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The Distributional Effects of Lobbying

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Abstract

How can firms influence the underlying economic structure in which they operate? To address this question we modify an heterogeneous firms model of monopolistic competition to include a lobbying game. In this setup, the market entry fee and the operating fixed cost are chosen by a non-benevolent policymaker that weights the potentially suboptimal welfare level its choice of policy entails against the political contributions it receives from special interest groups, a common agency problem. We identify differing policy objectives along the firm distribution which correspond to three distinct political scenarios. We describe the ex-ante market conditions that give rise to each case, characterize the resulting equilibrium policy induced by lobbying activity and contrast this political equilibrium with the baseline economy. Our results explain the reported link between lobby composition and levels of concentration within an industry, describe a political mechanism through which firms and policymakers are able to collude and capture economic rents and shed light on two outstanding puzzles in the literature - on the one hand, political contributions' apparent lack of an effect on firms' economic outcomes, and on the other, the increasing levels of concentration recently observed among industries in the American economy.

Keywords: Firm distribution, lobbying, political economy, monopolistic competition.

JEL-Codes: D72, L11, E23, E02, E60

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1. Introduction

That the level of concentration among US industries has increased considerably in the last decades is a well-established fact. Several theoretical explanations have been proposed in recent years, although the literature has yet to arrive at a consensus (H. Hopenhayn et al., 2022; Covarrubias et al., 2020).

In this paper, we explore a mechanism through which a policymaker could exchange favors with Special Interest Groups (SIGs) by enacting policy that distorts the underlying conditions in which firms operate and effectively shifting the firm-size - a theory of "bad" concentration.

To do so, we present a standard model of monopolistic competition with heterogeneous firms that incorporates a lobbying game. In exchange for political donations from SIGs, the policymaker distorts market fundamentals (an entry fee and a fixed operating cost) by enacting SIG-favorable policy. Similar to Jung (2012), we distinguish between exogenous costs, due to the nature of the sector firms participate in, and effective costs, the amount actually due because of the policymaker's decision.

Using the underlying economic environment as a guide, we are able to identify different incentives to lobby (i.e., different desired policy outcomes) among firms of different size, suggesting that firm heterogeneity gives rise to competing SIGs even within the same narrowly-defined industry[1]. The strategic interaction between the lobbies and the policymaker is modelled as a common agency problem, with the lobbies as the principals and the policymaker as the agent.

We use the game's Truthful Nash equilibrium (TNE) (Bernheim and Whinston, 1986; Dixit et al., 1997) to derive a relationship between lobbying composition (which of the identified camps partake in the game), lobbying behaviour (the amount of political contributions) and the underlying market structure. In the first two

[1] Following the original approach by GH, we focus on the game and abstract away the free-rider problem, assuming that firms from each camp are able to exogenously coordinate to form their own "lobby", an entity that deals with the policymaker on behalf of its members to advance their agenda.

cases considered, a single participating lobby is able to extract positive rents for its members conditional on the policymaker being selfish enough. In the third case, two opposing lobbies compete and the policymaker uses the threat of supporting the opposing lobby to capture positive rents from both lobbies. Interestingly, in this case the equilibrium policy turns out to be unchanged from the baseline case, although both lobbies are obliged to provide political contributions. Together, these cases shed light on the puzzingly low weight empirical studies suggest policymakers give to political donations, since competing lobbies would yield an excessively high weight to consumer welfare if not considered in an empirical analysis (Gawande and Bandyopadhyay, 2000).

A key variable uncovered is the threshold that separates big firms, concerned more about aggregates (they are interested in less competition overall), from small firms, concerned more about their own costs. This threshold allows us to distinguish between two real-world lobbying types. The lobby for small, low productivity firms is an abstraction for an industry-wide organization that represents the interests of a large number of small establishments with similar political goals. The lobby for big, high productivity firms is an abstraction for a single (or small amount of) big industry player(s) that lobby individually. We find that in less concentrated, more competitive industries there is a larger incentive for small firms to lobby together, while the inverse is true for big firms lobbying individually.

The paper is organized as follows. The remaining of the introduction reviews the related literature. Section ~ 2 presents the baseline economic environment with firm heterogeneity. Section ~ 3 introduces the lobbying game and states the main results. Section ~ 4 concludes.

Related literature: Lobbying has been the focus of extensive research, especially in the context of international trade, although its effect on the underlying economic environment in which firms operate has not been the center of attention (Grossman and Helpman, 1994; Hillman and Ursprung, 1988). The Grossman-Helpman

(GH) " Protection for Sale" model of lobbying provided a convenient framework to study the interactions between politicians and SIGs that give rise to suboptimal policy. It has since been extended to tackle a variety of related cases[2]. Ansolabehere et al. (2003) scrutinizes the nature of U.S. campaign contributions with a focus on how unexpectedly low they are, compared to the amount of resources at play, and on how they are shaped by regulation.

Cooper et al. (2010) show a strong link between political participation and future stock earnings, also suggesting exorbitantly high rates of return for relatively small political investments. As a possible explanation Ansolabehere et al. (2003) hypothesize that campaign contributions serve for most individuals as a consumption good rather than as an investment, which agrees with the empirical observation that most contributions come from private individuals (Evers-Hillstrom, 2020). Cooper et al. (2010) notes that this contrast can also be explained by under-the-table deals or outright bribes.

Bombardini (2008) presents evidence that industry characteristics, in particular the level of firm productivity dispersion, is positively correlated with that industry's level of protection. In her main finding, she shows that industries with high levels of product market competition and low levels of concentration tend to lobby through industry-wide organizations, while larger firms are more likely to partake in lobbying, and when they do so they tend to contribute more than smaller firms. To rationalize these findings she extends the GH framework by incorporating an oligopolistic manufacturing sector.

In a related paper, Bombardini and Trebbi (2012) ask why some firms lobby together and why some firms lobby by themselves. The authors find that greater elasticity of substitution, lower market concentration and a higher capital-to-labor ratio are positively correlated with lobbying through trade associations, as opposed to lobbying individually, which is less profitable in scenarios of limited market in-

[2] Most notably Baron and Hirsch (2012) for parliamentary systems of government and Chambers (2007) for "vote stealing".

fluence.

Huneeus and Kim (2021) focus on the misallocation of resources that results from lobbying. They start from a baseline model of monopolistic competition with heterogenous firms and extend it with the GH framework.

Assessing the effects of lobbying can be a nuanced task. As remarked by Bombardini and Trebbi (2019), quantifying a firm's payoff after lobbying is a highly complex issue, given the natural non-transparency of the lobbying endeavor. They review distinct empirical strategies to identify the demand of policy, on the part of interest groups, and the supply of policy, on the part of the policymaker.

Plouffe (2012) questions the traditional political economy approach of focusing on export propensity at the industry level. He points out that engaging in trade is inherently a firm-level activity and notes that while higly productive firms benefit from access to foreign markets, unproductive firms may be harmed by competition from foreign producers, deducing that different productivity levels in the same industry imply different target policies. He finds that it is mostly highly productive firms that export who mostly lobby, and do so in favor of trade liberalization, and highlights the importance of entry barriers to lobbying.

Regarding opposing lobbies competing for policy outcomes, Egerod and Junk (2000) explores this issue empirically and indeed finds evidence of zero-sum-like behaviour.

It has become standard to introduce agent heterogeneity in macroeconomic theory. Our baseline model features heterogeneity among firms and was first introduced by Melitz (2003) to explain the intra-industry reallocation of resources following trade liberalization. Jung (2012) studies optimal regulatory policy in a Melitz model in a similar fashion to out paper, without the political mechanism, and characterizes optimal subsidies in such an economy.

Costinot et al. (2020) start from a generalized version of Melitz (2003) and study optimal trade policy, also without considering political mechanisms that might be at work. A prominent feature of the firm-size distribution is its heavy right tail

(Gabaix, 2016; Kondo et al., 2021). This feature is usually captured either through a micro-level foundation for firm growth, such as in Gabaix (2016) and H. A. Hopenhayn (2014), or by assuming a specific functional form for the distribution of a key variable, such as in Melitz and Redding (2015) and Melitz and Redding (2013), where firm prodictivty is assumed to be Pareto-distributed.

Rossi-Hansberg and Wright (2007) stress the effect a sector's production technology, quantified as that industry's physical-to-human capital average ratio, has on the resulting distribution. This is important to the political process here studied since Bombardini and Trebbi (2012) report factor intensity is correlated to trade unions' campaign contributions.

2. Economic Environment

2.1 Demand

The underlying economic environment is similar that of Melitz (2003). A representative consumer has preferences over a continuum of differentiated varieties $\omega \in \Omega$ of the single consumption good, with Ω the set of all available varieties. Consumer preferences exhibit constant elasticity of substitution σ among varieties. Given her level of income, the consumer optimally chooses the level of consumption for each variety. Utility maximization yields the demand curve for each variety ω (see Appendix ~ A.1):

$$q(\omega) = Q\left[\frac{p(\omega)}{P}\right]^{-\sigma} \tag{1}$$

where $Q, P, p(\omega)$ are, respectively, a composite good, the economy's price index and the price of variety ω.

2.2 Production

Varieties are produced by firms with idiosyncratic productivity z using only labor as input. The aggregate amount of labor in the economy is exogenous and denoted by N. Firms face the representative agent's demand curve (equation ~ 1), the price of labor w, engage in monopolistic competition, and they set prices to maximize per-period profits. Production technology is modelled through a labor demand function that features increasing returns to scale because of per-period fixed costs, f, and an associated policy instrument τ_f. The policy instrument is set by a policy maker and represents any industry-wide policy that increases the cost of conducting business. Operating costs are modelled as additional labor and paid in wages.

Given each individual variety's demand curve (equation ~ 1), a firm with productivity level z maximizes profits by setting its price according to:

$$p(z) = \frac{w}{\rho z} \tag{2}$$

where $\rho = \frac{\sigma-1}{\sigma} < 1$ a firm's markup over marginal cost. Productivity is used to describe firm behaviour from now on. Output, revenue and profits are increasing in productivity (see Appendix ~ A.2).

At the start of each period, firms face a positive probability δ of experiencing a productivity shock that takes them out of business. In the economy's steady state equilibrium, shortly defined, there is a flow of entrants substituting dying firms. Entrants draw their productivity after paying an entry cost $\tau_e f_e$, where f_e is the natural startup cost and τ_e is an associated policy instrument. The policy instrument is set by the policymaker and represents regulation that makes market entry more costly for potential entrepreneurs. Similarly to the fixed operating cost we regard the entry cost to be paid in units of labor.

Increasing returns to scale imply a low productivity cutoff level z^*. This endogenous quantity is used to characterize the industry equilibrium, defined as mass of active firms M and a density $g(z)$ that describes their productivity. Let $H(z)$ be the exogenous productivity distribution of potential entrants and $h(z)$ its associated density. Then $g(z)$ is such that:

$$g(z) = \begin{cases} \frac{h(z)}{1-H(z^*)} & z \geq z^* \\ 0 & z < z^* \end{cases}$$

The endogenous cutoff productivity level is found by using two equilibrium conditions, free entry (firms' expected entry value equals costs) and zero cutoff profit (firms with productivity at exactly the low productivity cutoff earn zero profits. See Appendix ~ A.4). The relevant measure of welfare in this economy is wages' purchasing power:

$$W = wP^{-1} = \left[\frac{N}{\sigma \tau_f f}\right]^{1/(\sigma-1)} \rho z^* \tag{3}$$

2.2.1 Welfare and Policy Instruments without Lobbying

Following GH we assume the policymaker's objective function to be:

$$G(\tau, \boldsymbol{C}) = aW(\tau) + \sum_{j \in J} C_j \tag{4}$$

where $\boldsymbol{C} = (C_j)_{j \in J}$ is a vector representing political Contributions for each lobby $j \in J$ and a is the relative weight assigned to consumer welfare. In the absence of lobbies $J = \varnothing$, the policymaker is de-facto benevolent and chooses both policy instruments to maximize equation ~ 4. The solution to this problem requires specifying a functional form for the productivity distribution of potential entrants. We follow the common approach of assuming a Pareto distribution (Helpman et al., 2004; Melitz and Redding, 2015) with shape parameter $\alpha > (\sigma - 1)$, which quantifies the thickness of the distribution's right tail and scale parameter normalized to one[3]. The support of $H(z)$ is $[1,\infty)$.

The solution to the program in equation ~ 4 is $\tau = (1,1)$ when the productivity of potential entrants follows a Pareto distribution (Jung, 2012). From now on, w is used as the numeraire price and set equal to one.

3. The Lobbying Game

We focus on two channels through which lobbies alter the economic environment, the entry and fixed operating costs. Starting from an industry equilibrium we identify firms that benefit from modifying either of these costs in order to pinpoint special interests in this economy.

3.1 Special Interests

Modifying either policy instrument τ_e or τ_f away from the baseline policy $\tau = (1,1)$ affects the cutoff productivity level through the free entry (equation ~ 10) and the zero cutoff profits (equation ~ 9) conditions (see Appendix ~ A.4).

On the one hand, increasing the entry cost $\tau_e f_e$ by increasing τ_e has a direct distributional effect by protecting incumbents from potential entrants. An increase in τ_e results in a decrease of successful market entry for new firms and a decrease in exit for incumbent firms, both of which have been observed in selected industries in the U.S. economy during the 1980's and 1990's, and across sectors more recently during the 2000's (Philippon, 2019; Decker et al., 2016). It also lowers the cutoff productivity level, as less competition from entrants grants small firms better operating conditions that allow them to artificially remain in operation - a type of factor misallocation similar to Huneeus and Kim (2021).

On the other hand, distorting operating costs has an ambiguous effect. Raising operating costs increases the minimum amount of revenue needed to maintain

[3] The inequality is a technical requirement to guarantee existence of the industry equilibrium.

non-negative profits, which raises both the cutoff and average productivity levels and the price index. Firms that do not make sufficient revenue are forced to exit the market. Interestingly, a higher price index results in a substitution effect at the firm level. Recall that demand for any given variety, given by equation ~ 1, is increasing in the price index P since a rise a rise in the general price level makes any single price relatively cheaper. For a sufficiently productive firm, the increase in revenue from the substitution effect dominates the increase in operating costs, resulting in a net profit increase.

This analysis suggests competing incentives for modifying the policy vector associated with the operating cost (τ_f) depending on firm productivity. In Appendix ~ B.1 we show that in the Pareto case there is a threshold productivity level z_H such that firms with higher productivity ($z > z_H$) benefit from an increase in operating costs while firms with lower productivity ($z < z_H$) see their profits reduced. The ratio of z_H to the low cutoff productivity can be expressed as (see Appendix ~ B.1):

$$\left(\frac{z_H}{z^*}\right)^{\sigma-1} = \frac{r(z_H)}{r(z^*)} = \frac{\alpha}{\alpha+1-\sigma} = \frac{1}{1-\frac{\sigma-1}{\alpha}} \tag{5}$$

The middle step comes from the revenue ratio of two arbitrary firms (see ~ A.2). Equation ~ 5 is surprisingly deep. It rationalizes the empirical observation that lower market concentration is positively correlated with lobbying through industry-wide associations, while lobbying individually is mostly profitable and observed in scenarios of considerable market influence (Bombardini and Trebbi, 2012). It does so by linking lobby composition to the shape of the underlying firm-size distribution. To see this, note that it has shape parameter $\alpha/(\sigma-1)$[4]. The shape parameter in Pareto distributions, and more generally the Power Law (PL) exponent in PL distributions, quantifies the level of *inequality* between the low and top quantiles of the distribution - a measure of concentration at the top (Gabaix, 2009).

[4]The endogenous distribution of active firms is $G(z) = 1-(z/z^*)^{-\alpha}$. Using that plus the definition of $r(z)$ suffices to see that the cdf for r is also Pareto with shape parameter $\alpha/(\sigma-1)$, as claimed.

As $\alpha/(\sigma-1)$ goes further away from 1 the distribution becomes *less concentrated* at the top, the share of firms that have a similar policy goal increases and hence the incentive to lobby through a trade-wide association becomes dominant.

On the other hand, as $\alpha/(\sigma-1)$ gets closer to one and the firm revenue distribution becomes more concentrated at the top, the incentive for the top(s) firm(s) to lobby individually increases.

3.2 The Lobbies

Each of the previously identified subset of firms are each represented by a political lobby. Lobby S represents small, low productivity firms. Lobby B represents big, high productivity firms. Formally:

$$S = \{z \mid z \in [z^*, z_H)\}$$
$$B = \{z \mid z \in (z_H, \infty)\}$$

Each lobby $j \in J = \{S, B\}$ has an associated welfare function $U_j[\tau, C_j]$ that depends on the policy instrument and the amount of political contributions C_j, decreasing in the latter. Their objective is to maximize aggregate profits of member firms, net of political contributions.

Political contributions C_j from lobby j depend on the choice of τ. We use the concept of a contribution schedule $C_j = C_j(\tau)$, a map from every policy instrument τ to a non-negative political contribution. At this point, we have not imposed any structure on $C_j(\tau)$ besides the non-negativity requirement. However, in practice, political contributions are highly regulated and there are limits on how much lobbies can contribute (Ansolabehere et al., 2003). Furthermore, sufficiently big contributions would trigger general equilibrium effects and break down the underlying industry equilibrium. We focus on feasible contribution schedules:

Feasible Contribution Schedule: A contribution schedule is said to be feasible if $\bar{C} \geq C_j(\tau) \geq 0$ for all possible policy instruments τ. Denote by $\mathscr{C} = \{C_j(\tau)|0 \leq C_j(\tau) \leq \bar{C}\ \forall \tau\}$ the set of feasible contribution schedules under τ for lobby j.

$\bar{C}$ is an exogenous upper bound on contributions small enough that even if both lobbies contributed the maximum amount allowed it would not have an effect on the economy's aggregates.

3.3 The Policymaker

The policy instrument (τ) is chosen by a non-benevolent policymaker that maximizes a weighted sum of consumer welfare W and political contributions. Let $\boldsymbol{C} = (C_j)_{j\in J}$ be the vector of political contributions received by the policymaker. Recall the policymaker's objective function in equation ~ 4:

$$G(\tau, \boldsymbol{C}) = aW(\tau) + \sum_{j\in J} C_j$$

where a captures the relative weight assigned by the policymaker its main role as welfare maximizer. Feasible policies $\tau \in \boldsymbol{T}$ are such that an industry equilibrium is achievable (see Appendix ~ B.2).

3.4 The Game

The lobbying game takes the following form:

i Each lobby j presents a contribution schedule C_j to the policymaker.

ii The policymaker chooses a policy instrument (τ) and collects contributions from each lobby.

Each lobby keeps presenting their contribution schedule as long as the policymaker accepts them. If, in a particular period, any lobby decides to present a zero contribution schedule, the policymaker will optimize accordingly and the game will either become a traditional principal-agent game, if one lobby keeps its own contribution schedule in place, or the policymaker will revert to its ex-ante status and set the policy instrument back to $\tau = (1,1)$ if no lobby participates.

To solve the game we use the notion of a Truthful Nash equilibrium (TNE), a refined version of Subgame Perfect Nash equilibria (SPNE) first introduced by Bernheim and Whinston (1986)[5]. Let us first introduce the following definition:

Truthful Contribution Schedules: A contribution schedule $C_j^T(\tau, U^\circ)$ from lobby j is said to be truthful relative to welfare level U° if:

$$C_j^T(\tau, U^\circ) = \min\left\{\bar{C}, \max\{0, \theta(\tau, U^\circ)\}\right\}$$

with θ such that:

$$U_j[\tau, \theta(\tau, U^\circ)] = U^\circ$$

A truthful contribution schedule relative to a welfare level U° embodies the idea that, whatever policy instrument is chosen, the lobby will contribute the amount necessary to achieve precisely this welfare level so long as the contribution required to achieve it is feasible.

Truthful Nash Equilibrium: A Truthful Nash Equilibrium (TNE) of the lobbying game is a policy vector τ° and a collection of contribution schedules $\{C_j^T(\tau^\circ, U_j^\circ)\}_{j\in J}$ from each lobby that are truthful relative to the lobbies' equilibrium welfare levels. We use **Proposition 3** of Dixit et al. (1997), adapted to our notation, to characterize such an equilibrium:

Let $(\tau^\circ, \{C_j^T(\tau^\circ, U_j^\circ)\}_{j\in J})$ be a truthful tash equilibrium of the lobbying game with equilibrium lobby welfare levels $\{U_j^\circ\}_{j\in J}$. Then $\tau^\circ, \{U_j^\circ\}_{j\in J}$ are such that:

(i) $\tau^\circ = \underset{\tau\in\mathcal{T}}{\arg\max}\left\{G\left[\tau, \{C_j^T(\tau, U_j^\circ)\}_{j\in J}\right]\right\}$

[5]See Appendix ~ B.2 for the formal definition of a SPNE.

(ii) For each $j \in J$

$$\max_{\tau \in \mathcal{T}} G\left[\tau, \{C^T_{-j}(\tau, U^\circ_{-j}), 0\}\right] = G\left[\tau^\circ, (C^T_j(\tau^\circ, U^\circ_j))_{j \in J}\right]$$

Condition (i) is easy to grasp. The policymaker takes the (truthful) contribution schedules as given and optimally chooses the equilibrium policy vector τ° that maximizes its own objective function.

Condition (ii) is more subtle. Let us take a single lobby's point of view. It proposes a contribution schedule, aware that there is another lobby participating in the game and that, once both lobbies have presented their contribution schedules, it is the policymaker who chooses the policy instrument by maximizing his own objective function. Thus, whatever contribution schedule the lobby proposes to the policymaker, it must ensure it is big enough to guarantee the policymaker will achieve at least the same level of welfare it achieves in his outside option of ignoring him (or equivalently the case where the lobby proposes a zero contribution schedule).

It is in the lobby's interest to contribute just enough to guarantee this level of welfare for the policymaker, and nothing more. To see this, consider the case where lobby $-j$ proposes a truthful contribution at the equilibrium welfare level U°_{-j} but lobby j proposes an excessively generous contribution schedule $C_j(\tau)$, making the policymaker choose $\tau = \tau^*$. By assumption, the welfare level that the policymaker achieves under $(\tau^*, C^T_{-j}(\tau^*, W^\circ_{-j}), C_j(\tau^*))$ is higher than what he would achieve if lobby j did not participate:

$$G[\tau^*, (C^T_{-j}(\tau^*, W^\circ_{-j}), C_j(\tau^*))] > \max_{\tau \in \mathcal{T}} G[\tau, (C_{-j}(\tau), 0)]$$

From the point of view of lobby j this is not the best response, as the lobby could switch to a lower (in a pointwise sense) contribution schedule without fear of retaliation, a net improvement. If, on the contrary, lobby j decides to lowball the policymaker, it is in the best interest of the policymaker to simply ignore this contribution schedule[6]. Lobby j can go as low as lobby $-j$ allows him to, and should not go any higher.

When the lobbying game is played it induces a new policy vector that distorts the underlying industry equilibrium. I refer to this new equilibrium as a political equilibrium, defined as follows:

Political Equilibrium: A political equilibrium is composed of a policy vector τ°, the *action* chosen by the policymaker in a TNE of the lobbying game (see definition ~ 3.4), and the low cutoff productivity $z^\circ \equiv z^*(\tau^\circ)$ induced by such policy.

Let us analyze the effect a political equilibrium has on the baseline economy. We start with the case of a single lobby. Let $\Pi_j(\tau)$ be the aggregate profits of members of lobby j and let $\Delta W(\tau^\circ) = W(1,1) - W(\tau^\circ) \geq 0$ be the change in consumer welfare generated induced by implementing policy vector τ°. The following proposition characterizes the TNE of the lobbying game when only lobby j participates.

Proposition 1 *Suppose only lobby j participates in the lobbying game. In a truthful nash equilibrium, the policymaker's welfare level remains unchanged and, if the policymaker is selfish enough, the lobby's equilibrium welfare level is strictly positive. Such a TNE is characterized by a pair (τ°, U°), composed of a policy vector and a welfare level, such that:*

$$\tau^\circ = \underset{\tau \in \mathcal{T}}{\arg\max}\{aW(\tau) + \Pi_j(\tau)\}$$

$$U^\circ = \Pi_j(\tau^\circ) - a\Delta W(\tau^\circ)$$

Proof of Proposition 1. Single lobby game TNE

See Appendix ~ B.3. ■

Proposition 1 can be used to derive the lobby's contribution schedule. Using the lobby's welfare function and the definition of a truthful equilibrium we show in

[6]Dixit et al. (1997) features a more in- depth discussion of the arguments behind this claim.

Appendix ~ B.3 that the equilibrium contribution schedule is:

$$C^T(\tau^\circ, U^\circ) = a\Delta W(\tau^\circ) \tag{6}$$

Equation ~ 6 states that the policymaker's welfare is not affected. This result is linked to the nature of Truthful Nash Equilibria - the lobby compensates the policymaker for every unit of welfare she loses when shifting the policy instrument away from its optimal laissez-faire status by providing her with just enough political contributions to maintain the same ex-ante welfare level. In effect, the lobby pays the policymaker to not have a conscience, but no more.

There are two possible scenarios, as either the small firm (S) lobby or the big firm lobby (B) can participate. In both scenarios there is a direct effect on the firm distribution, although the qualitative nature of this effect is different.

3.5 Small firm lobby

The small firm lobby exchanges political contributions for an increase in the entry fee, which shifts the cutoff productivity level to the left (see Figure ~ 1)

A classic example of this type of lobbying is the taxi market, ubiquitous in urban environments around the world. While there are some economic rationale to impose artificially high barriers to entry into this market (Schaller, 2007), the supply of taxi licences is highly regulated in a high number of urban areas, most notably New York City. Focusing on this case, a political economy model of the persistence of taxi medallions suggests that an industry wide lobby manages to sustain an artificially high barrier to entry by applying political pressure on policymakers (Wyman, 2013).

Before the advent of ride-sharing apps, taxi technology was largely similar, corresponding to the case where α is relatively high, with most firms (individual taxi drivers) exhibiting similar performance. This makes it relatively easy to coordinate and apply pressure to a policymaker, as most market participants want the same

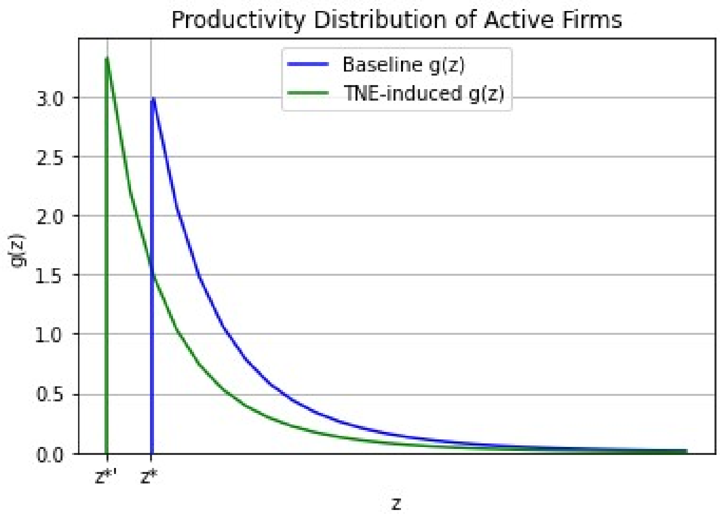

Figure 1: Firm productivity distribution shift under the influence of the small firm lobby. *Baseline parameter values:* $\delta = 0.15$, $\alpha = 4.25$, $\sigma = 5$, $f_e = f = 1$. In this example the TNE doubles τ_e.

policy outcome and there is no credible big firm special interest group - that is, until the advent of ride-sharing apps.

3.6 Big firm lobby

The lobby for big firms exchanges political contributions for an increase in the operating cost, which shifts the cutoff productivity level to the right.

Let us review an example from a slightly different perspective. Suppose a highly regulated industry with artificially high operating costs achieved by a big firm lobby already in a political equilibrium, and further suppose a new policymaker takes office. If the new policymaker is more benevolent then his predecessor we can expect the low cutoff productivity level to decrease, reducing the level of market concentration. As reviewed in Philippon (2019), during the deregulation episode experienced in the late 20$^{\text{th}}$ century in the American aviation industry, deregulation had a remarkably similar effect. Average price-per-mile flown decreased, new airlines

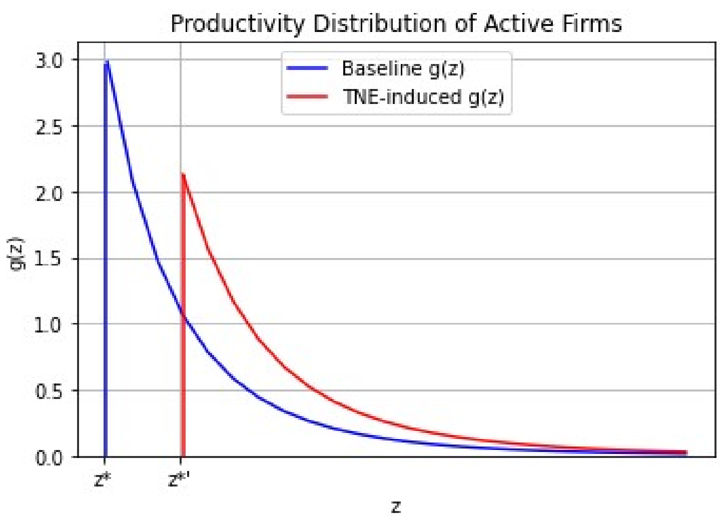

Figure 2: Firm productivity distribution shift in an industry equilibrium (blue) and in a political equilibrium (red). *Baseline parameter values:* $\delta = 0.15,\ \alpha = 4.25,\ \sigma = 5,\ f_e = f = 1$. In this example the TNE doubles τ_f.

entered the market and overall consumer welfare improved, implying that a sizable portion of regulation in place was benefitting airlines and not consumers, an indication of lobbying incentives.

3.7 Multiple Lobbies

Let $\Pi_B(\tau)$ and $\Pi_S(\tau)$ be, respectively, lobby B and lobby S aggregate profits. By definition their sum equals aggregate industry profits, $\Pi_B(\tau) + \Pi_S(\tau) = \Pi(\tau)$. As shown in Appendix ~ A.5, this quantity is independent of τ as long as the underlying industry equilibrium is well-defined. Each lobby tries to influence the policymaker to modify τ so as to increase the share of profits that accrue to its members. There is a key in this case, as now the policymaker can leverage the credible threat to ignore any one of them and only accept political contributions from the other. The next proposition and corollary characterize the game's TNE:

Proposition 2 *Multiple lobby game TNE*

Suppose $J = \{B, S\}$ is the set of lobbies participating in the lobbying game. Then

(i) The TNE policy vector τ° will be equal to the welfare maximizing policy vector $\tau^\circ = (1, 1)$.

(ii) The policymaker has a higher welfare level compared to the baseline case.

Proof of Proposition 2. Multiple lobby game TNE

See Appendix ~ B.4. ■

Corollary 1 *Let $(\tau^\circ, \{U_j^\circ\}_{j\in J})$ be the TNE of a lobbying game with both lobbies and let τ'' and τ' be such that:*

$$\tau' = \arg\max_{\tau\in\mathcal{T}}\{aW(\tau) + C_S^T(\tau, U_S^\circ)\}$$
$$\tau'' = \arg\max_{\tau\in\mathcal{T}}\{aW(\tau) + C_B^T(\tau, U_B^\circ)\}$$

Then the contribution schedules are such that:

$$C_S^T(\tau^\circ, U_S^\circ) = \Pi_S(\tau^\circ) - \Pi_S(\tau'') - a[W^\circ - W(\tau'')]$$
$$C_B^T(\tau^\circ, U_B^\circ) = \Pi_B(\tau') - \Pi_B(\tau^\circ) - a[W^\circ - W(\tau')]$$

Proof of Corollary 1. Contribution schedules under multiple lobbies

See Appendix ~ B.4. ■

Let us focus on the right hand side of the TNE contributions from the lobby for small firms. The difference of the first two terms corresponds to the potential loss of aggregate profit small firms would suffer if only the big firm lobby participated. Recall that the potential loss it faces is also decreasing in a. For a sufficiently selfish policymaker, the political contributions cap becomes binding, since as $a \to 0$ the potential loss it faces is the totality of its profit share. The next term corresponds to the aggregate welfare loss the policymaker is willing to tolerate, which is exactly the level of contributions the policymaker would receive from the other lobby.

On the other hand, the big firm lobby does not face the same threat, as the potential loss it faces is simply the added aggregate profits low productivity firms would capture if $z^* \to 1$. It is the underlying economic structure, namely the level of market concentration α, the elasticity of substitution between varieties σ and the industry's "natural" cost structure f_e, f that dictate the ex-ante low cutoff productivity, the high productivity threshold z_H and their ratio, thus dictating each lobby's relative market share and the a priori magnitude of the incentive to lobby. These fundamentals, together with the degree of benevolence a, determine the level of political contributions from each lobby by delimiting the risk associated with not lobbying. Thus, in a highly competitive market there is barely any incentive to lobby individually, as either the potential benefit is relatively low, the risk associated with not lobbying is relatively low, or most firms share the same target policy in the first place.

To see this more clearly, note that contributions are a weighted sum of losses: the lobbies contribute as much as they are risking to lose minus the amount of welfare the policymaker would lose, the second difference term, weighted by the policymaker's degree of benevolence. Because contributions are bound from below by zero, the amount at stake (for the lobby) if it decides not to participate must be higher than what the policymaker would lose if the lobby did not participate, which is the amount of political contributions he would receive anyway from the other lobby. Hence, the mere presence of a second lobby completely changes the game's structure: because each lobby has opposite, conflicting goals, the mere possibility of lobby $-j$ participating in the game introduces a new dimension in the strategy set.

Let us retake the first scenario, where the small-firm lobby is by itself. Suppose that a technological breakthrough occurs and a new, highly productive firm manages to enter the market and decides to form a lobby. From the policymaker's point of view there is a new outside option - which she can leverage in her interaction with the small loby. Hence, the small firm lobby now has to match this new outside option, at the expense of its previous rents, or risk the potentially devastating new policy vector the policymaker could set by only accepting political contributions from the newly-formed lobby.

Ride-sharing mobile applications are a great real-world anecdote of this happening. Since the inception of Uber (the most widely used ride-hailing mobile application), the price of taxi licences, the main barrier to entry in the taxi industry, has plummeted (Martini, 2017). Lobbying against the status quo is part of Uber's business. The effect on consumer behaviour has been remarkable, with both total number of rides and uber's share of the market steadily increasing between 2013 and 2015[7].

Another illustrative example is the 2018 legislative episode in the U.S. Congress regarding a modification of the Dodd–Frank Wall Street Reform and Consumer Protection Act, originally enacted to protect consumers from financial institutions' irresponsible behaviour (Miller and Ruane, 2012). During the rewriting procedures, small banks actively fought and lobbied against their big counterparts, reflecting the conflicting nature of the multiple lobby game [8].

4. Concluding Remarks

Researchers interested in understanding how economic forces shape policy outcomes should account for political influence in their study. In this paper, we have described a mechanism through which political forces can shape the economic environment in which firms operate. Our analysis provides a sharp prediction: lobbying can have tangible effects on economic aggregates. Indeed, in the two single lobby cases considered, consumer welfare was negatively affected. While that

[7] For anecdotical background see these articles in The Economist:
https://www.economist.com/graphic-detail/2015/08/10/substitutes-or-complements
https://www.economist.com/united-states/2015/08/15/a-tale-of-two-cities

[8] "Small banks trump Wall Street on Dodd-Frank rewrite", retrieved from https://www.reuters.com/article/us-usa-house-banks-lobbying/small-banks-trump-wall-street-on-dodd-frank-rewrite-idUSKCN1IN328 on 2021-25-11

was not the case in the third case considered, the multiple lobby game, this result hinges on the self-similar property of the Pareto distribution, not on the lobbying process itself.

In a more positive note, an implicit policy implication is how lobbying can be beneficial in specific circumstances, namely if it is able to counteract entrenched rent-seeking[9].

Legislative outcomes are very complex and neither the single lobby game nor its multiple lobbies counterpart necessarily map to a single or two opposing lobby(ies). Nevertheless, our approach exemplifies the importance of heterogeneity, as even within narrowly defined industries, SIGs can, and often do, operate with opposing policy goals in mind (Egerod and Junk, 2000).

A remarkable fact is that PACs and lobbies represent a relatively modest amount of total campaign funding (Ansolabehere et al., 2003). This suggests politicians may experience hidden costs to political contributions. A potential question to ask is what influences the policymaker's weight on political donations.

Two other potential and highly relevant venues for future work within the same research line are trade-related lobbying, and the effect lobbying has on long-run outcomes such as growth, income and wealth inequality and technological or institutional development (Philippon, 2019).

Statements and Declarations

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[9]See, for example, the case of ride- sharing apps versus taxis in the U.S. in Tzur (2019) and Collier et al. (2018).

Competing Interests

The authors have no competing interests to declare that are relevant to the content of this article.

References

Ansolabehere, S., J. M. de Figueiredo, and J. M. Snyder (2003). “Why Is There so Little Money in U.S. Politics?” In: *The Journal of Economic Perspectives* 17.1, pp. 105–130.

Baron, D. and A. Hirsch (2012). “Common agency lobbying over coalitions and policy”. In: *Econ Theory* 49, pp. 639–681.

Bernheim, B. D. and M. D. Whinston (1986). “Menu Auctions, Resource Allocation, and Economic Influence”. In: *The Quarterly Journal of Economics* 101.1, pp. 1–31.

Bombardini, M. (2008). “Firm heterogeneity and lobby participation”. In: *Journal of International Economics* 75.2, pp. 329–348.

Bombardini, M. and F. Trebbi (2012). “Competition and political organization: Together or alone in lobbying for trade policy?” In: *Journal of International Economics* 87.1. Symposium on the Global Dimensions of the Financial Crisis, pp. 18–26.

Bombardini, M. and F. Trebbi (Sept. 2019). *Empirical Models of Lobbying*. Working Paper 26287. National Bureau of Economic Research.

Chambers, C. P. (2007). “Citizen-candidates, lobbies, and strategic campaigning”. In: *Econ Theory* 33, pp. 285–309.

Collier, R. B., V. Dubal, and C. L. Carter (2018). “Disrupting Regulation, Regulating Disruption: The Politics of Uber in the United States”. In: *Perspectives on Politics* 16.4, pp. 919–937.

Cooper, M. J., H. Gulen, and A. V. Ovtchinnikov (2010). “Corporate Political Contributions and Stock Returns”. In: *The Journal of Finance* 65.2, pp. 687–724. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/j.1540-6261.2009.01548.x.

Costinot, A., A. Rodríguez-Clare, and I. Werning (2020). “Micro to Macro: Optimal Trade Policy With Firm Heterogeneity”. In: *Econometrica* 88.6, pp. 2739–2776.

Covarrubias, M., G. Gutiérrez, and T. Philippon (2020). “From Good to Bad Concentration? US Industries over the Past 30 Years”. In: *NBER Macroeconomics Annual* 34, pp. 1–46.

Decker, R. A., J. Haltiwanger, R. S. Jarmin, and J. Miranda (May 2016). “Declining Business Dynamism: What We Know and the Way Forward”. In: *American Economic Review* 106.5, pp. 203–07.

Dingel, J. I. (June 2009). *The basics of “Dixit-Stiglitz lite”*. Lecture Note. Columbia University.

Dixit, A. K., G. M. Grossman, and E. Helpman (1997). “Common Agency and Coordination: General Theory and Application to Government Policy Making”. In: *Journal of Political Economy* 105.4, pp. 752–769. eprint: https://doi.org/10.1086/262092.

Dixit, A. K. and J. E. Stiglitz (1977). “Monopolistic Competition and Optimum Product Diversity”. In: *The American Economic Review* 67.3, pp. 297–308.

Egerod, B. C. K. and W. M. Junk (2000). “Competitive lobbying in the influence production process and the use of spatial econometrics in lobbying research”. In: *Public Choice* 82, pp. 193–215.

Evers-Hillstrom, K. (Jan. 2020). *More money, less transparency:A decade under Citizens United*. Report. OpenSecrets.org.

Gabaix, X. (2009). “Power Laws in Economics and Finance”. In: *Annual Review of Economics* 1, pp. 255–293.

Gabaix, X. (Feb. 2016). “Power Laws in Economics: An Introduction”. In: *Journal of Economic Perspectives* 30.1, pp. 185–206.

Gawande, K. and U. Bandyopadhyay (2000). "Is Protection for Sale? Evidence on the Grossman-Helpman Theory of Endogenous Protection". In: *The Review of Economics and Statistics* 82.1, pp. 139–152.

Grossman, G. M. and E. Helpman (1994). "Protection for Sale". In: *The American Economic Review* 84.4, pp. 833–850.

Helpman, E., M. J. Melitz, and S. R. Yeaple (Mar. 2004). "Export Versus FDI with Heterogeneous Firms". In: *American Economic Review* 94.1, pp. 300–316.

Hillman, A. L. and H. W. Ursprung (1988). "Domestic Politics, Foreign Interests, and International Trade Policy". In: *The American Economic Review* 78.4, pp. 729–745.

Hopenhayn, H. A. (2014). "Firms, Misallocation, and Aggregate Productivity: A Review". In: *Annual Review of Economics* 6.1, pp. 735–770.

Hopenhayn, H., J. Neira, and R. Singhania (2022). "From Population Growth to Firm Demographics: Implications for Concentration, Entrepreneurship and the Labor Share". In: *Econometrica* 90.4, pp. 1879–1914. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.3982/ECTA18012.

Huneeus, F. and I. S. Kim (Aug. 2021). *The Effects of Firms' Lobbying on Resource Misallocation.* Working Papers Central Bank of Chile 920. Central Bank of Chile.

Jung, B. (2012). "Optimal fixed cost subsidies in Melitz-type models". In: *Empirica* 39.1, pp. 87–108.

Kondo, I., L. T. Lewis, and A. Stella (July 2021). *Heavy Tailed, but not Zipf: Firm and Establishment Size in the U.S.* Working Paper 21-15. US Census Bureau.

Martini, D. J. (2017). "International Regulatory Entrepreneurship: Uber's Battle with Regulators in France". In: *San Diego International Law Journal* 19.1, p. 127.

Melitz, M. J. (2003). "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity". In: *Econometrica* 71.6, pp. 1695–1725. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/1468-0262.00467.

Melitz, M. J. and S. J. Redding (Apr. 2013). *Firm Heterogeneity and Aggregate Welfare.* CEP Discussion Papers dp1200. Centre for Economic Performance, LSE.

Melitz, M. J. and S. J. Redding (Mar. 2015). “New Trade Models, New Welfare Implications”. In: *American Economic Review* 105.3, pp. 1105–46.

Miller, R. S. and K. A. Ruane (Nov. 2012). *The Dodd-Frank Wall Street Reform and Consumer Protection Act: Title VII, Derivatives*. CRS Report for Congress R41398. Congressional Research Service.

Philippon, T. (2019). *The Great Reversal: How America Gave Up on Free Markets*. Harvard University Press.

Plouffe, M. (2012). “Liberalization for Sale: Heterogeneous Firms and Lobbying Over FTAs”. In: *Revision of paper prepared for the 2012 APSA annual meeting*. New York, NY: SSRN.

Rossi-Hansberg, E. and M. L. J. Wright (Dec. 2007). “Establishment Size Dynamics in the Aggregate Economy”. In: *American Economic Review* 97.5, pp. 1639–1666.

Schaller, B. (2007). “Entry controls in taxi regulation: Implications of US and Canadian experience for taxi regulation and deregulation”. In: *Transport Policy* 14.6, pp. 490–506.

Tzur, A. (2019). “Uber Über regulation? Regulatory change following the emergence of new technologies in the taxi market”. In: *Regulation & Governance* 13.3, pp. 340–361. eprint: https://onlinelibrary.wiley.com/doi/pdf/10.1111/rego.12170.

Wyman, K. M. (2013). “Problematic Private Property: The Case of New York Taxicab Medallions”. In: *Yale Journal on Regulation* 30.1.

A. Appendix A: The Economic Environment

A.1 Demand in the Benchmark Economy

Let us review the basic results in this economy. Further discussion can be found on Dixit and Stiglitz (1977), Dingel (2009), and Melitz (2003). The representative agent’s preferences over varieties $\omega \in \Omega$ are represented by the following utility funtion

$$U = \left[\int_{\Omega} q(\omega)^{\rho} d\omega \right]^{1/\rho}$$

The representative agent maximizes her utility subject to her budget constraint. Let I be the agent's income and define the price index P in the usual way:

$$P = \left[\int_{\Omega} p(\omega)^{1-\sigma} d\omega \right]^{1/1-\sigma}$$

where $\sigma = \frac{1}{1-\rho} > 1$[10] is the elasticity of substitutions between varieties. Utility maximization implies the budget constraint is binding, total consumer income will be her total expenditure and hence $I = \int_{\Omega} p(\omega) q(\omega) d\omega = PQ$, where $Q = U$ is a composite good with price P. Solving for each variety's optimal level of consumption yields their demand curve as a function of its price as shown in equation ~ 1. It also yields expenditure level for each variety:

$$r(\omega) = I \left[\frac{p(\omega)}{P} \right]^{1-\sigma} \tag{7}$$

A.2 Production in the Benchmark Economy

Each variety is produced by a single firm and each firm produces a single variety. Firms are heterogeneous in their productivity z, which is reflected in their labor demand function $n(q,z) = \frac{q}{z} + \tau_f f$, where τ_f is a policy instrument and f is an industry wide operating cost. Let w be the wage rate. The profit maximizing problem of the firm is

$$\max_{p} \pi(z) = pq(p) - wn(q(p))$$

Taking first order conditions on the previous programme yields the pricing rule each firm follows as a function of its individual productivity:

[10] Conversively, $\rho = \frac{\sigma-1}{\sigma}$

$$p(z) = \frac{w}{\rho z}$$

Let $q(z) \equiv q(p(z)), n(z) \equiv n(q(z), z), r(z) \equiv p(z)q(z)$ and $\pi(z) \equiv r(z) - wn(z)$ respectively denote output, labor demand, revenue and profits of a firm with productivity z. . To see this, take individual firms' demand (equation ~ 1) and revenue (equation ~ 7), plug in the optimal pricing rule (equation ~ 2) and consider any two arbitrary firms with productivities $z_1 > z_2$. Taking the ratio of their output and revenue we can see that the firm arbitrary more productive firm will produce and sell more:

$$q(z_1) = \left[\frac{z_1}{z_2}\right]^{\sigma} q(z_2) > q(z_2), \quad r(z_1) = \left[\frac{z_1}{z_2}\right]^{\sigma-1} r(z_2) > r(z_2)$$

Where in both cases the inequality comes from the fact that $\sigma - 1 > 0$.

A.3 Aggregates in the Benchmark Economy

Define $\tilde{z}$ by

$$\tilde{z} = \left[\int_{Z} z^{\sigma-1} dg(z)\right]^{1/\sigma-1} \tag{8}$$

As in Melitz (2003), we use $\tilde{z}$ as a notion of average industry productivity. We also use it to express the price index and the other aggregate quantities in short expressions. Recall that the price index is integrating over all available varieties. We instead integrate over the set of incumbent productivities. Since every variety is manufactured using a specific productivity, the integral is over the support of z using the density $g(z)$:

$$P = \left[\int_{\Omega} p(\omega)^{1-\sigma} d\omega\right]^{1/1-\sigma} = \left[\int_{Z} p(z)^{1-\sigma} M dg\right]^{1/(1-\sigma)}$$

where M denotes the total mass of incumbent firms. Substituting the inside of the second integral and rearranging suffices to arrive at the expression. The expressions for average firm revenue $\tilde{r}$ and average firm profit $\tilde{\pi}$ are derived the same way.

A.4 Equilibrium in the Benchmark Economy

The value of an active firm is defined as its net present value at entry:

$$v(z) = \sum_{t=0}^{\infty}(1-\delta)^t \pi(z) = \frac{\pi(z)}{\delta}$$

The lack of time discounting is for simplicity as the probability of death each period effectively discounts future profits.

The cutoff productivity level $z^* = \inf\{z : v(z) > 0\}$ is the lowest level of productivity so that firm value (and profit) is bigger than zero. As claimed, it can be used to characterize the industry's productivity distribution $g(z)$. The equilibrium productivity distribution of incumbents $g(z)$ is thus a scaled version of the exogenous productivity distribution of potential entrants $h(z)$:

$$g(z) = \begin{cases} \frac{h(z)}{1-H(z^*)} & z \geq z^* \\ 0 & z < z^* \end{cases}$$

Profits can be rearrange to $\pi(z) = r(z)/\sigma - w\tau_f f$ (To see this plug the definition of demand and labor demand into the definition of profits $\pi(z) = p(z)q(z) - wn(z)$, note that $\frac{1}{\sigma} = 1-\rho$ and rearrange). By definition $\pi(z^*) = 0$ and thus $r(z^*) = \sigma w \tau_f f$. Using this information, $\tilde{z}, \tilde{\pi}$ and $\tilde{r}$ can be written in terms of the low cutoff productivity z^*:

$$\tilde{z} = \left[\frac{1}{1 - H(z^*)} \int_Z z^{\sigma-1} dh \right]^{1/\sigma-1}$$
$$\tilde{r} = \left[\frac{\tilde{z}(z^*)}{z^*} \right]^{\sigma-1} r(z^*)$$

and thus average firm profit can be expressed as a function of z^*:

$$\tilde{\pi} = \frac{\tilde{r}}{\sigma} - w\tau_f f = \left[\frac{\tilde{z}(z^*)}{z^*} \right]^{\sigma-1} \frac{r(z^*)}{\sigma} - w\tau_f f = w\tau_f f k(z^*) \tag{9}$$

where $k(z^*) = \left[\frac{\tilde{z}(z^*)}{z^*} \right]^{\sigma-1} - 1$. Equation ~ 9 is the previously-mentioned zero cutoff profit condition.

To pinpoint the equilibrium cutoff productivity z^* another condition is needed, namely free entry. Free entry implies that the expected value of creating a new firm must be equal to zero. Let $\tilde{v} = \frac{\tilde{\pi}}{\delta}$ denote the expected value of successful entry and let p_{en} be the associated probability. Then:

$$p_{en}\tilde{v} = [1 - H(z^*)] \frac{\tilde{\pi}}{\delta} = w\tau_e f_e$$
$$\tilde{\pi} = \frac{\delta w \tau_e f_e}{1 - H(z^*)} \tag{10}$$

Equation ~ 10 is the free entry condition, which, independently from the zero cutoff profit condition links average firm profit and the low cutoff productivity level. Define $\zeta(z) \equiv k(z)[1 - H(z)]$. In equilibrium the low cutoff productivity adjusts so that both conditions hold:

$$\tilde{\pi} = w\tau_f f k(z^*) = \frac{w\delta\tau_e f_e}{1 - H(z^*)}$$

Which implicitely defines the equilibrium low cutoff productivity level by

$$z^* = \zeta^{-1}\left(\frac{\delta\tau_e f_e}{\tau_f f}\right)$$

As shown in the appendix of Melitz (2003), $\zeta(z)$ is strictly decreasing in Z and thus is invertible, only leaving the issue of existence to address. While it might be the case that the point where the inverse function ζ^{-1} is being evaluated is not on its domain, this question depends on the specification of $H(z)$ and thus for the moment we assume z^* exists.

Let us now analyze an industry equilibrium in the absence of lobbying. Each period δM firms die, potential entrants pay the entry fee and, if their productivity is high enough, they start operations. Denote the mass of potential entrants by M_e. Successful entrants are those who have a higher productivity than z^*, that is, $[1 - H(z^*)]M_e$ successfully enter the market. Stability of the firm distribution implies that $[1-H(z^*)]M_e = \delta M$. This flux of firms entering and exiting at the same time does not influence $g(z)$, since both groups of firms have the same productivity distribution. Let $N_e = M_e\tau_e f_e$ be aggregate entry investment paid by entrants in terms of labor. The free entry condition (equation ~ 10) implies that total payments to entry labor equal aggregate industry profit:

$$wN_e = wM_e\tau_e f_e = \frac{\delta M}{1-H(z^*)} w\tau_e f_e = M\tilde{\pi} = \Pi$$

Where the last equation comes from the definition of $\tilde{\pi}$. The only resource constraint in this economy is the economy's labor force N, allocated between production and investment labor. Let N_p denote the total amount of labor allocated towards production, such that $N = N_p + N_e$. By definition, aggregate industry profits must equal the difference between total industry revenue and total payments to labor:

$$\Pi = I - wN_p \iff I = w(N_p + N_e)$$

Hence, total worker income is equal to total industry expenditure in labor. The price index and the equilibrium mass of firms can be written in terms of the wage rate w, total labor supply N, industry's per-period fixed cost f and the elasticity of substitution σ:

$$M = \frac{wN}{\sigma[\tilde{\pi} + w\tau_f f]}, \quad P = \frac{wM^{1/(1-\sigma)}}{\rho\tilde{z}(z^*)} = \frac{w}{\rho z^*}\left[\frac{N}{\sigma\tau_f f}\right]^{1/(1-\sigma)}$$

Per-capita welfare defined as wages' purchase power:

$$W = wP^{-1} = \left[\frac{N}{\sigma\tau_f f}\right]^{1/(\sigma-1)} \rho z^*$$

For the rest of the appendix we set $w = 1$.

A.5 Introducing the Pareto Productivity Distribution

We follow the common approach of assuming a Pareto distribution for the pool of potential entrants (Helpman et al., 2004) with parameter $\alpha > (\sigma - 1)$[11]. The support for z is then $[1, \infty)$. The parameter α defines the "thickness" of the distribution's right tail. The cumulative distribution function is:

$$H(z) = \begin{cases} 1 - z^{-\alpha} & 1 \le z \\ 0 & z < 1 \end{cases}$$

The associated density is $h(z) = \alpha z^{-(\alpha+1)}$. We can now evaluate the integral in the definition of average industry productivity in equation ~ 8:

$$\tilde{z}(z^*) = \left[\frac{\alpha}{\alpha - \sigma + 1}\right]^{1/(\sigma-1)} (z^*)$$

[11]The requirement on α is in order to guarante the existence of average firm productivity $\tilde{z}$.

Implicitely, $\tilde{z}$ depends on τ through z^*. We use this expression to evaluate $k(z)$ and subsequently solve for $z^*(\tau)$:

$$k(z) = \left[\frac{\tilde{z}(z)}{z}\right]^{\sigma-1} - 1 = \frac{\alpha}{\alpha+1-\sigma} - 1 = \frac{\sigma-1}{\alpha+1-\sigma}$$
$$\zeta(z) = k(z)[1-H(z)] = \left(\frac{\sigma-1}{\alpha+1-\sigma}\right) z^{-\alpha}$$
$$\zeta(z^*) = \left(\frac{\delta\tau_e f_e}{\tau_f f}\right) \implies z^*(\tau) = k_0 \left(\frac{\tau_f}{\tau_e}\right)^{1/\alpha}$$

where $k_0 = \left[\frac{f(\sigma-1)}{\delta f_e(\alpha+1-\sigma)}\right]^{1/\alpha} > 0$ is a constant made u of the model's fundamentals. We state a lemma regarding these fundamentals and the existence of an industry equilibrium under a Pareto distribution:

Lemma 1 *Let the productivity distribution of potential entrants be a Pareto distribution with shape parameter $\alpha > \sigma - 1$. Then a sufficient condition for an industry equilibrium pair $(\tilde{\pi}, z^*)$ to exist is:*

$$\left[\frac{(\sigma-1)f}{(\alpha+1-\sigma)\delta f_e}\right] \geq \frac{\tau_e}{\tau_f}$$

which is equivalent to requiring the equilibrium low cutoff productivity z^* be within the support of H(z). In particular, for the equilibrium pair $(\tilde{\pi}, z^*)$ to exist when $\tau = (1,1)$ it must be that:

$$\left[\frac{(\sigma-1)f}{(\alpha+1-\sigma)\delta f_e}\right] \geq 1$$

Proof of Lemma 1. Existence of an industry equilibrium under a Pareto distribution

The equilibrium pair is realized at the point, in $(\tilde{\pi}, z^*)$- space, where both the zero cutoff profit (equation ~ 10) and the free entry (equation ~ 9) conditions hold. The zero cutoff profit curve must be intercepted from below by the free entry condition curve, which is a strictly increasing function of z^*. For this to happen it must be that equation ~ 10 is smaller or equal than equation ~ 9 evaluated at the lowest z^* possible, namely at the boundary of the support of $H(z)$, $[1,\infty)$:

$$\tilde{\pi}_{FE} = \tau_e f_e \delta \, (z^*)^\alpha \big|_{z^*=1} = \tau_e f_e \delta \leq \frac{(\sigma-1)\tau_f f}{(\alpha+1-\sigma)} = \tilde{\pi}_{ZCP}$$

Rearranging is enough to prove the first claim and plugging in the welfare maximizing policy $\tau = (1,1)$ proves the second. ■

Lemma ~ 1 helps understand the set of feasible policy instruments. The low productivity threshold determines the equilibrium distribution of productivities and thus the aggregate quantities of interest in the economy. By assuming firm productivity is Pareto-distributed we are making an implicit assumption on the support of z and thus must ensure that the resulting z^* is within this support.

Now, for completeness, let us substitute z^* into the expressions derived for the mass of active firms M, the industry's profit level Π and aggregate welfare W:

$$M = \frac{(\alpha+1-\sigma)N}{\alpha\sigma\tau_f f}, \;\; \Pi = \frac{\rho N}{\alpha}, \;\; W = \left[\frac{N}{\sigma f}\right]^{-1/(\sigma-1)} \rho k_o \left(\frac{\tau_f^{\frac{1}{\sigma-1}}}{\tau_e}\right)^{1/\alpha}$$

A.6 Optimality of a Laissez-faire Policy

We now tackle the issue of how the policymaker sets $\tau = (\tau_e, \tau_f)$ in the absence of lobbying. We follow GH and let go of the assumption of a completely benevolent policymaker. Instead, we assume it has an objective function of the form:

$$G(\tau, \boldsymbol{C}) = aW(\tau) + \sum_{j\in J} C_j$$

Political contributions sum to 0 in the absence of lobbying and thus the policy-maker is de-facto benevolent:

$$G(\tau, \boldsymbol{C}) = aW(\tau)$$

To see that $\tau = (1,1)$ is the optimal policy vector note that if we were to allow for subsidies, the problem is exactly the same as that in Jung (2012) (see Proposition 4), where the author shows that in the close economy it is not welfare maximizing to subsidize neither the entry nor the operating costs. Optimality of $\tau = (1,1)$ follows by showing that $W(\tau)$ is decreasing in both arguments. Since we are working under the assumption that productivity follows a pareto distribution, we know that the equilibrium low cutoff productivity z^* is:

$$z^*(\tau) = k_0 \left(\frac{\tau_f}{\tau_e}\right)^{1/\alpha}$$

Plugging this into the expression for W from Appendix ~ A.5 we see that indeed welfare is decreasing in both τ_e and τ_f.

B. Appendix B: The Lobbying Game

B.1 Lobby Threshold productivity

Let z and z^* be respectively the productivity of an arbitrary incumbent firm and the low cutoff productivity, noting that by definition $z \geq z^*$. A firm with productivity level z has profits:

$$\pi(z) = \tau_f f \left[\left(\frac{z}{z^*}\right)^{\sigma-1} - 1\right]$$

Taking derivatives with respect to τ_f:

$$\begin{aligned}
\frac{\partial \pi}{\partial \tau_f} &= f\left[\left(\frac{z}{z^*}\right)^{\sigma-1} - 1\right] + \tau_f f z^{\sigma-1}(1-\sigma)(z^*)^{-\sigma}\frac{\partial z^*}{\partial \tau_f} \\
&= f\left(\frac{z}{z^*}\right)^{\sigma-1} - f - \tau_f f(\sigma-1) z^{\sigma-1}(z^*)^{-\sigma}\frac{\partial z^*}{\partial \tau_f}\frac{z^*}{z^*} \\
&= f\left[\left(\frac{z}{z^*}\right)^{\sigma-1}(1-\varepsilon_{\tau_f}^{z^*}(\sigma-1)) - 1\right]
\end{aligned}$$

where

$$\varepsilon_{\tau_f}^{z^*} = \frac{\partial z^*}{\partial \tau_f}\frac{\tau_f}{z^*}$$

The derivative's sign depends on the expression inside the brackets. Suppose that $\varepsilon_{\tau_f}^{z^*}(\sigma-1) > 1$. Then firm profits are decreasing in τ_f at all productivity levels. Alternatively, suppose that $\varepsilon_{\tau_f}^{z^*}(\sigma-1) < 1$. We are interested in locating the productivity level at which profits become increasing in τ_f. Let that productivity level be z_H - it is cleat that at that point the derivative becomes zero:

$$\begin{aligned}
\left(\frac{z_H}{z^*}\right)^{\sigma-1}(1-\varepsilon_{\tau_f}^{z^*}(\sigma-1)) - 1 &= 0 \\
\left(\frac{z_H}{z^*}\right)^{\sigma-1} &= \frac{1}{1-\varepsilon_{\tau_f}^{z^*}(\sigma-1)}
\end{aligned}$$

We know that $z_H \geq z^*$ and by assumption $\varepsilon_{\tau_f}^{z^*}(\sigma-1) < 1$, implying that for all $z > z_H$ ($z^* \leq z < z_H$), profits are increasing (decreasing) in τ_f. Under the Pareto assumption used in our paper we can show that the inequality $\varepsilon_{\tau_f}^{z^*}(\sigma-1) < 1$ does hold:

$$0 < \varepsilon_{\tau_f}^{z^*}(\sigma - 1) = \frac{\sigma - 1}{\alpha} < 1$$

The middle step follows from the definition of $\varepsilon_{\tau_f}^{z^*}$ and the equilibrium expression for z^* (Appendix ~ A.5). Hence in the baseline economy with a pareto productivity distribution we have the following relationship between the marginal incumbent (small) firm, and the marginal big firm:

$$\left(\frac{z_H}{z^*}\right)^{\sigma-1} = \frac{r(z_H)}{r(z^*)} = \frac{\alpha}{\alpha + 1 - \sigma} = \frac{1}{1 - \frac{\sigma-1}{\alpha}}$$

B.2 Equilibrium in the Lobbying Game

The set of admissible policy instruments is denoted by $\mathcal{T}$. Any $\tau \in \mathcal{T}$ must obey the following constraints:

i We do not allow for subsidies and hence limit our policy instruments to be bounded below by one.

ii Total labor supply is constant and thus total labor demand cannot exceed it.

iii The policy instrument must be such that an industry equilibrium as defined in the previous section is achieved.

The first constraint is for simplicity's sake. Any increase in either policy vector from (1,1) will result in higher labor demand from either production or investment and hence not revenue collected by the government. This avoids redistribution and the required modifications to the agent's budget constraint, so that we can focus on loss of welfare and changes in the firm distribution.

The second constraint is just formally stating the economy's labor constraint. Since the supply of labor is fixed, any policy change must shift labor between production and investment without increasing the total amount of labor demanded.

The third constraint is related to the choice of a pareto distribution and follows from Lemma 1, stating that the set of feasible policy instruments must respect it.

Best Response A feasible contribution schedule $C_j^{\circ}(\tau)$ and admissible policy vector τ° are said to be a best response to $C_{-j}^{\circ}(\tau)$ if

(i) $\tau^{\circ} \in \underset{\tau \in \boldsymbol{\mathcal{T}}}{\arg\max}\{G[\tau, \boldsymbol{C}^{\circ}(\tau)]\}$, where $\boldsymbol{C}^{\circ} = (C_j^{\circ}(\tau), C_{-j}^{\circ}(\tau))$

(ii) There is no other feasible contribution schedule $C_j(\tau)$ and admissible policy vector τ such that $U_j[\tau, C_j(\tau)] > U_j[\tau^{\circ}, C_j^{\circ}(\tau^{\circ})]$

and

$$\tau \in \underset{\tau \in \boldsymbol{\mathcal{T}}}{\arg\max}\{G[\tau, (C_{-j}^{\circ}(\tau), C_j(\tau))]\}$$

In the definition of a best response it is made explicit that although the contribution schedule presented by lobby j holds fixed the other lobby's contribution schedule, in a second stage the policymaker will optimize with respect to both of these and choose the policy instrument that maximizes his own objective function. Let us now introduce the relevant notion of equilibria for the lobbying game

Subgame Perfect Nash Equilibrium An equilibrium of the lobbying game consists of a vector of feasible contribution schedules $\boldsymbol{C}^{\circ}(\tau)$ and a policy vector $\tau^{\circ} \in \boldsymbol{\mathcal{T}}$ such that, for every lobby j, $C_j^{\circ}(\tau)$ and τ° are a best response to $C_{-j}^{\circ}(\tau)$.

Problematically, common agency problems can have a multiplicity of subgame perfect equilibria. This is the motivation behind the notion of TNE, introduced in the main text.

B.3 Proof of Proposition 1

Proof of Proposition 1. Single lobby game TNE

Let us first define lobby j's welfare function. We consider the case where the lobby maximizes the aggregate profits of its members net of contributions. Let Π_j denote aggregate profits of members of lobby j. Then the lobby's welfare function is given by:

$$U_j[\tau, C_j] = \Pi_j - C_j$$

A first step in proving **Proposition 1** is solving the lobbying game for the equilibrium policy vector and contribution level. Let us first focus on the policy vector. We can exploit the lobby welfare function's linearity in the contributions to use **Corollary 1** to **Proposition 4** of Dixit et al. (1997), which provides a close form expression for the agent's equilibrium action in a TNE:

$$\tau^\circ = \arg\max_{\tau \in \mathcal{T}} \{aW(\tau) + \Pi_j(\tau)\} \tag{11}$$

Directly as claimed in the proposition. The policy vector maximizes the joint welfare of the lobby and the consumer, with a representing the weight placed on consumer welfare by the policymaker. For the moment, suppose that an interior solution to the previous program exists.

Now let us denote the equilibrium welfare of the participating lobby by:

$$U^\circ \equiv U_j[\tau^\circ, C_j^T(\tau^\circ, U^\circ)]$$

Evaluating the truthful contribution schedule (definition ~ 3.4) at the equilibrium pair (τ°, U°) and rearranging yields[12]

$$\bar{C} \geq C_j^T(\tau^\circ, U^\circ) = \Pi_j(\tau^\circ) - U^\circ \geq 0$$

Define $\Delta W(\tau^\circ) \equiv [W(1,1) - W(\tau^\circ)]$, the loss of consumer welfare brought about by implementing τ°, and note that $\Delta W(\tau^\circ) \geq 0$. Using the optimality of the laissez-

[12] Note we are taking care of not violating the feasibility constraints the contribution schedule is subject to.

faire policy vector in the absence of lobbying and the characterization of a TNE presented in definition ~ 3.4, we know that

$$\max_{\tau\in\mathcal{T}}\{G(\tau,C_j)\}\Big|_{C_j=0} = \max_{\tau\in\mathcal{T}}\{aW(\tau)\} = aW(1,1) = G[\tau^\circ, C_j^T(\tau^\circ,U^\circ)]$$

Plugging in the expression for $C_j^T(\tau^\circ,U^\circ)$ into the policymaker's objective function in the rightmost expression from the previous equation and rearranging we have

$$aW(\tau^\circ) + \Pi_j(\tau^\circ) - U^\circ = aW(1,1)$$
$$U^\circ = \Pi_j(\tau^\circ) - a\Delta W(\tau^\circ)$$

The equilibrium contribution level is:

$$\bar{C} \geq C_j^T(\tau^\circ,U^\circ) = a\Delta W(\tau^\circ) \geq 0$$

We still have to show that τ° is an interior solution to the policymaker's program and that there is an upper bound on the level of selfishness $\bar{a}$ such that it is not profitable to engage in lobbying.

We focus on τ° first. Recall that M is the equilibrium mass of firms. By definition:

$$\Pi_j(\tau) = \int_j \pi(z) M \frac{dh(z)}{1-H(z^*)}$$

We have suppressed the dependance of z^* and $\pi(z)$ on τ for notational simplicity. We can show that under the Pareto distribution, total industry profit Π is constant and independent of τ, as long as an industry equilibrium is achieved[13]. Thus any change in aggregate profit that lobby j experiences must come form lobby $-j$. Evaluating the previous integral for each lobby yields:

$$\Pi_L(\tau) = \frac{(\alpha+1-\sigma)N}{\sigma}\left[\frac{\sigma-1}{\alpha(\alpha+1-\sigma)} + \frac{1}{\alpha}\left(\frac{z^*}{z_H}\right)^{\alpha} - \frac{1}{\alpha-(\sigma-1)}\left(\frac{z^*}{z_H}\right)^{\alpha-(\sigma-1)}\right]$$
$$\Pi_B(\tau) = \frac{(\alpha+1-\sigma)N}{\sigma}\left[\frac{1}{\alpha-(\sigma-1)}\left(\frac{z^*}{z_H}\right)^{\alpha-(\sigma-1)} - \frac{1}{\alpha}\left(\frac{z^*}{z_H}\right)^{\alpha}\right]$$

We see that Π_S, small firms' aggregate profits, is strictly decreasing in z^*, while the opposite holds true for Π_B. Note that τ° must be feasible: it must respect the resource constraint on labor, it must imply the resulting z^* is within the support of z and each policy instrument cannot be lower than one. Recall from Appendix ~ A.5 that z^* is decreasing in τ_e and increasing in τ_f, implying that each lobby will focus on lobbying for a change to a single policy instrument.

Let us analyze each lobby separately, starting with the small firm lobby, lobby S. The lobby will try to push τ_e as high as possible. Assuming the resource constraint is non-binding, first order conditions imply that either

$$\frac{\partial \Pi_S}{\partial \tau_e} = -a\frac{\partial W}{\partial \tau_e}$$

Or

$$\tau_e^\circ = \bar{\tau}_e$$

Where the top bound $\bar{\tau}_e \equiv (\sigma-1)f/(\alpha+1-\sigma)\delta f_e$ ensures that the resulting z^* is within the support of z and thus an industry equilibrium can be achieved.By construction, the resulting equilibrium policy instrument $\tau^\circ = (\tau_e^\circ, 1)$ is feasible:

[13] Appendix ~ A.5.

The resource constraint is already plugged-in into $W(\tau)$[14], the industry equilibrium can be achieved since the resulting policy respects the bounds set by the support of z by construction and of course subsidies are being avoided.

Now let us focus on the big firm lobby. The lobby's interest lies in making τ_f as high as necessary to push (potential) members of lobby S completely out of the market, which it perfectly achieves if $z^*(\tau^\circ) = z_H$. Hence, τ_f° is such that either:

$$\frac{\partial \Pi_B}{\partial \tau_f} = -a\frac{\partial W}{\partial \tau_f}$$

Or

$$z^*(\tau^\circ) = z_H$$

The derivative of Π_B with respect to z^* disappears at $z^* = z_H$, so unless the policymaker is perfectly selfish, it is the first condition that holds.

Note that by construction the policy vector chosen is feasible: the resource constraint has been substituted into $W(\tau)$ from the start, $z^*(\tau^\circ)$ is within the support of z and of course no subsidies are being given.

Now we show that lobbying is only profitable if the policymaker is selfish enough. Let $\Delta\Pi_j(\tau^\circ) \equiv \Pi_j(\tau^\circ) - \Pi_j(1,1)$ denote the change in aggregate profits experienced by members of lobby j. In the single lobby game, a perfectly selfish policymaker will try to maximize this quantity by adjusting τ, while a benevolent policymaker will not care about it and set $\tau = (1,1)$.

Change in lobby welfare is given by

$$\Delta U_j(\tau^\circ) \equiv \Delta\Pi_j(\tau^\circ) - a\Delta W(\tau^\circ) \tag{12}$$

The first element of the last equation's right hand side is increasing in τ_e for

[14]Indeed, the expression derived for M comes from the resource constraint. The price index P and W follow.

$j = L$ (increasing in τ_f for $j = H$), while $W(\tau)$ is decreasing in both arguments. Fix $\tau^\circ \neq (1,1)$ such that τ° is feasible, and without loss of generality suppose that $\Delta\Pi_j(\tau^\circ) > 0$. Then $\Delta U_j(\tau)\big|_{\tau=\tau^\circ} = \Delta U_j(a)$ is a continuous function of a with the followingcharacteristics:

$$\lim_{a\to\infty} \Delta U_j(a) = -\infty$$

$$\lim_{a\to 0} = \Delta\Pi_j(\tau^\circ)$$

By the Intermediate Value Theorem there is an $\bar{a}$ such that $\Delta U_j(\bar{a}) = 0$.

To conclude the proof, recall that the level of contributions is capped at $\bar{C}$. Now fix an arbitrary $a > 0$. As long as a is low enough, the cap on contributions is not violated, τ°, the solution to the program in equation ~ 11 is achieved and $\Delta U_j > 0$. ■

B.4 Proof of Proposition 2 and Corollary 1

Proof of Proposition 2. Multiple lobby game TNE

First let us show that the TNE policy vector in this case coincides with the welfare maximizing $(1,1)$. Recall that the objective function of lobby j is given by $U_j[\tau, C_j] = \Pi_j(\tau) - C_j$. Once again we exploit linearity in the contributions to use Corollary 1 to Proposition 4 of Dixit et al. (1997):

$$\tau^\circ = \arg\max_{\tau\in\boldsymbol{\tau}} \{aW(\tau) + \Pi_B(\tau) + \Pi_S(\tau)\}$$

$$= \arg\max_{\tau\in\boldsymbol{\tau}} \{aW(\tau) + \Pi\}$$

As shown in Appendix ~ A.5, aggregate industry profits are constant, as long as an industry equilibrium is attained. It follows that the previous program yields the same maximizing argument. Before showing that the policymaker is able to increase his welfare in the multiple lobbies game let us first prove the stated corollary about lobby contributions.

Condition (ii) of the TNE characterization presented in definition ~ 3.4 we know that each lobby non-cooperatively chooses its own ex-post welfare level U_j° such that the policymaker achieves the same welfare level whether the other lobby contributes or not:

- Lobby B chooses U_B° such that $\max_\tau\{aW(\tau)+C_S^T(\tau,U_S^\circ)\} = G^\circ[\tau^\circ,\{C_j^T(\tau^\circ,U_j^\circ)\}_{j\in J}]$
- Lobby S chooses U_S° such that $\max_\tau\{aW(\tau)+C_B^T(\tau,U_B^\circ)\} = G^\circ[\tau^\circ,\{C_j^T(\tau^\circ,U_j^\circ)\}_{j\in J}]$

Let τ',τ'' be, respectively, the maximizing argument in the first and in the second case above. Expanding each equation on both sides and rearranging yields the lobbies' ex-post welfare levels:

$$U_S^\circ = a[W^\circ - W(\tau'')] + \Pi_S(\tau'')$$
$$U_B^\circ = a[W^\circ - W(\tau'] + \Pi_B(\tau')$$

Plugging the previous expressions into the definition of the TNE contributions we see that

$$\begin{aligned}\bar{C} \geq C_S^T(\tau^\circ,U_S^\circ) &= \Pi_S(\tau^\circ) - U_S^\circ \\ &= \Pi_S(\tau^\circ) - \Pi_S(\tau'') - a[W^\circ - W(\tau'')] \geq 0 \\ \bar{C} \geq C_B^T(\tau^\circ,U_B^\circ) &= \Pi_B(\tau^\circ) - U_B^\circ \geq 0 \\ &= \Pi_B(\tau^\circ) - \Pi_B(\tau') - a[W^\circ - W(\tau')] \geq 0\end{aligned}$$

As stated in **Corollary 1**. We now have to show that the policymaker's welfare is higher in this TNE. To see that, let us expand G° with the expressions for equilibrium contributions we just derived:

$$G^{\circ}[\tau^{\circ}, \{C_j^T(\tau^{\circ}, U_j^{\circ})\}_{j\in J}] = aW(1,1) + \Pi - (U_S^{\circ} + U_B^{\circ}) \geq aW(1,1)$$

With a strict inequality whenever contributions are positive, namely whenever the policymaker is not completely benevolent. ■