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Metafrontier Productivity Indices: Questioning the Common Convexification Strategy

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Abstract

While the construction of metafrontiers based on the union of underlying group frontiers normally yields a non-convex metaset, a large majority in the literature seems to assume that a convexification strategy leads to a reasonable convex approximation of this non-convex metafrontier. However, Kerstens, O'Donnell, and Van de Woestyne (2019) recently deliver new results on the union operator on technologies under a variety of assumptions and empirically illustrate that such a convexification strategy is doubtful. The purpose of this contribution is to verify to which extent such a convexification strategy is tenable when computing the Malmquist and Hicks-Moorsteen productivity indices with respect to a metafrontier. This methodology is empirically applied on a secondary data set under a wide variety of assumptions: we explore balanced and unbalanced data as well as constant and variable returns to scale. Anticipating our key results, we do establish a potential bias of the convexification strategy for the metafrontier productivity indices.

Keywords: Data Envelopment Analysis, Metafrontier, Malmquist, Hicks-Moorsteen.

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1 Introduction

Different organisations across industries, regions and countries may face different production possibilities at a given point in time as well as over time. Heterogeneity in performance can be due to differences in available technologies (i.e., the ways inputs can be transformed into outputs) and/or to differences in environments (e.g., economic infrastructure, regulation, geography, climate, etc.). There have been a variety of alternative proposals around to account for heterogeneity in production models. Some rather popular methods include the use of latent class models (e.g., Orea and Kumbhakar (2004)), the aggregation over groups or industries (e.g., Mayer and Zelenyuk (2014), but see Balk (2016) for some caveats), among others. To the best of our knowledge, no theoretical or empirical review has ever compared these different methods to account for heterogeneity in production.

This contribution focuses on one particular method to account for heterogeneity when estimating production relations. One historically important literature was initiated by Hayami and Ruttan (1970) who proposed and estimated a kind of meta-production function. This meta-production function concept has been empirically applied mainly in agriculture and for country-level data: an empirical survey is found in Trueblood (1989). Hayami and Ruttan (1970, p. 898) "call the envelope of all known and potentially discoverable activities a secular or "meta-production function"." This secular production function indicates the maximum output obtainable from given inputs and from a given stock of knowledge. Thus, all organisations have access to the same set of input-output combinations, but each may choose a different input-output combination from that set depending on specific circumstances (e.g., regulation, relative prices, etc.). Some of this literature takes the possibility of inefficiency into account (e.g., Lau and Yotopoulos (1989)).

These basic ideas have initially been transposed into a stochastic production frontier framework by Battese and Rao (2002) and Battese, Rao, and O'Donnell (2004). Thereafter, O'Donnell, Rao, and Battese (2008) refined the loose ends in the methodology and finalised the formal metafrontier framework for making efficiency comparisons across groups of firms using both stochastic frontier analysis and nonparametric deterministic frontiers. This seminal article defines a meta-production possibility set (or metaset) as the union of underlying group-specific sets. These authors refer to the boundary of the metaset as a metafrontier, and to the boundaries of the group-specific sets as group-specific frontiers (or group frontiers).

This so-called metafrontier approach has meanwhile been amply applied across sectors and disciplines. Examples are production studies from agriculture (e.g., Latruffe, Fogarasi, and Desjeux (2012)), banking (e.g., Casu, Ferrari, and Zhao (2013)), fisheries (e.g., Lee and Midani (2015)), hotels (e.g.,Huang, Ting, Lin, and Lin (2013)), schools (e.g., Thieme, Prior, and Tortosa-Ausina (2013)), and wastewater treatment plants (e.g., Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)) to name but a few. This basic metafrontier concept has found its way in a variety of other literatures: one example is its transposition to a cost frontier framework (e.g., Huang and Fu (2013)); another example is the computation of productivity indices relative to metafrontiers (see, e.g., Casu, Ferrari, and Zhao (2013) and Huang, Juo, and Fu (2015) for a primal respectively a dual Malmquist index); a final example is the development of more elaborate efficiency decompositions (see Kounetas, Mourtos, and Tsekouras (2009) and Tsekouras, Chatzistamoulou, and Kounetas (2017)).

Basic group-specific frontier models tend to make a series of standard assumptions, one of which is convexity. This convexity assumption can only be justified by a time divisibility argument (see Shephard (1970, p. 15) and Hackman (2008, p. 39)). But, even if group-specific sets are convex, then the metaset defined by their union is normally nonconvex (see O'Donnell, Rao, and Battese (2008)). Despite this basic mathematical fact that convex group-specific sets yield a nonconvex metaset, the seminal article of O'Donnell, Rao, and Battese (2008) adopts a convexification strategy by estimating the metafrontier as a boundary of a convex metaset (see also, e.g., Battese and Rao (2002) and Battese, Rao, and O'Donnell (2004)). Since this convexification strategy is normally not true, estimates of the metafrontier are potentially biased. While the large majority of articles adopting a metafrontier approach seem to follow such a convexification strategy, one should stress that some articles do not adopt such a strategy: examples include Huang, Ting, Lin, and Lin (2013), Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)), Tiedemann, Francksen, and Latacz-Lohmann (2011), and (partially) Walheer (2018) among others. Kerstens, O'Donnell, and Van de Woestyne (2019) elaborate on the union operation on technologies under various assumptions and find empirically convincing evidence that a convexification strategy leads to statistically significant biases.

The purpose of this contribution is to investigate the impact of a convexification strategy on the estimation of metafrontier-based productivity indices. A variety of productivity indices have been computed using metafrontiers: examples include the popular primal Malmquist index (e.g., Casu, Ferrari, and Zhao (2013)) as well as the primal Luenberger indicator (e.g., Zhang and Wang (2015)), the dual (most often cost-based) Malmquist index (e.g., Huang, Juo, and Fu (2015)), the primal Färe-Primont (e.g., Dakpo, Desjeux, Jeanneaux, and Latruffe (2019)), Hicks-Moorsteen (e.g., Verschelde, Dumont, Rayp, and Merlevede (2016)) and Lowe (e.g., O'Donnell, Fallah-Fini, and Triantis (2017)) Total Factor Productivity (TFP) indices, among others.[1]

This contribution is structured as follows. The next section 2 develops the geometric intuition why a convexification strategy may potentially lead to biases in the estimation of the metafrontier. Section 3 introduces notation and formally outlines the metafrontier methodology. Thereafter, section 4 defines the productivity indices that are computed relative to the metafrontier: on the one hand the input-oriented Malmquist productivity index, and on the other hand the Hicks-Moorsteen TFP index that combines input- and output-oriented efficiency measures. Section 5 specifies the details about the deterministic nonparametric frontier technologies employed in computing the productivity indices relative to the metafrontier. Then, the next section 6 offers an empirical illustration using a secondary data set of hydroelectric power plants from Chile. Finally, the last section 7 summarizes results and draws some conclusions.

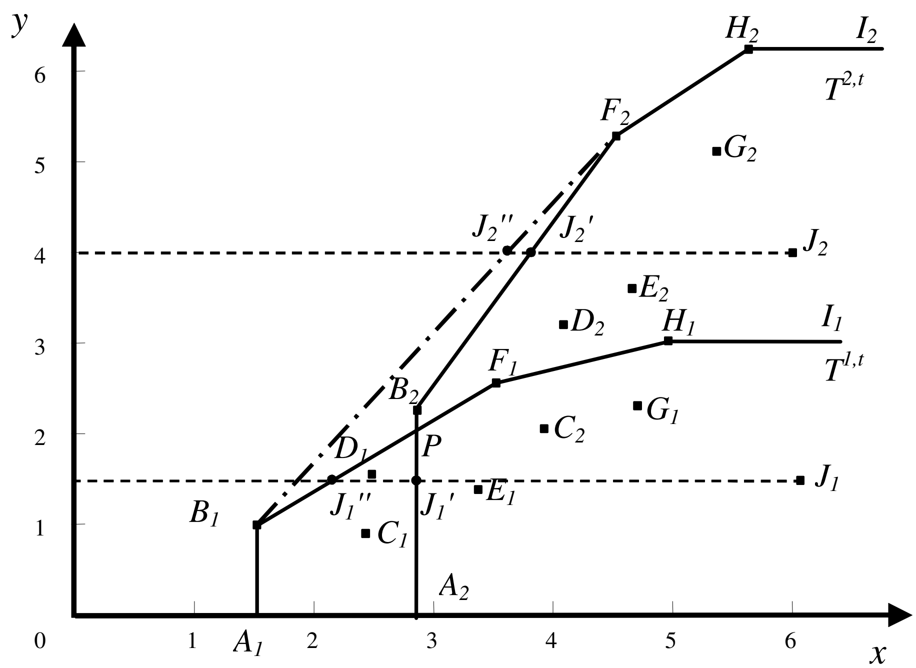
2 Metafrontier and Convexification Strategy: A Clarification

It is essential to remind the reader about the intuition underlying the metafrontier approach. To fix our ideas, we start by an example taken from wastewater treatment plants (Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)). There are four main technologies for wastewater treatment: activated sludge, aerated lagoon, trickling filter, and rotating biological contactor (biodisk). Suppose we focus on two such technologies to simplify matters.

Figure 1 illustrates the single-input-single-output case when only two technologies exist. Group technology $T^{1,t}$ consist of 8 observations $(B_1, C_1, D_1, E_1, F_1, G_1, H_1, J_1)$ denoted by square dots and represented by the polyline $A_1B_1F_1H_1I_1$ and the horizontal axis. Group technology $T^{2,t}$ consist of 8 observations $(B_2, C_2, D_2, E_2, F_2, G_2, H_2, J_2)$ also denoted by square dots and represented by the polyline $A_2B_2F_2H_2I_2$ and the horizontal axis. The metatechnology $T^{\Gamma,t}$ is now the union of technologies $T^{1,t}$ and $T^{2,t}$: it is clearly nonconvex. $T^{\Gamma,t}$ consists of all points between the polyline $A_1B_1PB_2F_2H_2I_2$ and the horizontal axis. The convexification strategy consists in convexifying this nonconvex metatechnology $T^{\Gamma,t}$ by adding the region denoted by the polyline $B_1PB_2F_2B_1$.

[1]Recent surveys on productivity indices and indicators are found in O'Donnell (2018) and Russell (2018), among others.

Figure 1: Group technologies and metatechnology: the single-input-single-output case



Let us now explore what happens when we project inefficient observations with respect to the frontiers of these technologies. From the 8 observations in group technology $T^{1,t}$, 5 observations are situated below the frontier and are therefore inefficient. Let us focus on inefficient observation J_1: it is projected onto projection point J_1'' on the line segment B_1F_1 of the group technology $T^{1,t}$ and can learn to position itself onto this frontier from combining somehow the inputs and outputs of observations B_1 and F_1. The same observation J_1 can also be projected with respect to the other group technology at projection point J_1'.

From the 8 observations in group technology $T^{2,t}$, 5 observations are also situated below the frontier and are therefore inefficient. Let us focus on inefficient observation J_2: it is projected onto projection point J_2' on the line segment B_2F_2 of the group technology $T^{2,t}$ and can learn to position itself onto this frontier from combining somehow the inputs and outputs of observations B_2 and F_2.

However, the same observation J_2 can now be projected with respect to the other group technology at projection point J_2'' depending on whether or not we adopt a convexification strategy. If we do not adopt a convexifixation strategy, then the metatechnology is nonconvex and the projection point J_2'' is simply infeasible. The distance to the metatechnology coincides with the distance to its own group technology and the distance to the other group technology is simply undefined. The reason why the projection point J_2'' is deemed infeasible is because this presupposes making a linear combination of point B_1 from group technology $T^{1,t}$ and point F_2 from group technology $T^{2,t}$. While we allow for convex combinations within each group technology, we normally rule out taking convex combinations across group technologies.

If we adopt a convexifixation strategy, then the metatechnology becomes convex again and the projection point J_2'' can be achieved. It should be realised that this convexification strategy is to some extent self-contradictory, because it destroys the very idea of distinguishing between different group technologies and only allowing for convexity per group technology. In other words, the union operator on group technologies does not normally preserve the convexity axiom on the resulting metatechnology.

The large majority of articles adopting a metafrontier approach seem to follow such a convexification strategy: a benign interpretation is that most authors just follow O'Donnell, Rao, and Battese (2008) and assume that such a strategy is rather harmless. Kerstens, O'Donnell, and Van de Woestyne (2019) cite a handful of articles that do not adopt such a strategy. Walheer (2018, p. 1015) states in this context: "All in all, the safest option is to assume a non-convex envelopment, as in the original definition of O'Donnell et al. (2008), while assuming a convex envelopment should be well-motivated."

Our reading of the metafrontier productivity literature also confirms this tendency: the large majority of articles adopts a convexification strategy. We are only aware of three exceptions. First, Verschelde, Dumont, Rayp, and Merlevede (2016) compute a metafrontier Hicks-Moorsteen TFP index starting from nonconvex group technologies: this choice for nonconvex technologies automatically leads to a nonconvex metatechnology. However, most researchers seem reluctant to give up the convexity of the group technologies and therefore we also maintain the convexity of the group technologies in this contribution.

Second, Afsharian, Ahn, and Harms (2018) report a metafrontier Malmquist productivity index starting from convex group technologies and report differences when computing a wrong convexified metatechnology rather than a correct nonconvex metatechnology. However, our work differs from these authors in two respects. First, we use formal statistical test procedures to verify whether a convexification strategy is innocuous or not (instead of a mere comparison). Second, we use the standard Malmquist productivity index computed over a two year time window, while these authors compute a so-called overall Malmquist index relative to one global technology computed over all available time periods (see Afsharian and Ahn (2015) for more details).

Third, Walheer (2018) focuses on the aggregation of metafrontier technology gap ratios and contrasts the results of wrong convexified and correct nonconvex metatechnologies, but this author reports no formal statistical test procedure.

3 Metafrontier Methodology

3.1 Technology and Technology-Specific Frontier, Metatechnology and Metafrontier

In this methodological section, we follow closely the notation and terminology introduced in Kerstens, O'Donnell, and Van de Woestyne (2019). O'Donnell (2016, p.328) defines a *technology* as "a technique, method or system for transforming inputs into outputs .... For most practical intents and purposes, it is convenient to think of a technology as a book of instructions, or recipe". This definition is adopted here: we perceive a technology as a kind of intellectual capital.

Technology is represented by a *technology-specific production possibilities set* (TPPS), which is a set containing all possible combinations of inputs and outputs using a given technology. Let $x^t \in \mathbb{R}^M_+$ denote vectors of inputs and let $y^t \in \mathbb{R}^N_+$ denote vectors and outputs at time period t. The set of all pairs of input and output vectors that can be produced at time period t using technology g is described as follows:

$$T^{g,t} = \{(x^t, y^t) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ : x^t \text{ with technology } g \text{ can produce } y^t\}. \tag{1}$$

The boundary of this TPPS is called a *technology-specific frontier*. Commonly, one makes the following assumptions on the TPPS:

(T.1) $(x^t, 0) \in T^{g,t}$ for all $x^t \in \mathbb{R}^M_+$.

(T.2) If $(0, y^t) \in T^{g,t}$, then $y^t = 0$.

(T.3) $T^{g,t}$ is a closed subset of $\mathbb{R}^M_+ \times \mathbb{R}^N_+$.

(T.4) If $(x^t, y^t) \in T^{g,t}$ and $(x', -y') \geq (x^t, -y^t)$, then $(x', y') \in T^{g,t}$.

(T.5) $T^{g,t}$ is a convex set.

(T.6) If $(x^t, y^t) \in T^{g,t}$, then $\delta(x^t, y^t) \in T^{g,t}$ for all $\delta \geq 0$.

These traditional axioms concerning technology g in period t state that: (i) inaction is possible, (ii) there is no free lunch, (iii) the set of feasible input-output combinations contains all the points on its boundary (closedness), (iv) inputs and outputs are strongly (or freely) disposable, (v) the technology is convex, and (vi) the technology satisfies constant returns to scale in that observations can be scaled down or up at will. For more details on these axioms: see, for instance, Hackman (2008).

Note that the first assumption (T.1) is not always maintained in this contribution. Furthermore, in the empirical illustration we impose either constant returns to scale (T.6) or the more traditional variable returns to scale assumption (which amounts to the absence of any scaling: $\delta = 1$).

When axiom (T.4) is maintained, then $T^{g,t}$ is represented by the following technology-specific input distance function:

$$d_I^{g,t}(x^t, y^t) = \sup_{\lambda \in \mathbb{R}_+} \left\{ \lambda : (x^t/\lambda, y^t) \in T^{g,t} \right\}. \tag{2}$$

This function is (i) non-negative, (ii) linearly homogeneous in inputs, and (iii) no less than unity for all $(x^t, y^t) \in T^{g,t}$.

The *technology set* or *metatechnology* Γ is the set of all technologies g that exist for all time periods. If a technology is seen as a recipe, then following Caselli and Coleman (2006, p. 509) one can view a technology set as "a library, containing blueprints, or recipes to turn inputs into outputs". The set of all input and output vectors that are feasible using a given technology set Γ (i.e., using some technology that is contained in Γ) is labelled a *metatechnology-specific production possibilities set* (MTPPS). Mathematically, this MTPPS is defined as

$$T^{\Gamma,t} = \{(x^t, y^t) \in \mathbb{R}_+^M \times \mathbb{R}_+^N : \exists g \in \Gamma : x^t \text{ and } g \text{ can produce } y^t\}. \tag{3}$$

Obviously, we have that $T^{\Gamma,t} = \cup_{g \in \Gamma} T^{g,t}$. The boundary of a MTPPS is called a *metafrontier*.

When strong disposability applies (i.e., (T.4) is true), then the MTPPS $T^{\Gamma,t}$ can be represented using the metatechnology-specific input distance function:

$$D_I^{\Gamma,t}(x^t, y^t) = \max_{g \in \Gamma} \{d_I^{g,t}(x^t, y^t)\}. \tag{4}$$

Equivalently, $D_I^{\Gamma,t}(x^t, y^t) = \sup_{\lambda \in \mathbb{R}_+} \{\lambda : (x^t/\lambda, y^t) \in T^{\Gamma,t}\}$. This function is non-negative, linearly homogeneous in inputs, and no less than unity for all $(x^t, y^t) \in T^{\Gamma,t}$.

Instead of using the technology-specific input distance function (2), $T^{g,t}$ can also be represented by the technology-specific output distance function:

$$d_O^{g,t}(x^t, y^t) = \inf_{\lambda \in \mathbb{R}_+} \left\{ \lambda : (x^t, y^t/\lambda) \in T^{g,t} \right\}. \tag{5}$$

This function is (i) non-negative, (ii) linearly homogeneous in outputs, and (iii) less than unity for all $(x^t, y^t) \in T^{g,t}$. Under the strong disposability assumption (T.4), the MTPPS $T^{\Gamma,t}$ can then be represented using the metatechnology-specific output distance function:

$$D_O^{\Gamma,t}(x^t, y^t) = \min_{g \in \Gamma} \{ d_O^{g,t}(x^t, y^t) \}. \tag{6}$$

Equivalently, $D_O^{\Gamma,t}(x^t, y^t) = \inf_{\lambda \in \mathbb{R}_+} \{ \lambda : (x^t, y^t/\lambda) \in T^{\Gamma,t} \}$. This function is non-negative, linearly homogeneous in outputs, and less than unity for all $(x^t, y^t) \in T^{\Gamma,t}$.

3.2 Technical Efficiency

In this contribution, the input-oriented metatechnology-specific technical efficiency (ITE) of an organization using inputs x^t to produce outputs y^t using some technology $g \in \Gamma$ at time period t is defined as the reciprocal of the metatechnology-specific input distance function (4):

$$\mathrm{ITE}^{\Gamma,t}(x^t, y^t) = 1/D_I^{\Gamma,t}(x^t, y^t). \tag{7}$$

This radial technical efficiency measure lies in the closed unit interval and indicates the maximum proportional reduction in x^t that still allows production of y^t by some technology $g \in \Gamma$.

If Γ contains more than one technology, then the measure of ITE (7) can be written as the product of an input-oriented metatechnology ratio (IMR) and a measure of residual input-oriented technical efficiency ($RITE$). Mathematically, the IMR relative to the technology set Γ of a firm that uses inputs x^t and technology g to produce outputs y^t is

$$\mathrm{IMR}^{g\Gamma,t}(x^t, y^t) = d_I^{g,t}(x^t, y^t)/D_I^{\Gamma,t}(x^t, y^t). \tag{8}$$

Also this measure lies in the closed unit interval. It can be interpreted as an input-oriented technical efficiency measure of whether a firm has chosen the best technology that is available.

The associated measure of $RITE$ is

$$\text{RITE}^{g,t}(x^t, y^t) = 1/d_I^{g,t}(x^t, y^t). \tag{9}$$

This measure also lies in the closed unit interval and indicates the maximum proportional reduction in x^t that allows production of y^t when using technology g for time period t. It can also be interpreted as the component of ITE that remains after accounting for the IMR (whence the term "residual"). Obviously, equations (4), (7) and (9) imply that

$$\text{ITE}^{\Gamma,t}(x^t, y^t) = \min_{g\in\Gamma}\{\text{RITE}^{g,t}(x^t, y^t)\}. \tag{10}$$

Note that some of the components in (10) can be undefined for some input-output combinations that are not contained in the group technology composing the technology or metatechnology (see Briec and Kerstens (2009) for details on infeasibilities). Finally, equations (7), (8) and (9) imply that

$$\text{ITE}^{\Gamma,t}(x^t, y^t) = \text{IMR}^{g\Gamma,t}(x^t, y^t) \cdot \text{RITE}^{g,t}(x^t, y^t). \tag{11}$$

Hence, technical efficiency can be decomposed into the product of a metatechnology ratio and a measure of residual technical efficiency: the first measures how close a technology-specific frontier is to the metafrontier, while the second measures how close a firm is operating to the technology-specific frontier.

By analogy with the former, the output-oriented metatechnology-specific technical efficiency (OTE) of an organization using inputs x^t to produce outputs y^t using some technology $g \in \Gamma$ at time period t is defined as the reciprocal of the metatechnology-specific output distance function (6):

$$\text{OTE}^{\Gamma,t}(x^t, y^t) = 1/D_O^{\Gamma,t}(x^t, y^t). \tag{12}$$

This radial technical efficiency measure results in values larger than or equal to one and indicates the maximum proportional expansion in y^t that is still achievable with x^t inputs by some technology $g \in \Gamma$.

If Γ contains more than one technology, then the measure of OTE (12) can be written as the product of an output-oriented metatechnology ratio (OMR) and a measure of residual output-oriented technical efficiency ($ROTE$). Mathematically, the OMR relative to the technology set Γ of a firm that uses inputs x^t and technology g to produce outputs y^t is

$$\text{OMR}^{g\Gamma,t}(x^t, y^t) = d_O^{g,t}(x^t, y^t)/D_O^{\Gamma,t}(x^t, y^t). \tag{13}$$

The associated measure of $ROTE$ is

$$\text{ROTE}^{g,t}(x^t, y^t) = 1/d_O^{g,t}(x^t, y^t). \tag{14}$$

This measure also has values greater than or equal to one and indicates the maximum proportional expansion in y^t that can still be realized with inputs x^t when using technology g for time period t. It can also be interpreted as the component of OTE that remains after accounting for the OMR (whence the term "residual"). Obviously, equations (6), (12) and (14) imply that

$$\text{OTE}^{\Gamma,t}(x^t, y^t) = \max_{g\in\Gamma}\{\text{ROTE}^{g,t}(x^t, y^t)\}. \tag{15}$$

Like in the input orientation, some of the components in (15) can be undefined for some input-output combinations that are not contained in the group technology composing the technology or metatechnology (see Briec and Kerstens (2009) for details on infeasibilities).

4 Metafrontier Productivity Indices

The measurement of productivity has in the last 25 years or so often been analysed using a technology-based, discrete-time Malmquist productivity index. Initially defined by Caves, Christensen, and Diewert (1982) as a ratio of distance functions, this index has become increasingly popular due to the innovations of Färe, Grosskopf, Lindgren, and Roos (1995). The latter authors have shown how: (i) to relax the hypothesis of technical efficiency maintained in Caves, Christensen, and Diewert (1982); (ii) to decompose this index into technology shifts and technical efficiency changes; and (iii) to compute this index relative to multiple inputs and outputs technologies by exploiting the relationship between distance functions and technical efficiency measures. O'Donnell (2012) argues rather convincingly that the Malmquist productivity index is not a TFP index. This same position is also found in, among others, O'Donnell (2018, p. 120-121) and Russell (2018). It is rather a technology index aimed at mainly measuring local technical change (see Grosskopf (2003)). Therefore, we also compare the Malmquist productivity index with the Hicks-Moorsteen TFP index, one among the several available TFP indices (see O'Donnell (2018) and Russell (2018)).

To the best of our knowledge, this is the first comparison of the Malmquist productivity index and the Hicks-Moorsteen TFP index within a metafrontier context. Earlier, such a comparison using standard technologies has already been published in the literature (see, e.g., Kerstens and Van de Woestyne (2014)).[2]

4.1 Metafrontier Malmquist Productivity Index

One can define the input-oriented metafrontier Malmquist productivity index ($IMMI$) in base period t as follows:

$$IMMI^{\Gamma,t}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{ITE^{\Gamma,t}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t}(x^t, y^t)}. \tag{16}$$

Values of this base period t input-oriented $IMMI$ above (below) unity reveal productivity growth (decline). Similarly, a base period $t+1$ input-oriented $IMMI$ is defined as follows:

$$IMMI^{\Gamma,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{ITE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t+1}(x^t, y^t)}. \tag{17}$$

Again, values of this base period $t+1$ input-oriented $IMMI$ above (below) unity reveal productivity growth (decline).

To avoid an arbitrary selection among base years, the input-oriented $IMMI$ is defined as a geometric mean of a period t and a period $t+1$ index:

$$IMMI^{\Gamma,t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \left[\frac{ITE^{\Gamma,t}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t}(x^t, y^t)} \cdot \frac{ITE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t+1}(x^t, y^t)}\right]^{1/2}. \tag{18}$$

Note again that when the geometric mean of the $IMMI$ is larger (smaller) than unity, then it points to a productivity growth (decline).

Remark that the above definitions deviate from the original ones in Caves, Christensen, and Diewert (1982) in that the ratios have been inverted. This ensures that productivity indices above (below) unity reveal productivity growth (decline), which is in line with traditional TFP indices.

Following Färe, Grosskopf, Lindgren, and Roos (1992) and Färe, Grosskopf, Norris, and

[2] A similar comparison using difference-based indicators rather than ratio-based indices is available in the literature: see, e.g., Kerstens, Shen, and Van de Woestyne (2018).

Zhang (1994), an equivalent way of writing this $IMMI$ index is

$$IMMI^{\Gamma,t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \underbrace{\frac{ITE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t}(x^t, y^t)}}_{(IMEC)} \cdot \underbrace{\left[\frac{ITE^{\Gamma,t}(x^t, y^t)}{ITE^{\Gamma,t+1}(x^t, y^t)} \cdot \frac{ITE^{\Gamma,t}(x^{t+1}, y^{t+1})}{ITE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}\right]^{1/2}}_{(IMTC)}, \quad (19)$$

where the ratio outside the brackets represents the relative input-oriented metatechnology efficiency change ($IMEC$) from period t to $t+1$. The part inside the brackets captures the shift of the metafrontiers between two periods. Due to the treatment of avoiding an arbitrary selection among years in (18), the geometric mean of the two ratios evaluated at (x^t, y^t) and (x^{t+1}, y^{t+1}) is specified as the input-oriented metatechnology change ($IMTC$).

Referring to (11), $IMEC$ can be further represented in terms of IMR and $RITE$, which can be written:

$$IMEC^{\Gamma,t,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \underbrace{\frac{RITE^{g,t+1}(x^{t+1}, y^{t+1})}{RITE^{g,t}(x^t, y^t)}}_{(ITEC)} \cdot \underbrace{\frac{IMR^{g\Gamma,t+1}(x^{t+1}, y^{t+1})}{IMR^{g\Gamma,t}(x^t, y^t)}}_{(IMRC)}, \quad (20)$$

where the first part measures the efficiency changes with respect to the technology-specific frontier (i.e., input-oriented technology-specific efficiency change ($ITEC$)), whereas the second part depicts the input-oriented change of IMR ($IMRC$) between periods t to $t+1$. The latter describes whether the distance between technology-specific frontier and metafrontier in period $t+1$ is getting smaller or larger than that in period t.

In conclusion, the proposed $IMMI$ is represented by means of the following decomposition:

$$IMMI = ITEC \cdot IMRC \cdot IMTC. \quad (21)$$

Specifically, $ITEC > 1$ (< 1) indicates that the unit under evaluation is approaching (moving away from) the corresponding technology-specific frontier from period t to $t+1$ in the input-orientation. $IMRC > 1$ (< 1) shows that the technology-specific frontier is approaching (moving away from) its metafrontier from period t to $t+1$ in the input-orientation. Finally, the last component implies a metatechnology progress (regress) from period t to $t+1$ if $IMTC > 1$ (< 1).

Note that the $IMMI$ and its components can all be affected by the convexification strategy applied to the computation of the metatechnology, except the $ITEC$ component that is evaluated with respect to the group-specific frontiers only. Furthermore, note that more elaborate decompositions of the metafrontier Malmquist index have been proposed in the literature: see, e.g., Chen and Yang (2011).

4.2 Metafrontier Hicks-Moorsteen Productivity Index

The seminal article by Bjurek (1996) introduces a Hicks-Moorsteen TFP index with a base period t as the ratio of a Malmquist type output quantity index (MO) in base period t over a Malmquist type input quantity index (MI) in the same base period t. Applied to a metatechnology Γ, this period t based metatechnology Hicks-Moorsteen productivity (MHM) index boils down to the following:

$$MHM^{\Gamma,t}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO^{\Gamma,t}(x^t, y^t, y^{t+1})}{MI^{\Gamma,t}(x^t, x^{t+1}, y^t)}, \tag{22}$$

with

$$MO^{\Gamma,t}(x^t, y^t, y^{t+1}) = \frac{OTE^{\Gamma,t}(x^t, y^t)}{OTE^{\Gamma,t}(x^t, y^{t+1})}$$

and

$$MI^{\Gamma,t}(x^t, x^{t+1}, y^t) = \frac{ITE^{\Gamma,t}(x^t, y^t)}{ITE^{\Gamma,t}(x^{t+1}, y^t)}.$$

A metatechnology Hicks-Moorsteen productivity index larger (smaller) than unity indicates a gain (loss) in productivity.

Similarly, the period $t+1$ based metatechnology Hicks-Moorsteen TFP index is defined as follows:

$$MHM^{\Gamma,t+1}(x^t, y^t, x^{t+1}, y^{t+1}) = \frac{MO^{\Gamma,t+1}(x^{t+1}, y^{t+1}, y^t)}{MI^{\Gamma,t+1}(x^t, x^{t+1}, y^{t+1})}, \tag{23}$$

with

$$MO^{\Gamma,t+1}(x^{t+1}, y^{t+1}, y^t) = \frac{OTE^{\Gamma,t+1}(x^{t+1}, y^t)}{OTE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}$$

and

$$MI^{\Gamma,t+1}(x^t, x^{t+1}, y^{t+1}) = \frac{ITE^{\Gamma,t+1}(x^t, y^{t+1})}{ITE^{\Gamma,t+1}(x^{t+1}, y^{t+1})}.$$

Again, a metatechnology Hicks-Moorsteen productivity index larger (smaller) than unity points to a productivity gain (loss).

To avoid an arbitrary choice of base year, it is customary to take a geometric mean of these two Hicks-Moorsteen TFP indices (22) and (23) (see Bjurek (1996)):

$$MHM^{\Gamma,t,t+1}(x^t,y^t,x^{t+1},y^{t+1}) = \left[MHM^{\Gamma,t}(x^t,y^t,x^{t+1},y^{t+1}) \cdot MHM^{\Gamma,t+1}(x^t,y^t,x^{t+1},y^{t+1})\right]^{1/2}. \tag{24}$$

Note once more that a geometric mean metatechnology Hicks-Moorsteen productivity index larger (smaller) than unity indicates a productivity gain (loss).

Note that decompositions of the Hicks-Moorsteen TFP index are still rare in the literature. A recent proposal for a decomposition is the one by Diewert and Fox (2017), but is has rarely if ever been applied. The sole study applying the metatechnology Hicks-Moorsteen index (i.e., Verschelde, Dumont, Rayp, and Merlevede (2016)) did not decompose. The comparison study of the Malmquist and Hicks-Moorsteen indices with standard technologies of Kerstens and Van de Woestyne (2014) did not decompose as well. Therefore, we do not develop a decomposition of the Hicks-Moorsteen index here.

5 Nonparametric Frontier Technologies

If each TPPS is convex and exhibits VRS at time period t, then this TPPS is defined as:

$$T^{g,t}_{C,VRS} = \Big\{(x^t,y^t) \in \mathbb{R}^M_+ \times \mathbb{R}^N_+ : \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x^t_{\phi_g(i)} \leq x^t, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y^t_{\phi_g(i)} \geq y^t, \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \mathbb{R}_+\Big\}, \tag{25}$$

with $s^{g,t} = \{(x^t_{\phi_g(i)}, y^t_{\phi_g(i)}) : i = 1,\ldots,n^g\}$ the set of n^g initial observations at time period t determining technology g. The associated MTPPS is simply defined as the union of the above TPPS:

$$T^{\Gamma,t}_{C,VRS} = \cup_{g\in\Gamma} T^{g,t}_{C,VRS}. \tag{26}$$

The convexified version of (26) yields the following metatechnology:

$$H_{C,VRS}^{\Gamma,t} = \Big\{(x^t, y^t) \in \mathbb{R}_+^M \times \mathbb{R}_+^N : \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)}^t \leq x^t, \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)}^t \geq y^t,$$
$$\sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} = 1, \lambda_{\phi_g(i)} \in \mathbb{R}_+\Big\}. \quad (27)$$

O'Donnell, Rao, and Battese (2008, p. 238) employ this specification to determine an estimate of metatechnology-specific technical efficiency.[3] However, note that in general $T_{C,VRS}^{\Gamma,t} \subseteq H_{C,VRS}^{\Gamma,t}$ where equality holds only for restrictive special cases (e.g., if only one group technology exists), as follows from Proposition 5.5 in Kerstens, O'Donnell, and Van de Woestyne (2019). The basic question we address in this contribution is whether the use of (27) leads to efficiency estimates that are close to the estimates obtained using the unbiased estimator (26).

If each TPPS is convex and exhibits CRS at time period t, then this TPPS is defined as:

$$T_{C,CRS}^{g,t} = \Big\{(x^t, y^t) \in \mathbb{R}_+^M \times \mathbb{R}_+^N : \sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)}^t \leq x^t, \sum_{i=1}^{n^g} \lambda_i y_{\phi_g(i)}^t \geq y^t, \lambda_{\phi_g(i)} \in \mathbb{R}_+\Big\}, \quad (28)$$

with $s^{g,t} = \{(x_{\phi_g(i)}^t, y_{\phi_g(i)}^t) : i = 1, \ldots, n^g\}$ the set of n^g initial observations at time period t determining technology g. The associated MTPPS is again defined as the union of the previous TPPS:

$$T_{C,CRS}^{\Gamma} = \cup_{g\in\Gamma} T_{C,CRS}^{g,t}. \quad (29)$$

The convexified version of (29) defines the following metatechnology:

$$H_{C,CRS}^{\Gamma,t} =$$
$$\Big\{(x^t, y^t) \in \mathbb{R}_+^M \times \mathbb{R}_+^N : \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} x_{\phi_g(i)}^t \leq x^t, \sum_{g\in\Gamma}\sum_{i=1}^{n^g} \lambda_{\phi_g(i)} y_{\phi_g(i)}^t \geq y^t, \lambda_{\phi_g(i)} \in \mathbb{R}_+\Big\}. \quad (30)$$

Proposition 5.5 of Kerstens, O'Donnell, and Van de Woestyne (2019) implies that $T_{C,CRS}^{\Gamma,t} \subseteq H_{C,CRS}^{\Gamma,t}$, whereby equality only holds for restrictive special cases. Again, the basic question is whether the use of (30) generates efficiency estimates that are close to the estimates obtained using the unbiased estimator (29).

[3] In fact, O'Donnell, Rao, and Battese (2008) compute an output-oriented metatechnology-specific technical efficiency under the assumption that there is no technical change.

The convex technology specifications with variable (i.e., (25) and (27)) and constant (i.e., (28) and (30)) returns to scale are commonly known as data envelopment analysis (moniker DEA) models. These technology specifications have been introduced to the literature by Banker, Charnes, and Cooper (1984) and Färe, Grosskopf, and Lovell (1983). If the TPPS exhibit VRS (respectively CRS), then the technology specification (25) (respectively (28)) can be used to compute the measure of $RITE$ (9) by solving a linear program for each evaluated observation (see Hackman (2008) or Ray (2004)). The associated MTPPS (26) and (29) can be used to compute the measure of ITE (7) by solving for each evaluated observation several linear programs: one per TPPS. Recently, Afsharian and Podinovski (2018) show how to achieve the measure of ITE (7) relative to the MTPPS by solving a single LP problem. The convexification strategy embodied in the technologies (27) and (30) normally leads to a biased estimator for the ITE measure: it is an open empirical question how biased these computations exactly are when computing productivity indices with respect to a metafrontier.

Figure 2: Group technologies and metatechnologies over time

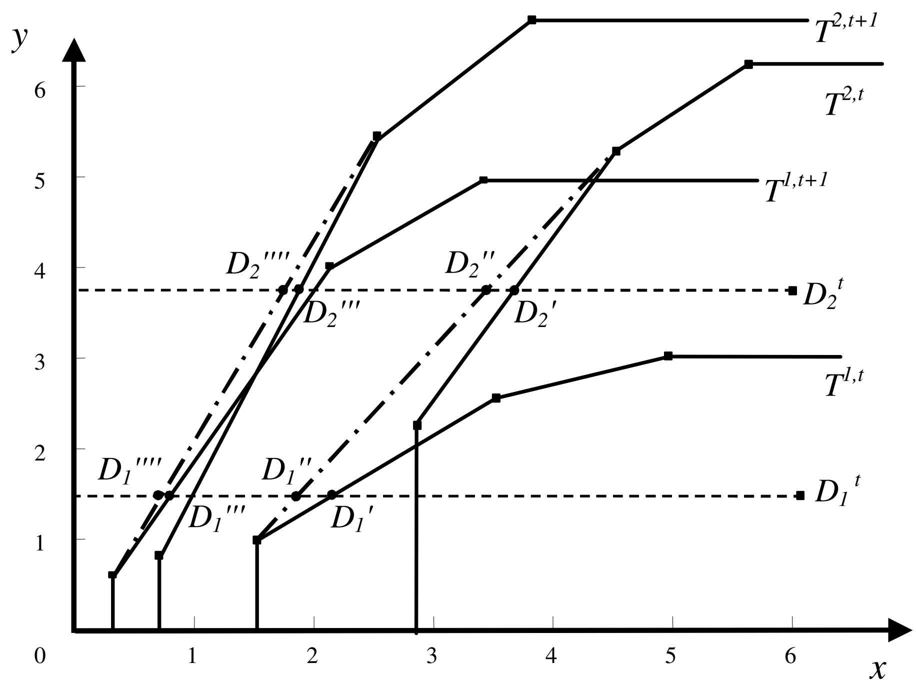


Figure 2 illustrates the issue at stake using variable returns to scale technologies (just as in Figure 1). The TPPS $T^{1,t}$ and $T^{2,t}$ make up the associated MTPPS in period t. Similarly, the TPPS $T^{1,t+1}$ and $T^{2,t+1}$ constitute the components of the associated MTPPS in period $t+1$. From the discussion in Section 4 we recall that $IMMI$ as well as its components can be affected by the convexification strategy applied to the MTTPS except for the $ITEC$ component (since it is based on $RITE$). For instance, when projecting observation D_2 in period t (D_2^t) to the MTTPS in period t, then we can either project on the true MTTPS at point D_2' or at the convexified MTTPS at point D_2''. When projecting this same observation to the MTTPS in period $t+1$, then we can either project on the true MTTPS at point D_2''' or at the convexified MTTPS at point D_2''''. Obviously, these measurements involving ITE may affect both the $IMRC$ and $IMTC$ components.

6 Empirical Illustration: Hydroelectric Power Plants

6.1 Secondary Data

This empirical section aims to illustrate the implications of a convexification strategy using secondary data that were previously used by Atkinson and Dorfman (2009) to evaluate the performance of an unbalanced panel of Chilean hydroelectric power plants. These data are publicly available on the data repository of the *Journal of Applied Econometrics*.[4] This sample comprises monthly data on $M = 3$ inputs and $N = 1$ output for 21 Chilean hydroelectric power plants over the period from April 1986 to December 1997. There are 7 dam plants and 14 run-of-river (ROR) plants in this sample. The three inputs are labor (in thousands of workers), capital (in real pesos), and water (in cubic meters). The single output is electricity generated (in gigawatt hours). More details regarding these data is found in Atkinson and Dorfman (2009) and Atkinson and Halabí (2005).

In Chile two main techniques (technologies) are employed to generate hydroelectric power. The first technology (index 1) builds a dam on a river to store water and releases this water from the dam to spin turbines generating electricity. The main advantage of dam systems is that electricity generation is uncoupled from river flows. The second hydroelectric power technology (index 2) involves merely diverting river flows through turbines. The advantage of these ROR or diversion systems is that these are relatively inexpensive and have relatively little impact on the environment. A key disadvantage of such systems is that these cannot be used to match electricity generation with consumer demand.[5] By construction, the technology set $\Gamma = \{1, 2\}$.

[4] Web site: `http://qed.econ.queensu.ca/jae/`

[5] A third hydroelectric power technology, called pumped storage, allows to match electricity generation with variations in consumer demand. But, there are no pumped storage plants in this sample.

Our understanding of hydroelectric power generation leads us to believe that it *may* be possible for the manager of a given dam (ROR) plant to use a given input vector to produce a given level of output for some time within the planning period, and then use a different input vector to produce a different level of output for the rest of the time. This suggests that each TPPS *may* be convex. Consequently, we begin by computing these TPPSs t^1 and t^2 using a convex nonparametric frontier technology. Given the different types of capital involved in constructing different plants, it is also our understanding that the manager of a given plant cannot learn how to improve the performance by convex combinations of dam systems and ROR systems. This suggests that the MTPPS should not be convexified. It is now an open question to check how a convexification strategy of the MTPPS approximates the true nonconvex MTPPS.

6.2 Empirical Results

Table 1 contains basic descriptive statistics of the $IMMI$ estimates and its components with balanced and unbalanced panel data of hydroelectric power plants under CRS and VRS technologies, respectively. This table is structured as follows: first we discuss the columns, then we explain the rows. The first four columns list the results under a CRS technology and the last four columns assume a VRS technology. Within each of these technologies, the first two columns report the results with unbalanced panel data while the last two columns develop the balanced panel data results. A further distinction is related to whether a convexification strategy is applied or not: a C indicates a convexification strategy is applied to calculate ITE, while NC reveals that this is not the case. Horizontally, the first block of rows contains the results of the $IMMI$ estimates. The following three horizontal blocks of rows reports the decomposition results of $IMMI$. Within each of these four horizontal blocks, we report results on geometric means, standard deviations, minimum and maximum values of the corresponding estimates. The use of geometric means guarantees the multiplicative decomposition of the $IMMI$.

Furthermore, a nonparametric Li-test is applied to test the null hypothesis that the distributions of the two C vs. NC $IMMI$ as well as its two components $IMRC$ and $IMTC$ are equal. Since the $ITEC$ component does not differ under C vs. NC, no test statistic is computed. This Li-test is first proposed by Li (1996) and has been refined by Fan and Ullah (1999) and others: one of the most recent developments is by Li, Maasoumi, and Racine (2009). This nonparametric test analyzes the differences between entire distributions by comparing the differences between two kernel-based estimates of density functions f and g

of a random variable x. The null hypothesis affirms that both density functions are almost everywhere equal ($H_0 : f(x) = g(x)$ for all x). The alternative hypothesis simply negates this equality of both density functions ($H_1 : f(x) \neq g(x)$ for some x). The test statistics marked with "***" means the null hypothesis is rejected at the 0.1% significance level. A large (resp. small) p value indicates that the null hypothesis should not be rejected (resp. be rejected).[6]

Finally, the number as well as the percentage of contradictory results implied by comparing estimates under C and NC strategies are reported in the last row of each block (denoted "#Contrad. Res."). Contradictory results arise when one estimate points to a productivity decline while the other shows a productivity growth, or the other way around.

Note that the computation of these descriptive statistics and tests is based on the productivity indicators available. Hence, the number of valid results may differ from case to case. Taking the case of balanced panel data as an example, there are 1085 valid $IMMI$ results under CRS. Due to the occurrence of computational infeasibilities, the valid number under VRS is only 933.

Several observations can be made with regard to the results in Table 1. First, the basic descriptive statistics for $IMMI$, $IMRC$ and $IMTC$ estimates all show certain differences when comparing between C and NC strategies. In addition, the $IMMI$ estimates under a C strategy are on average lower than the ones under a NC strategy in our data. Theoretically, estimates of these indices derived under C strategy can be either higher or lower than the ones derived under NC strategy. This is consistent with the observations for the $IMRC$ and $IMTC$ estimates. As for the $ITEC$ estimates, which capture the efficiency changes with respect to technology-specific frontiers, the convexification strategy makes no difference.

Second, the Li-test for the $IMRC$ estimates reveals that the C strategy leads to a significant difference in the distribution of the metafrontier compared with the NC strategy. This holds true for both CRS and VRS technologies. For the VRS technology, a statistically significant difference can be found for all $IMMI$ and $IMTC$ estimates. This indicates that the convexification strategy has a stronger influence on calculating these estimates under VRS than under CRS.

Third, all estimates under various cases offer some contradictory signs between applying C and NC strategies. On average, more contradictory signs are detected in the estimates of $IMRC$ and $IMTC$ than that of $IMMI$. The percentage of contradictory signs reaches

[6]The Matlab code for the Li-test adopted here has been developed by P.J. Kerstens based on Li, Maasoumi, and Racine (2009). The code file is found at: `https://github.com/kepiej/DEAUtils`.

Table 1: Descriptive statistics and Li-test for the estimates of $IMMI$ and its decompositions

		CRS				VRS			
		Unbalanced		Balanced		Unbalanced		Balanced	
		C	NC	C	NC	C	NC	C	NC
$IMMI$	Mean	1.0434	1.0443	1.0181	1.0191	1.0397	1.0461	1.0246	1.0361
	Std. Dev.	0.3486	0.3492	0.1911	0.1982	0.3084	0.3321	0.2348	0.3106
	Min	0.1687	0.1660	0.3970	0.3970	0.1853	0.1662	0.1859	0.2163
	Max	6.1295	6.0020	2.4833	2.4833	5.0747	5.3552	2.6642	2.6642
	Li-test		-2.4605		0.5387		4.3619***		7.3193***
	p-value		(0.999)		(0.2675)		(0.001)		(0.0005)
	#Inf.		0/2412		0/1085		185/2412		152/1085
	Res.		(0)		(0)		(7.67%)		(14.01%)
	#Contrad.		35/2412		29/1085		200/2227		73/933
	Res.		(1.45%)		(2.67%)		(8.98%)		(7.82%)
$ITEC$	Mean	1.0245		1.0028		1.0121		1.0013	
	Std. Dev.	0.2616		0.0774		0.1924		0.0531	
	Min	0.1705		0.5906		0.2134		0.5672	
	Max	4.3616		1.7857		5.0214		1.6507	
$IMRC$	Mean	1.0105	1.0099	1.0108	1.0116	1.0104	1.0061	1.0090	1.0040
	Std. Dev.	0.1694	0.1692	0.1516	0.1581	0.1561	0.1374	0.1376	0.0861
	Min	0.3107	0.2944	0.4916	0.4916	0.4072	0.2379	0.5020	0.5118
	Max	5.3189	5.7023	1.9405	1.9405	2.4700	4.2052	2.1427	2.0855
	Li-test		483.4166***		259.9905***		158.8927***		489.1541***
	p-value		(0.0000)		(0.0000)		(0.0000)		(0.0000)
	#Contrad.		731/2412		230/1085		873/2227		268/933
	Res.		(30.31%)		(21.20%)		(39.20%)		(28.72%)
$IMTC$	Mean	1.0130	1.0143	1.0119	1.0110	1.0211	1.0331	1.0141	1.0269
	Std. Dev.	0.1431	0.1472	0.1480	0.1440	0.1859	0.2530	0.1751	0.2742
	Min	0.5246	0.5215	0.5246	0.5215	0.2157	0.2220	0.1962	0.2163
	Max	2.0983	2.1508	1.8789	1.8789	2.7293	3.1923	2.6642	2.6642
	Li-test		-2.0476		1.0960		6.3767***		6.5930***
	p-value		(0.9930)		(0.1245)		(0.0000)		(0.0000)
	#Contrad.		127/2412		57/1085		349/2227		153/933
	Res.		(5.27%)		(5.25%)		(15.67%)		(16.40%)

39.2% for $IMRC$ estimates and 16.40% for $IMTC$. The contradictory signs appear more frequently under the VRS technology than under CRS. This also testifies that the bias of applying the convexification strategy is more evident under VRS than under CRS.

The descriptive statistics results, results for the nonparametric Li-test, and contradictory signs for the metafrontier Hicks-Moorsteen (MHM) TFP estimates are displayed in Table 2. This table is similar in structure to Table 1, except that only the MHM estimate is reported and no decomposition.

Table 2: Descriptive statistics and Li-test for the estimate of MHM

	CRS				VRS			
	Unbalanced		Balanced		Unbalanced		Balanced	
	C	NC	C	NC	C	NC	C	NC
Mean	1.0434	1.0443	1.0181	1.0191	1.0496	1.0489	1.0254	1.0262
Std. Dev.	0.3486	0.3492	0.1911	0.1982	0.4033	0.3924	0.2484	0.2586
Min	0.1687	0.1660	0.3970	0.3970	0.1040	0.1040	0.1774	0.1774
Max	6.1295	6.0020	2.4833	2.4833	8.3617	8.0871	4.6910	4.6910
Li-test		-2.4605		9.3788***		-2.6610		-1.9210
p-value		1		0		1		0.9995
# Inf.		0/2412		0/1085		0/2412		0/1085
Res.		(0)		(0)		(0)		(0)
# Contrad.		31/2412		29/1085		52/2412		39/1085
Res.		(1.29%)		(2.67%)		(2.16%)		(3.59%)

The following observations can be made regarding this Table 2. First, minor differences are observed between the C and NC strategies from the basic descriptive statistics of the MHM estimates. Second, a statistically significant difference in distributions is detected by the Li-test for the case with balanced data and under the CRS assumption only. Third, contradictory results exist while comparing between C and NC strategies for all four cases. Furthermore, more opposite signs show up under the VRS assumption than under the CRS assumption. Fourth, no infeasibilities are recorded (see Briec and Kerstens (2011) who prove that the Hicks-Moorsteen index does not yield any infeasibilities under standard assumptions on technology).

In general, we can conclude that applying a convexification strategy for both productivity indices shows quite a difference from the original non-convex metafrontier productivity indices. The contradictory results also underscore the drawback of applying a convexification strategy. More specifically, there is a non-negligible possibility that the suggestions based on the estimates obtained by applying a convexification strategy lead to opposite conclusions and policy recommendations.

Table 3: Descriptive statistics and Li-test for the estimates of $IMMI$ and MHM

	VRS			
	Unbalanced		Balanced	
	IMMI	MHM	IMMI	MHM
Mean	1.0461	1.0489	1.0361	1.0262
Std. Dev.	0.3321	0.3924	0.3106	0.2586
Min	0.1662	0.1040	0.2163	0.1774
Max	5.3552	8.0871	2.6642	4.6910
Li-test		3.8946**		13.7819***
p-value		0.002		0.0000
# Inf.	185/2412	0/2412	152/1085	0/1085
Res.	(7.67%)	(0)	(14.01%)	(0)
# Contrad.		281/2227		145/933
Res.		(12.62%)		(15.54%)

In Table 3 the degree of similarity between the metafrontier Malmquist and metafrontier Hicks-Moorsteen productivity indices -both obtained under the correct NC strategy- is further investigated. First, note that the empirical study here contains a single output and multiple inputs. This makes both metafrontier productivity indices coincide under the CRS assumption (see Bjurek, Førsund, and Hjalmarsson (1998)). However, both indices potentially remain to show differences under VRS. Second, we can observe that all descriptive statistics somewhat vary between $IMMI$ and MHM. There is no infeasibilities detected in calculating MHM, while infeasibilities arise in calculating $IMMI$ under VRS. Accordingly, the results where the distributions and contradictory signs are examined are based on available indices. Third, the Li-test shows that the distributions of $IMMI$ and MHM are significantly different for the balanced data under VRS at the significance level of 0.1%. For the case with unbalanced data under VRS, their distributions are significantly different at the significance level of 1%, which is marked with "**" in Table 3. Finally, the percentage having contradictory results between $IMMI$ and MHM reaches 15.54% and 12.62% under balanced and unbalanced panel data, respectively. Therefore, the above observations all imply that the $IMMI$ and MHM indices under VRS are empirically distinct.

7 Conclusions

In their seminal article, O'Donnell, Rao, and Battese (2008) define a nonconvex metatechnology as the union of two or more underlying group-specific technologies. They suggest estimating the metafrontier (i.e., the boundary of the metatechnology) under the assumption that the metatechnology is convex. If this assumption yields a poor approximation of the true nonconvex metatechnology, then their convexification strategy yields a biased estimator. Kerstens, O'Donnell, and Van de Woestyne (2019) develop some new results on the union operation on technologies under various assumptions of returns to scale and convexity and are the first to statistically test that a convexification strategy leads to significant biases.

This is the first empirical contribution that explores the impact of a convexification strategy on a Malmquist productivity index evaluated relative to a metafrontier methodology. Theoretically, we find that the input-oriented Malmquist productivity index itself as well as its components input-oriented metatechnology change ($IMTC$) and input-oriented metatechnology ratio change ($IMRC$) are potentially affected by the choice of a convexification strategy. However, this is not the case for the input-oriented technical efficiency change ($ITEC$) component. Empirically, our results for a secondary date set reveal that a convexification strategy leads to statistically significant differences for the input-oriented Malmquist productivity index and its components. Furthermore, at the level of the individual observations it can lead to opposite signs for a substantial fraction of the sample.

Equally so, this is the first contribution that investigates the impact of a convexification strategy on a metafrontier Hicks-Moorsteen TFP index. Though at the level of the individual observations, we can observe some opposite signs for some fraction of the sample, our empirical results show that a convexification strategy only leads to statistically significant differences for the balanced CRS case.

Finally, to the best of our knowledge, this is the first study comparing the Malmquist technology index with the Hicks-Moorsteen TFP index in a metafrontier setting. Just as in the case of a standard technology, the CRS case in our setting being identical, we do find contradictory results and statistically significant differences for the VRS case. This confirms earlier comparative results on standard technologies (e.g., Kerstens and Van de Woestyne (2014)).

In conclusion, we can state that this contribution has shown that a convexification strategy threatens to undermine the metafrontier methodology by yielding biased results. We may safely assume that these conclusions also transpose to alternative productivity indices such as, e.g., the Luenberger indicator and the dual Malmquist index, and to the primal TFP indices (such as the Färe-Primont and Lowe indices, among others).

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