predetermined variable.
Patterns of interference	Distinguish between different forms of interference, to estimate the relevant causal estimate.
Interaction scheme definition	Conceptualize the possible way(s) individuals may affect each other to define the adequate exposure mapping function or the relevant group within which individuals might interact.
Selection bias based on spatially observable variables	Regression adjustments that include spillovers in potential outcomes.
	Propensity Score Matching (PSM) that directly account for location proximity in the matching.
	(Generalized) Inverse probability weighted estimator, Multilevel PSM, or generalized PSM.
Selection bias based on spatial confounding	Difference-in-Differences (DiD) estimator based on either linear or nonlinear exposure mapping, DiD on aggregate data where spillovers only occurs within aggregated groups, or DiD estimator for spatially targeted treatments.
	Geographic Regression Discontinuity Designs (RDD), multiscore RDDs or RDDs which explicitly account for spillovers between treated and control groups.
	Under the noncompliance to treatment take-up framework, use instrumental variables to compute Local Average Treatment Effect allowing for direct interference in the treatment intake and/or in treatment assignment (encouragement).

Interestingly, this classification is similar to the typology of Manski (1993), of, respectively, contextual effects, endogenous effects, and correlated effects. Disentangling the sources of interference requires more structured approaches relying on models and identification issues that are close to those detailed in Section 3, and include the need to control for confounding and homophily (Tchetgen & VanderWeele 2012, VanderWeele, Tchetgen & Halloran 2012, Ogburn & VanderWeele 2017, Liu & Tchetgen Tchetgen 2021, and Ogburn & Shpitser 2021). In the tax competition literature, an example of such an approach is Agrawal (2016) who shows how spillovers drive the differences in equilibrium tax rates between neighboring jurisdictions and then uses these differences to identify tax competition.

These developments point to a form of convergence between structural and reduced form approaches. In the former, we saw that some papers use exogenous institutional or environmental shocks to improve identification even when starting from a structural model. In the latter, exploring the underlying mechanisms and the sources of interference necessitates that some structure is added in the model. As stated by Lewbel (2019, p.872), "best practice will often be to combine features of both methodologies".

5 Recommendations for applied spatial econometrics

In this section, we propose some recommendations for good practice in applied spatial econometrics, which might help to identify spatial spillover effects. These recommendations include a clear discussion on the parameters of interest, the internal logic of using a structural versus a reduced approach, the unit of decision, and, related to this, the choice of interaction scheme, the issue of spatial heterogeneity and spatial confounding, and the nature of inference.

Parameters of interest. It is fundamental to clearly define the parameters of interest, i.e. those for which one wishes to provide a causal interpretation. These parameters might be spatial spillovers (endogenous and/or contextual spillovers), or they might also be the impact of changes in variables on the outcome. While this recommendation might seem trivial or unnecessary, one has to admit that many applied spatial econometric papers are not explicit enough, especially when the identification strategy is blurred with spatial spillovers solely obtained as a side-product of a data-driven chosen specification. When spatial spillovers are the main parameters of interest, a structural approach with special care devoted to the identification conditions and adequate considerations of the spatial threats discussed in section 3 might be appropriate. Otherwise, a reduced-form approach, where identification, interference and confounding are carefully discussed, should be considered.

Methodological approaches. The necessity of a clear discussion of the parameters of interest leads to our second recommendation, which is to keep in mind the specific features of identification in each of the structural and reduced-form frameworks. On the one hand, in structural models, the mechanisms of underlying interactions should ideally be set up. For instance, in the context of our motivating example, one can rely on the various models of local government policy interactions: fiscal competition, bidding for firms, yardstick competition, expenditure spillovers and Tiebout sorting (Agrawal et al. 2022). A purely structural approach is not without difficulties, as these various mechanisms yield the same reduced reaction function. Likewise, in the peer effects literature, it has only very recently been proposed a structural model allowing to distinguish between the behavioral sources of interactions (conformism or spillovers) within a linear-in-means model (see Boucher et al. 2024).

On the other hand, in causal reduced forms, an important first step consists in precisely defining the causal estimands of interest. As we point out above, some papers introduce a spatial lag term of the exogenous or the endogenous variables *ex-post* in PSM, or DiD designs without defining the appropriate causal estimands at hand, resulting in ambiguity about what is truly being identified. Moreover, except in very specific cases, such as in Auerbach et al. (2024) or Vazquez-Bare (2023), it is not possible to obtain a causal interpretation of the endogenous effect ($\mathbf{Wy}$) since it is impossible to exogenously manipulate it while keeping all other variables constant.

Linked to this, we point out several major issues in the practice of applied spatial econometrics, which heavily relies on the spatial Durbin model. This model is widely used in applied spatial econometrics on the argument that, in the presence of spatially autocorrelated omitted variables, this model provides consistent estimates of the parameters (LeSage & Pace 2009). However, it should now be apparent that this strategy is only valid if all other threats to identification are adequately considered. Furthermore, it assumes that the exogenous variables and their spatial lags are plausibly exogenous, a major aspect that is rarely discussed. In other words, the standard spatial Durbin model is an adequate answer to identification in very restrictive cases only, characterized by assumptions that are rarely encountered in observational data.

As a consequence, the impacts (total, direct, indirect impacts, developed by LeSage & Pace 2009) traditionally computed in SAR or SDM specifications can only be considered as impacts in the causal sense when all the possible threats to identification have been addressed. Otherwise, they should only be viewed as marginal changes without any causal interpretation.

Finally, as the presence of an endogenous spatial lag term introduces many complexities (in terms of identification, interpretation, and estimation), which is not appropriate in a purely reduced-form model, the spatial Durbin model should ideally be rooted in an underlying theory.

For all the reasons set above, when the main parameter of interest is not some spillover effect (or if the theory does not clearly state that structural spillovers should be accounted for), reduced forms with appropriate treatment of interference might be more adequate.

Unit of decision vs unit of observation. Identification of spatial spillovers also requires that the unit of observation (the geographical level at which the empirical analysis is conducted) matches the unit of decision. When it is not the case, a specification including spillovers (endogenous spatial lag, neighborhood characteristics or both) is mostly not relevant, unless there are strong theoretical foundations that imply interdependence at equilibrium (for instance, microfounded gravity models).

Moreover, despite conducting empirical research centered on decision units, Gibbons et al. (2015) point out that the scope for including an endogenous spatial lag is generally more restricted than commonly applied. For instance, in the context of hedonic price models, it is generally assumed that a house price is a function of the price at which neighboring houses were sold. However, this cannot be a structural model as the price of houses already sold cannot be dependent on the price of future houses (Gibbons et al. 2015, p134). As such, the simultaneous nature of decisions should be discussed much more.

Interaction matrix. In the context of a structural model, identification of spillover effects is tightly linked to the definition of the interaction scheme. Defining links between observations based on geographical proximity avoids Manski's reflection problem. However, as pointed out by Gibbons et al. (2015), in a two-stage least squares framework, this definition of neighborhood might generate weak instruments if the matrix is not sufficiently sparse. Besides, identification of spillovers also requires the connectivity matrix to capture the causal transmission channel of interactions. This hinders the consideration of changes in public policy that affect the framework of interactions.

As we argued above, an a-theoretical geographically based matrix precludes the identification of the causal mechanism underlying interactions, as it acts as a catch-all of different interaction theories (see Neumayer & Plumper 2016 and Agrawal et al. 2022 for examples). In addition, the effects of a change in a public policy variable on the structure of interactions cannot be accounted for, leading to potential biases in measuring the impact of a public policy. As such, if the causal mechanism of interactions is not based on geographic similarity, alternative definitions of neighborhood, possibly based on socioeconomic proximities, should be considered. Evidently, these economically driven interactions present numerous econometric challenges, yet they are beneficial from the identification perspective.

Spatial heterogeneity. An important point is that spatial heterogeneity should be carefully accounted for, both in structural and reduced forms approaches. In the structural framework, heterogeneous effects of spillovers (or other variables of interest) could be considered, using the QML approach developed by Aquaro et al. (2021), the GMM approach of Konstantinidi et al. (2023), a combination of GMM and nonparametric methods such as in Sun & Malikov (2018) and Hong et al. (2024) or the Bayesian framework of Pace & LeSage (2004) and Cornwall & Parent (2017). Theoretical models developed by Ertur & Koch (2007) and Behrens et al. (2012) imply heterogeneous parameters for spillovers and determinants between countries. Also, Agrawal (2016) studies the influence of local tax rates on the location of retail activity, with an emphasis on cross-border shoppers. He shows that in the presence of multiple counties, the strength of town strategic interactions can be heterogeneous with respect to distance to the county's border. Within a reduced-form approach, spatially differentiated treatment effects may be obtained by interacting the treatment with a spatial indicator. Spatial heterogeneity can also act as a confounder of spillovers, notably through sorting or hierarchical structure (common factors). This form of spatial heterogeneity is typically overlooked in applied spatial econometrics papers, preventing clear interpretations of findings. Several approaches have been developed in the statistical literature to mitigate this form of unobserved spatial heterogeneity, such as relying on flexible spline functions of latitude-longitude coordinates (see Reich et al. 2021, for a recent overview). Related to this, we emphasize that the econometric consequences of spatial sorting have received a lot of attention in the urban economics literature, but less so in applied spatial econometrics. This problem might arise due to spatially mobile units of decision or whether some control variables are the results of outcomes of spatially mobile decision units, such as the share of each economic activity in a given country, leading to the bad controls issue.

Statistical inference. Unless theoretical reasons imply the parametrization of spillovers in the error terms, we follow Kelejian (2016) who states p.115: "errors are the *unknown* part of the model; we should not model them!" (Highlights from Kelejian). In the context of gravity equation, the model developed by Behrens et al. (2012) leads to an econometric model that includes spatial moving average errors. Another example is Calvò-Armengol et al. (2009), who develop a model of peer effects in education characterized by a spatial autoregressive process for the error terms, aimed at capturing spillovers effects between unobserved efforts.

A large battery of tests have been developed in spatial econometrics. However, contrary to the usual approach that consists in using LM-based tests for specification search, we contend that it is preferable to use diffuse statistics such as nonparametric tests (López et al. 2010) or scan tests (Kulldorff & Nagarwalla 1995, López et al. 2015, and Chasco et al. 2018) for diagnostic purposes. These tests are indeed very powerful in detecting the remaining structure in the residuals. For example, in studying a Spanish hedonic price model, Chasco et al. (2018) use the scan test to uncover local spatial clusters of low or high values in the residuals, pinpointing to spatial omitted variables. This information has been then integrated in the main model to improve the specification. Due to their diffuse alternative hypothesis, these aforementioned tests are uninformative with respect to the causes of the violations of *i.i.d.'ness*, but should invite to revise the whole estimation strategy. Indeed, as seen earlier, cross-sectional dependence in the error term might result from unobserved confounding unobserved variables, local common factors, misspecification of the interaction matrix, or a combination of these elements.

To wrap up, we advocate the use of general spatial diagnostic tests to check that no remaining spatial structure is present in the error terms and then apply some robust inference method (see Conley & Molinari 2007 and Kelejian & Prucha 2007) rather than specifying a strong parametric process in the errors (such as spatially correlated errors).

6 Conclusion

Our objective is to provide insights into the conditions under which the widely used notion of spatial spillovers has a meaningful empirical content, by looking at this question from the identification perspective.

First, we argue that one should stop systematically considering spillovers as side effects of specifications and either see them as the parameters of interest or consider them as interference that need to be accounted for. Indeed, modeling interactions through the use of a matrix imposes a lot of structure on the model, such as explicitly specifying the channel(s) through which interactions occur as well as the functional form of the links between observations. This also implies that these models necessarily fall into the structural econometrics approach as endogenous spatial lags are generally not compatible with a causal inference framework.[32] If spillovers are considered only as confounding the identification of another parameter of interest, then causal reduced forms approaches should be preferred. Second, scaling up the importance of spillover effects in the analysis requires giving up selecting the econometric model based on a mechanical application of specification tests. Indeed, this selection procedure, borrowed from time series analysis, is not compatible with a causal interpretation of spillovers, as the model is selected according to data considerations rather

[32]Exogenous spatial lags play a different role as they might be viewed as a particular way of modeling exposure mapping, a topic that needs to be further investigated.

than from an economic model. Third, a careful attention to the matching between the observational unit and the unit of decision is important to provide sound interpretations. Fourth, except when the economic model involves cross-sectional dependence in the error term, we also recommend to first using diffuse tests to model the potential residual cross-sectional dependence and then use a robust approach to account for the non-iid behavior of the error term, rather than trying to parametrize it. Fifth, the identification of spillovers requires a thorough investigation of the multiple facets of spatial heterogeneity to control for all potential confounding effects and be more confident about the interpretations to be extracted from the estimated model. Last but not least, much more thought should be given to the selection of the interaction matrix and except in some very specific cases, avoid using geographic proximity to construct the links scheme between observations. As discussed above, the costs associated with the use of geographical space far outweigh the benefits of a simplified econometric estimation.

To conclude somewhat provocatively, we advocate to stop using the confusing name of *spatial econometrics*. In fact, this term encompasses both the methodological tool (spillover modeling) and a particular type of spillover (based on locational similarity). We believe that *econometrics of spatial interactions* would be a much better name since it would focus on the identification challenges inherent to the use of spatial data. This term would also be in line with the econometrics of social networks (or peer effects), which is understood to deal with all the difficulties raised by interactions between individuals. We also suggest using spatial data econometrics to refer to all methods specific to spatial data.

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