Document de travail du LEM / Discussion paper LEM 2019-07

The estimation of pesticides marginal products: A Bayesian multi-output stochastic frontier with effective inputs

Kassoum AYOUBA
CESAER, AgroSup Dijon, INRA / ayouba.kassoum@inra.fr

Stéphane Vigeant
LEM UMR 9221 / stephane.vigeant@univ-lille.fr

http://lem.cnrs.fr/

Les documents de travail du LEM ont pour but d'assurer une diffusion rapide et informelle des résultats des chercheurs du LEM. Leur contenu, y compris les opinions exprimées, n'engagent que les auteurs. En aucune manière le LEM ni les institutions qui le composent ne sont responsables du contenu des documents de travail du LEM. Les lecteurs intéressés sont invités à contacter directement les auteurs avec leurs critiques et leurs suggestions.

Tous les droits sont réservés. Aucune reproduction, publication ou impression sous le format d'une autre publication, impression ou en version électronique, en entier ou en partie, n'est permise sans l'autorisation écrite préalable des auteurs.

Pour toutes questions sur les droits d'auteur et les droits de copie, veuillez contacter directement les auteurs.

The goal of the LEM Discussion Paper series is to promote a quick and informal dissemination of research in progress of LEM members. Their content, including any opinions expressed, remains the sole responsibility of the authors. Neither LEM nor its partner institutions can be held responsible for the content of these LEM Discussion Papers. Interested readers are requested to contact directly the authors with criticisms and suggestions.

All rights reserved. Any reproduction, publication and reprint in the form of a different publication, whether printed or produced electronically, in whole or in part, is permitted only with the explicit written authorization of the authors.

For all questions related to author rights and copyrights, please contact directly the authors.

The estimation of pesticides marginal products: A Bayesian multi-output stochastic frontier with effective inputs

Kassoum Ayouba
CESAER, AgroSup Dijon, INRA,
University Bourgogne Franche-Comté,
21000 Dijon, France
ayouba.kassoum@inra.fr
and

Stéphane Vigeant
Université de Lille, LEM – CNRS (UMR 9221)
Faculté des Sciences Économiques et Sociales
Université de Lille : Sciences et Technologies
59655 Villeneuve d'Ascq Cédex, France
and
IESEG School of Management
E-mail: Stephane.Vigeant@univ-lille.fr

April 2019

Abstract

Pesticide marginal products inform us, on the one hand, on the intensity of their use (allowing us to determine over- or under-optimal utilization) and, on the other hand, it tells us what the cost of a quantity regulation of pesticides would be in terms of lost production. The objective of this paper is to measure the marginal product of pesticides in order to guide pesticide regulation. Measuring marginal products requires that we estimate the production frontier of the underlying production process. In this paper, we propose a method that takes into account two major features of agricultural production processes that are rarely addressed together. First, we consider a multi-output production process, which is a standard characteristic of the production process of the vast majority of farms, but that is often neglected. We also reckon with the fact that pesticides are not standard productive inputs, as they do not directly produce outputs. Pesticides are pest managers and used to reduce the damages caused to the crops and so their role is to protect the potential output. This leads us to make the distinction between effective inputs and standard inputs. This allows us to capture the interaction between all production factors in a realistic way. We adapt this approach to pesticides to a flexible functional form (translog) that we estimate using a multi-output Bayesian stochastic production frontier. We derive the marginal product of pesticides for the farms located on the frontier. This methodology is applied to a sample of farms from the Eure-et-Loir *département* (French administrative region) for the period 2005-2008. The results show that farms in this area tend to use a suboptimal level of pesticides.

Keywords: Pesticides, marginal products, stochastic frontier analysis

JEL classification: Q12, Q18, Q52, C11

1. Introduction

Over the last fifty years, agriculture was successful at increasing global food production. Pesticides have played a major role in driving this growth, alongside with other technological innovations (e.g. development of irrigation and harvesting technologies). In particular, pesticides enhance production performance (prevent and/or reduce damage caused by pests and diseases) and allow farmers to reduce the use of other relatively more expensive inputs, such as labor. This has allowed the industry to spread their use and make pesticides uses a common practice.

However, the widespread use of pesticides was soon to raise some concerns on the side effects (verified empirically or not) they may have on wildlife and humans. We can trace these concerns back to 1960 with the publication of *Silent Spring* by Carson [1962] and *Pesticides and the living landscape* by Rudd [1964]. These authors have pointed out some of the potential risks related to pesticide use. Several studies have followed this path and have shown that, amongst others, pesticides play an important role in the pollution of surface water (Sharpley et al. [2001]) and may have a negative (lagged) effect on organisms that are beneficial for the farm (e.g. Wilson and Tisdell [2001], Skevas et al. [2013]). Verified negative side effects or precautionary principle have led governments to discuss and introduce policies with the objective of reducing pesticide use. For example, in France, the plan *Ecophyto2* aims to reduce pesticide consumption by 50% between 2015 and 2025.

The objective of this paper is to measure the marginal product of pesticides in order to give some guidance on the cost of pesticide regulation. Indeed, in order to implement a sound pesticides reduction plan, we need to understand the mechanism supporting their use at the farm level, and specifically their contribution to the farms' return. To this end, marginal products constitute a natural tool.

Measuring marginal products requires that we estimate the production frontier of the underlying production process. There are many problems to solve before achieving this objective. In this paper, we propose a method that takes into account two major characteristics of farming that are never or rarely considered jointly. First, we consider a multi-output production process, which is rarely addressed in the (parametric) marginal product literature of farms, despite the fact that it

characterizes the production process of the vast majority of farms. Second, we reckon with the fact that pesticides are not standard productive inputs, as they do not directly contribute to output production. Pesticides are pest managers and used to reduce the damages caused to the crops and so their role is to protect the potential output (see Lichtenberg and Zilberman [1986], Carpentier and Weaver [1997], Zhengfei et al. [2005], Karagiannis and Tzouvelekas [2011], Böcker et al. [2018] for applications with pesticides modeled as damage reducing input). Thus, in light of this literature, it is important to make the distinction between effective inputs and standard inputs while modelling the production process. In doing so, we capture the interaction between all production factors in what we believe to be a more realistic way.

To achieve this double objective, we specify a multi-input-multi-output production process, where the output aggregator is separable from the input management process. Following Carpentier and Weaver [1997], we propose an input specification that accounts for the difference between standard and damage reducing inputs. That is, the pesticides are introduced through a multiplicative input specific exponential quadratic function. This functional form allows us to identify downturn points (which characterize over-optimal use of pesticides). This function is inserted into a translog production function. We are able to show that, in spite of the nonlinear structure, the functional form linearizes so that all the parameters are identified, allowing us to infer pesticides specific effects for each individual input.

One more complication that we have to deal with is the fact that farms are not all using the best practice. When a farm is not efficient, the concept of marginal product is not well defined and usually cannot be measured meaningfully. In fact, marginal product is a concept well defined only on the production frontier. The only relevant information on the marginal product will derive from the behavior of the farms located on the frontier. To estimate the production frontier accounting for firms' inefficiency, we use a multi-output stochastic frontier. The estimation procedure derives from Fernandez et al. [2000] Bayesian estimation procedure. This methodology is applied to a sample of farms from the Eure-et-Loir *département* (French administrative region) for the period 2005-2008.

2. Pesticides and effective input

The estimation of pesticide marginal products in production processes started with Headley [1968] and has since raised many questions on how pesticides should be modeled. Essentially, the debate has been over the choice of functional form needed to capture the 'true' effect of pesticides. For a long time, these have been introduced in the specification as any other standard inputs. They were either substitutes or complements to the other inputs like any others and furthermore they had their own specific effect on the production. Lichtenberg and Zilberman [1986] claimed however, that when pesticides are introduced as standard input in the specification of the production function (as in Headley [1968], Carlson [1977], etc.) their marginal products are biased upward since in a world with no pests, pesticides are useless and do not contribute to production. Clearly, pesticides only protect the production and do not create it. The resulting strategy was to model the pesticides as damage reducing inputs, so they only indirectly affect the efficiency of the other inputs. Consequently, it created a real issue concerning the functional form of the damage reduction function and how it must be integrated into the input functional form.

2.1. Damage reduction function

The damage reduction function is part of a production process that we now describe. A farm produces outputs, $\mathbf{y} \in \mathbb{R}_+^n$, with standard inputs, $\mathbf{x} \in \mathbb{R}_+^m$ and pesticides, $\mathbf{z} \in \mathbb{R}_+^s$. We suppose that all inputs are under the control of the farmers at decision time. Before specifying the relationship between the inputs and outputs and how pesticides are related to the production process, we address the problem of interpreting formally the role of pesticides.

Following Lichtenberg and Zilberman [1986], we model pesticides as damage reducing inputs. These inputs do not have a direct effect on the production, as their role is to protect the crops from pests and other factors that may reduce the returns. Pesticides are usually modeled using a function that relates the quantity of pesticides used ($\mathbf{z}$) and pest pressure ($\mathbf{r}$). The damage reduction function, $\phi(\mathbf{z}, \mathbf{r})$, is a mapping defined as:

$$\phi: \mathbb{R}_+^s \times \mathbb{R}_+^m \rightarrow [0,1]. \qquad (1)$$

When the damage caused by the pest is maximal ϕ goes to zero, and when there is no damage the function is equal to one, $\phi(\mathbf{z}, \mathbf{r}) = 1$. It is assumed that there exists a quantity of pesticide, $\mathbf{z}^*$, such that $\phi(\mathbf{z}^*, \mathbf{r}) = 1$. Lichtenberg and Zilberman [1986] assumed that $\phi(\cdot)$ is an increasing function of $\mathbf{z}$ and tends to its maximum, $\phi(\boldsymbol{z}^*, \mathbf{r}) = 1$. This last assumption may be too strong however, as the pesticides may have a negative effect on the production past a given threshold, and thus may be decreasing in $\mathbf{z}$. Following Carpentier and Weaver [1997] such a function must satisfy the following requirements:

Property 1. *The damage reduction function* $\phi(\mathbf{z}, \mathbf{r})$ *satisfies:* (i) $\phi(\mathbf{z}, \mathbf{r})$ *is continuously differentiable;* (ii) $0 \leq \phi(\mathbf{z}, \mathbf{r}) \leq 1$ *and in particular* $0 \leq \phi(0, \mathbf{r}) \leq 1$*;* (iii) $\phi(0,0) = 1$ *and* $\phi(\mathbf{z}, 0) \leq 1$ *with* $\mathbf{z} \geq 0$ (iv) $\partial\phi(\mathbf{z}, 0)/\partial \mathrm{z}_k \leq 0$ *;* $\partial\phi(0, \mathbf{r})/\partial \mathrm{r}_o \leq 0$*;* (v) $\partial\phi(\mathbf{z}, \mathbf{r})/\partial \mathrm{r}_\mathrm{o} \leq 0$ *for all* $\mathbf{z}$ *and* $\partial\phi(\mathbf{z}, \mathbf{r})/\partial \mathrm{z}_k \geq 0$ *for some* $\mathbf{r}$*;* (vi) *For all* $\mathbf{r}$ *there exists a* $\mathbf{z}$ *such that* $\phi^\star = \max_z \phi(\mathbf{z}, \mathbf{r}) = \phi(\mathbf{z}^\star, \mathbf{r})$.

The first and second properties tell us that pests may reduce output, but not necessarily, and that damages are possible with and without pesticides. The third property tells us that when there is no pest, we get the full production if no pesticides are used, but using pesticides might reduce the production. The fourth property tells us that increasing pesticides with no pest present may reduce the output, while increasing pest pressure would result in damages. Note that $\partial\phi(\mathbf{z}, 0)/\partial \mathrm{z}_k \leq 0$ is often neglected. Property *(v)* tells us that, given a quantity of pesticide, increasing the pressure increases the damages and that, for some pest pressure, pesticides will always reduce the damage. In practice, a monotonic relationship is specified, as pesticides cannot damage the crops. In this paper, we will suppose that this relationship is not necessarily monotonic. That is, for a given pest pressure, we may reach a pesticide level that will damage the crop. This property is essentially summarized by property (vi).

In practice, the functional forms mostly used are the Pareto, exponential and Weilbull distribution functions. Lichtenberg and Zilberman [1986] and Carpentier and Weaver [1997], Karagiannis, and Tzouvelekas [2011], among others, used the exponential function. In the simple case of one type of pesticide and no pest pressure, we have:

$$\phi(\mathrm{z}) = 1 - \exp\{-\phi_0 - \phi_1 \mathrm{z}\} \text{ with } \phi_0 \geq 0, \phi_1 \geq 0. \tag{2}$$

This specification systematically returns a positive contribution to output, eliminating the possibility of pesticides overdosage that would destroy the crop. A possible alternative is the function proposed by Zhengfei *et al.* [2005]:

$$\phi(\mathrm{z}) = \exp\{-(\phi_0 + \phi_1 \mathrm{z})^2\}. \qquad (3)$$

This function is interesting because there is a threshold after which it is decreasing, reflecting the potential over dosage. It has also an inflection point, implying that pesticides can have an initial accelerating contribution and then a decreasing marginal effect until it becomes negative after the threshold. This function is illustrated on Figure 1. The parameters are free to take any value and the maximal dosage, before negative effect is reached, is $\mathrm{z}_{\max} = -\phi_0/\phi_1$. This function will be inserted in the production technology.

[INSERT Figure 1 ABOUT HERE]

2.2. Transformation function and pesticides

The next step is to introduce the damage reducing function into the production process. Lichtenberg and Zilberman [1986] have proposed to introduce pesticides as damage reducing inputs and to model this input asymmetrically with respect to the standard inputs. Essentially, in the case of a single output, we have:

$$\mathrm{y} = h(\mathbf{x}, \mathbf{z}, \mathbf{r}) = h^{\max}(\mathbf{x})\phi(\mathbf{z}, \mathbf{r}) \,, \qquad (4)$$

where $\phi(\mathbf{z}, \mathbf{r})$ is the output oriented damage reduction function, $h^{\max}(\mathbf{x})$ is the maximum output that can be obtained from $\mathbf{x}$. This is Lichtenberg and Zilberman [1986] standard specification. Although the specification permits interaction between inputs, as the marginal product of any standard inputs depends on the quantity of pesticides, the marginal rate of substitution between these standard inputs is independent of the quantity of pesticides which is clearly not a common feature of agricultural production processes.

Carpentier and Weaver [1997] have proposed a specification that does not carry this limitation. Instead of having pesticides affecting the production directly, they assume that the effectiveness of each standard input changes with the pest pressure and the quantity of pesticide used. That is, $\mathrm{x}_i^e = \phi_i(\mathbf{z}, \mathbf{r})\mathrm{x}_i$ where x_i^e is the i^{th} effective input when one uses the quantity x_i of regular input and pesticide quantities $\mathbf{z}$ and facing pest pressures $\mathbf{r}$. Thus, we have:

$$y = h(\mathbf{x}, \mathbf{z}, \mathbf{r}) = h^e(\phi_1(\mathbf{z}, \mathbf{r})x_1, \ldots, \phi_m(\mathbf{z}, \mathbf{r})x_m) = h^e(\mathbf{x}^e). \quad (5)$$

This specification is consistent with the expected results as the marginal products and the marginal rate of substitution all depends on the inputs and the pesticides used. This is a complete specification of an agricultural production process taking into account the specificity of the pesticides in the production process. However, farms are multi-output production units and this is not accounted for in this specification.

Agricultural production processes are almost always multi-crops, a feature that is often neglected. Practitioners were either aggregating all outputs implicitly reverting to a single output production process model or they simply modeled each crop individually. To take into account the multi-output features of the production process, we also have the option of modelling the entire production process at once. This is the approach we prefer and to do so we adapt the Carpentier and Weaver [1997] approach above to the multi-crop production process. The generalization is a transformation function:

$$H(\mathbf{y}, \mathbf{x}, \mathbf{z}, \mathbf{r}) = 0, \quad (6)$$

in which we introduce the effective inputs, $x_i^e = \phi_i(\mathbf{z}, \mathbf{r})x_i$, to obtain:

$$H^e(\mathbf{y}, \mathbf{x}^e) = H^e(\mathbf{y}, \phi_1(\mathbf{z}, \mathbf{r})x_1, \ldots, \phi_m(\mathbf{z}, \mathbf{r})x_m) = 0. \quad (7)$$

The estimation of this type of function represents a considerable challenge as the interaction between the individual outputs and the inputs increases in the dimensionality of the problem. One possible way out of this problem is to assume output separability. This supposes that we have an output aggregator on the one hand and an input aggregator on the other hand. Formally, we can write:

$$A(\mathbf{y}) = h^e(\phi_1(\mathbf{z}, \mathbf{r})x_1, \ldots, \phi_m(\mathbf{z}, \mathbf{r})x_m), \quad (8)$$

where $A(\mathbf{y})$ is the aggregator function, h^e is the effective production function and the functions $\phi_i(\mathbf{z}, \mathbf{r})$ are the input oriented damage reduction functions. This is the generalized Carpentier and Weaver specification used in our empirical analysis.

To obtain an empirical specification for our model of the production process, we need to specify the aggregator and the production function. We begin with the production side of the problem.

This implies obtaining an explicit specification for the estimated functional form that will encompass the standard inputs and the damage function.

We use a translog functional form for the production technology of the m effective inputs, x_i^e with $i = 1, \ldots, m$. Applying this to Equation (8) we get:

$$\ln[A(\mathbf{y})] = \ln[h^e(\phi_1(\mathbf{z},\mathbf{r})\mathrm{x}_1, \ldots, \phi_m(\mathbf{z},\mathbf{r})\mathrm{x}_m)] = \boldsymbol{\delta}(\mathbf{x},\mathbf{z})$$

$$= \beta_0 + \sum_{i=1}^{m} \beta_i \ln(\mathrm{x}_i^e) + \frac{1}{2}\sum\nolimits_{j=1}^{m}\sum\nolimits_{i=1}^{m} \beta_{ij} \ln(\mathrm{x}_i^e)\ln(\mathrm{x}_j^e). \quad (9)$$

For reasons that we will explain in the data section below, we assume that the pest pressure is uniform for all units, and can be ignored so that the damage reducing function depends only on the pesticides.[1] We also assume that $\mathbf{z}$ is a scalar (only one pesticide) so that $\phi_i(z)$ is a scalar function. The functional form for the damage reducing function is:

$$\mathrm{x}_i^e = \mathrm{x}_i\phi_i(z) = \mathrm{x}_i\exp\{-(\phi_{0i} + \phi_{1i}z)^2\} \qquad i = 1, \ldots, m. \quad (10)$$

Substitution of equation (10) into equation (9) gives:

$$\boldsymbol{\delta}(\mathbf{x},\mathrm{z}) = \beta_0 + \sum_{i=1}^{m} \beta_i \ln(\mathrm{x}_i) + \sum_{i=1}^{m} \beta_i \ln\big(\phi_i(z)\big) + \frac{1}{2}\sum_{i=1}^{m}\sum_{j=1}^{m} \beta_{ij}[\ln(\mathrm{x}_i) + \ln(\phi_i(z))]\big[\ln\big(\mathrm{x}_j\big) + \ln\big(\phi_j(z)\big)\big]. \quad (11)$$

Regrouping terms gives a translog function linear in the parameters:

$$\boldsymbol{\delta}(\mathbf{x},\mathrm{z}) = \vartheta_0 + \sum_{i=1}^{m} \gamma_i \ln(\mathrm{x}_i) + \sum_{i=1}^{m}\sum_{j\geq i}^{m} \pi_{ij} \ln(\mathrm{x}_i)\ln\big(\mathrm{x}_j\big) + \sum_{i=1}^{m} \psi_i \ln(\mathrm{x}_i)\mathrm{z} + \sum_{i=1}^{m} w_i \ln(\mathrm{x}_i)\mathrm{z}^2 + \sum_{i=1}^{m} \varphi_i \mathrm{z}^i, \quad (12)$$

where $\vartheta_0 = \beta_0 - \sum_{i=1}^{m} \beta_i\, \phi_{0i}^2 + \frac{1}{2}\sum_{i=1}^{m} \beta_{ii}\, \phi_{0i}^4 + \sum_{i=1}^{m-1}\sum_{j>i}^{m} \beta_{ij}\, \phi_{0i}^2\phi_{0j}^2$, $\gamma_i = \beta_i - \sum_{j=1}^{m} \beta_{ij}\, \phi_{0j}^2$, $w_i = -2\sum_{j=1}^{m} \beta_{ij}\, \phi_{0j}\phi_{1j}$, and $\psi_i = -\sum_{j=1}^{m} \beta_{ij}\, \phi_{1j}^2$, $\pi_{ii} = \frac{1}{2}\beta_{ii}$ for $i = 1, \ldots, m$, $\pi_{ij} = \beta_{ij}$ $i = 1, \ldots, m-1$ and $i < j = 2, \ldots, m$, $\varphi_1 = -2\sum_{i=1}^{m} \gamma_i\, \phi_{0i}\phi_{1i}$, $\varphi_2 = -\sum_{i=1}^{m} \gamma_i\, \phi_{1i}^2 - \sum_{i=1}^{m} \psi_i\, \phi_{0i}\phi_{1i}$, $\varphi_3 = -2\sum_{i=1}^{m} w_i\, \phi_{0i}\phi_{1i}$, and finally $\varphi_4 = -\frac{1}{2}\sum_{i=1}^{m} w_i\, \phi_{1i}^2$. It is not necessarily

[1] In our application we use data for a small region, so the farmers are all very likely to face a homogeneous pest pressure and this allows us to remove pest pressure from the specification.

obvious, but all parameters are identified, except for the sign of ϕ_{1i} and ϕ_{0i}. As it turns out, for all the quantities we need to estimate, the sign of these parameters does not interfere with the characterization of the results. That is, $\phi_i = \exp\{-(\phi_{0i} + \phi_{1i}\mathrm{z})^2\} = \exp\{-(\phi_{0i}^2 + \phi_{1i}^2\mathrm{z}^2 + 2\phi_{0i}\phi_{1i}\mathrm{z})\}$ and we directly get ϕ_{0i}^2, ϕ_{1i}^2 and $\phi_{0i}\phi_{1i}$. Consequently, we have all the necessary parameters to estimate the pesticide marginal product.

Since we have:

$$\frac{dA(\mathbf{y})}{d\mathrm{z}}\frac{1}{A(\mathbf{y})} = \frac{d\ln A(\mathbf{y})}{d\mathrm{z}} = \frac{d\boldsymbol{\delta}(\mathbf{x},\mathrm{z})}{dz}. \tag{13}$$

Using equation (12), the pesticide marginal product on the aggregate production is given by:

$$\frac{dA(\mathbf{y})}{d\mathrm{z}} = \left[\sum_{i=1}^{m}\psi_i\ln(\mathrm{x}_i) + 2\mathrm{z}\sum_{i=1}^{m}w_1\ln(\mathrm{x}_i) + \sum_{i=1}^{m}i\varphi_i\,\mathrm{z}^{i-1}\right]A(\mathbf{y}). \tag{14}$$

Using the same logic, the marginal product of pesticides on output y_j, is obtained as follows:

$$\frac{dA(\mathbf{y})}{d\mathrm{y}_j}\frac{d\mathrm{y}_j}{d\mathrm{z}} = \frac{d\boldsymbol{\delta}(\mathbf{x},\mathrm{z})}{d\mathrm{z}}A(\mathbf{y}), \tag{15}$$

so that:

$$\frac{d\mathrm{y}_j}{d\mathrm{z}} = \left[\sum_{i=1}^{m}\psi_i\ln(\mathrm{x}_i) + 2\mathrm{z}\sum_{\boldsymbol{i=1}}^{\boldsymbol{m}}w_1\ln(\mathrm{x}_i) + \sum_{i=1}^{m}i\varphi_i\,\mathrm{z}^{i-1}\right]A(\mathbf{y})\left[\frac{dA(\mathbf{y})}{d\mathrm{y}_j}\right]^{-1}. \tag{16}$$

The marginal product of the standard input i is given by:

$$\frac{dA(\mathbf{y})}{d\mathrm{x}_i} = \frac{d\boldsymbol{\delta}(\mathbf{x},\mathrm{z})}{d\ln(\mathrm{x}_i)}\frac{A(\mathbf{y})}{\mathrm{x}_i} = \left[\gamma_i + \sum_{j=1}^{m}\pi_{ij}\ln(\mathrm{x}_j) + \psi_i z + w_i\mathrm{z}^2\right]\frac{A(\mathbf{y})}{\mathrm{x}_i}. \tag{17}$$

The marginal product of the input x_i on the output y_j is obtained by multiplying equation (17) by $\left[dA(\mathbf{y})/d\mathrm{y}_j\right]^{-1}$.

A version of the *CES* functional form is our choice for the aggregator, $A(\mathbf{y})$, to model the output. That is:

$$A(\mathbf{y}) = \left(\sum_{j=1}^{n} \alpha_j^q \, \mathrm{y}_j^q \right)^{\frac{1}{q}}, \tag{18}$$

where $\alpha_j \in (0,1)$ for $j = 1, \ldots, n$ and $\sum_{j=1}^{n} \alpha_j = 1$. The elasticity of substitution is $1/(1-q)$. We impose $q > 1$, so that increasing one of the output necessitate a reduction of another output to keep $A(\mathbf{y})$ constant. Barnett (1985) has shown that the CES functional form is flexible for $n = 2$, so we feel that using this specific function is not hurting the generality of our specification since we will be using two outputs in our application.

3. Estimation procedure

Now that we have specified the technology, we have to complete it with the operating circumstances of the farms. It is common practice to allow farmers to operate under the production frontier, so that some inefficiency in the production process is possible. Clearly, the marginal product is meaningful only for efficient farms as, for any inefficient unit, it is possible to arbitrarily change its output in any direction by changing its inputs. For a firm located on the frontier, this is not possible however, as it is bounded by the frontier. That is, the output change induced by increasing an input cannot exceed what the production frontier allows the farmer to do, and this what we refer to as the marginal product. Consequently, the production frontier contains the only relevant information on the marginal products. A Stochastic Frontiers Analysis (*SFA*) framework, introduced by Aigner *et al.* [1977] and Meeusen and Van den Broeck [1977], is implemented to estimate the frontier. Essentially, we suppose that the model is made of three components: The frontier itself, an error term and technical inefficiency.

The biggest challenge is to account for the multi-output technology. The estimation procedure of such multi-output technologies typically involves factoring out one of the outputs or inputs and estimating the resulting equation using maximum likelihood methods (e.g. Lovell *et al*. [1994]). There is a problem of endogeneity in this setup, as the error terms might be correlated with the variables entering the frontier itself (see O'Donnell [2012]). Fernandez *et al.* [2000] propose a Bayesian approach that does not require the factorization stage and and treats directly the endogeneity problem without having to use an instrumental variable method. This procedure is applied to the transformation function specified in the preceding section. The method requires a

specification for the data density and a prior distribution for each parameter. The posterior distribution of each individual parameter is then deduced.

Using the production structure defined in the previous section and introducing efficiency, we obtain the following model:

$$A(\mathbf{y}) = h^e(\mathbf{x}, \mathrm{z})\tau, \tag{19}$$

where τ is the efficiency term, satisfying $0 \leq \tau \leq 1$, $h^e(\mathbf{x}, \mathrm{z})$ is a translog function representing the maximum output that can be produced using inputs $\mathbf{x}$ and pesticides z. The production surface is given by all possible production vectors $\mathbf{y}$ such that $A(\mathbf{y})$ equals a constant and is of the *CES* type, as defined above (Equation (18)). We are not restricted to this type of function for the aggregator, but this functional form allows us to specify independent prior distributions for q and $\boldsymbol{\alpha}$. For given values of $\boldsymbol{\alpha}$, q and $A(\mathbf{y})$, Equation (18) describes a surface of dimension $(n-1)$ corresponding to the vectors of dimension n used to construct the aggregate production. This aggregator allows us to reduce a multi-output problem to a single output problem so that a standard *SFA* specification can be implemented.

Suppose that the error term is multiplicative, *i.e.* $A(\mathbf{y}) = h^e(\mathbf{x}, \mathrm{z})\tau e^{\varepsilon}$, then applying the logarithmic transformation on both sides of this equation leads to a standard *SFA* model:

$$\ln A(\mathbf{Y}) = \mathbf{V}\boldsymbol{\beta} - \mathbf{u} + \boldsymbol{\varepsilon} \tag{20}$$

where $\ln(h^e(\mathbf{x_d}, \mathrm{z}_d)\tau_d e^{\varepsilon_d}) = \mathbf{V_d}\boldsymbol{\beta} - \mathrm{u}_d + \varepsilon_d$, $\mathbf{V_d} = \boldsymbol{v}(\mathbf{x_d})$ is a function (mostly logarithmic transformations and cross product) of the $m+1$ inputs (m standard inputs and one damage reducing input) for farm d, $\boldsymbol{\beta} \in \Im \subseteq \mathbb{R}^k$,[2] $\mathbf{V_d}\boldsymbol{\beta}$ is given by Equation (12), $\tau_d = \exp\{-u_d\}$, and ε_d is a standard noise defining the stochastic frontier. Since $d = 1, \ldots, D$, $\mathbf{V} = [v(\mathrm{x}_1), \ldots, v(\mathrm{x}_D)]^{\top}$ is a $(D \times k)$ matrix of independent variables, $\mathbf{u}$ and $\boldsymbol{\varepsilon}$ are $(D \times 1)$ vectors and $\mathbf{Y}$ is $(D \times n)$ output matrix. Technical inefficiency means that some observations are located under the production frontier. That is, $0 \leq \tau_d \leq 1$ and $\tau_d = \exp\{-u_d\}$, so that $u_d > 0$ and $\mathbf{u} > 0$.

We now have all the ingredients of the model (the production technology, the outputs and the inefficiency) so we have to construct the statistical model to estimate the parameters. Because

[2] Because $\pi_{ij} = \pi_{ji}$, there are 27 independent parameters in Equation (12), so *k=27*.

there is no restriction on the sign of $\boldsymbol{\varepsilon}$, we assume that ε_d is identically and independently normally distributed for $d = 1, \ldots, D$ with variance σ^2. The density function of $\mathbf{V\beta} - \mathbf{u} + \boldsymbol{\varepsilon}$, given $(\boldsymbol{\beta}, \mathbf{u}, \sigma)$, is:

$$p(\,\mathbf{V\beta} - \mathbf{u} + \boldsymbol{\varepsilon} \mid \boldsymbol{\beta}, \mathbf{u}, \sigma\,) = f_N^D(\mathbf{V\beta} - \mathbf{u}, \sigma^2 I_D) \tag{21}$$

where f_N^D is the multivariate normal distribution. In spite of the fact that we have extended the *SFA* model to the multi-output case, we have yet to specify the density and the data generating process of the outputs, $(\mathbf{y}_d = (y_{d,1}, \ldots, y_{d,n})^\top)$. That is, the model defined by (21) is good to characterize the likelihood of $\mathbf{lnA}(\mathbf{y_d}) = \mathbf{V_d\beta} - \mathrm{u}_d + \varepsilon_d$ but is silent on the distribution of the individual outputs, $\mathbf{y}_d$.

To complete the model we need to construct a relationship between the aggregate output and the individual components. Instead of dealing directly with the individual outputs, we can as well deal with their respective weights in the aggregate, this will convey just about the same information for our needs. Let these weights be denoted as $\boldsymbol{\eta}_d = (\eta_{d,1}, \eta_{d,2}, \ldots, \eta_{d,n})^\top$. There exists a correspondence between the observed output vector $\mathbf{y}_d \in \mathbb{R}_+^n$ $(\mathbf{y}_d = (y_{d,1}, \ldots, y_{d,n})^\top)$, and the n −vector $(\delta_d, \eta_{d,2}, \ldots, \eta_{d,n})^\top$ so that:

$$f_y(y_{d,1}, y_{d,2}, \ldots, y_{d,n}) = f_N^D(\delta_d) \times f(\,\eta_{d,2}, \ldots, \eta_{d,n} \mid \delta_d\,) \cdot |\mathbf{J}| \tag{22}$$

where $|\mathbf{J}|$ is the absolute value of the Jacobian of the transformation. Now, because $0 \leq \eta_{d,j} \leq 1$ and $\sum_{j=1}^n \eta_{d,j} = 1$, we assume that the $\eta_{d,j}$ are jointly Dirichlet distributed. The details of the derivation are given in Appendix A.

Under the assumption that all farms are identically and independently distributed, we have the likelihood function of the model:

$$p(\mathbf{Y} \mid \boldsymbol{\beta}, \mathbf{u}, \sigma, \boldsymbol{\alpha}, q, \mathbf{s}) = f_N(\mathbf{V\beta} - \mathbf{u}, \sigma^2 I_D) \times f_{Dir.}^{n-1}(\,\boldsymbol{\eta}_d \mid \mathbf{s}\,) \times \prod_{j=1}^{n} q^{n-1} \prod_{j=1}^{n} \frac{\eta_{d,j}}{y_{d,j}} \tag{23}$$

To proceed to the estimation step, the model must completed with the parameter prior distributions, $\boldsymbol{\beta}, \mathbf{u}, \sigma, \boldsymbol{\alpha}, q$ and $\mathbf{s}$. As shown by Koop *et al.* [1997], it is preferable to use proper priors because improper distributions may lead to problems for *SFA* models. Because we have virtually no information on the location of the parameters in the parameter space, we choose to be

as uninformative as possible on the distribution choice. As in Fernandez *et al.* [2000], we use independent priors:

$$p(\boldsymbol{\beta}, \mathbf{u}, \sigma, \boldsymbol{\alpha}, q, \mathbf{s}) = p(\boldsymbol{\beta})p(\mathbf{u})p(\sigma)p(\boldsymbol{\alpha})p(q)p(\mathbf{s}) \quad (24)$$

We use this independence assumption to choose parameter distributions that do not impose structure, so that no involuntary information is introduced into the inference procedure. We also choose proper non-informative priors for each parameter, as in Fernandez et al. [2000] (*i.e.* we choose the hyper parameters of the priors so that the prior is as uninformative as possible). The specific priors are listed in Appendix B.

The product of the likelihood (*i.e.* the density of the observations given by equation (24)) and the priors defines the posterior distribution of our Bayesian model. We cannot obtain a closed form solution for the posterior distribution, so a Monte Carlo by Markov Chain (*MCMC*) is used to approximate the posterior distribution and to obtain the empirical distributions of the parameters. We partition the posterior distribution for all the model's parameters and each group of parameters is simulated sequentially given the data and the values of the other parameters. This algorithm (Gibbs algorithm) converges to the true distribution of the entire set of parameters. The derivation of the posterior distributions and the simulation procedure we have adopted for the estimation are presented in Appendix C.

4. The data

The data used in this paper come from the *POPSY* (Arable Crop Production, Environment and Regulation) project database (*CERFRANCE Alliance Centre*). The individual units are crop farms in the Eure-et-Loir *département* in France, and cover the period 2005-2008. The data were obtained through a voluntary survey of farmers in this region. Farmers' participation in the survey was voluntary and the data collected are anonymous. After cleaning for missing and inconsistent data, an unbalanced panel of 3,462 arable farms is obtained. The data set contains farm-level information on physical characteristics (outputs and inputs) and economic-financial data (revenues from specific products and product groups, expenses related to input use, subsidies etc.). Four inputs (land, labor, capital and materials), one damage control input and three outputs characterize the production technology. Land is measured using the Utilized Agricultural Area

(UAA) of each farm in hectares (ha), labor is obtained by aggregating family labor and hired workers, and measured in Annual Work Units (AWU). Materials are the sum of intermediate consumption ($Int.Cons.$, which includes operational costs like fertilizers, and seeds) and other costs (water, gas, electricity, maintenance and repair work), and are measured in Euros. Depreciation ($Dep.$) approximates the capital stock (equipments ans buildings) of the farms and is also measured in Euros. Finally, the consumption of pesticides ($Pest.$) at the farm level (variable pesticides) is our damage control input.

The survey we used contains information on 24 crops, all grown in the *département*. This information is mostly related to the surface sown. We do not have the information on the production or the value for the specific crops, however. The information we can directly use is the value in Euros of three aggregate outputs: cereal crops, industrial crops and miscellaneous crops. The composition of these aggregated outputs is detailed in Appendix D. The Bayesian methodology we implement allows us to deal with multiple outputs but, unfortunately it does not work with farms that have zeros production for one or more outputs. In our sample, most farms (81.02%) have zeros for miscellaneous crops. Based on this observation, we have decided to aggregate industrial and miscellaneous crops together. Then we have in our sample two aggregate outputs: cereals (Cer.) and the industrial-miscellaneous (Ind-Oth) crops. Finally, we have dropped farms with zero values in one of these two aggregate outputs. Our final database contains information on $3{,}420$ arable farms, distributed over the years as follows: 937 farms in 2005, 916 in 2006, 913 in 2007 and 654 in 2008.

The variables measured in Euros (materials, depreciation, pesticides and the aggregated outputs) have been deflated using adequate price indexes from the French National Institute of Statistics and Economic Studies ($INSEE$) using 2005 as te base year. The exact construction of these price indexes is presented in Appendix D. Table 1 contains descriptive statistics of the variables used to characterize the technology.

[INSERT TABLE 1 ABOUT HERE]

5. The Results

The estimation involves simulating the posterior distribution using Gibbs algorithm. We simulate parameters $\boldsymbol{\beta}$, σ, λ and $\mathbf{u}$ from known distributions. As explained in Appendix C, the posterior distributions of $\boldsymbol{\alpha}$, q and $\mathbf{s}$ do not have standard forms so, following Fernandez et al. [2000], we simulated them using the Metropolis random walk algorithm. The components of $\boldsymbol{\alpha}$ sum to one so we only have to simulate one of the two components of this vector, say α_1, using a unidimensional transition density, and the other parameter is naturally deduced as $\alpha_2 = 1 - \alpha_1$. For q and the two independent components of the vector $\mathbf{s}$ we draw values from a unidimensional transition density. The mean of the density is given by the previous value in the chain. The simulated value, called a candidate, enters the algorithm with a given probability. In Table 2 we present the standard errors used to calibrate the algorithm and the corresponding acceptation rates. The parameter acceptance probabilities range from 26% to 48%, which is standard.

[INSERT TABLE 2 ABOUT HERE]

The starting values used to initialize the Gibbs algorithm are 0.5 for all components of $\boldsymbol{\beta}$, 0.5 for the standard error (σ), 10 for λ, 0.5 for both elements of $\boldsymbol{\alpha}$, 2 for q, 1 for both components of $\mathbf{s}$ and $\mathbf{u}$. For each year we conducted $260{,}000$ simulations. We have eliminated the first $10{,}000$ simulations to reduce the dependence of the results on the initial values. From the $250{,}000$ remaining simulations, we keep one value per cycle of ten iterations to avoid correlation between successive steps. Thus the results rest on $25{,}000$ simulations.

To verify that the algorithm has converged, we have performed standard tests. The plots of the simulations did not show any systematic pattern or trend, so the visual check seems to confirm convergence. The Geweke and the Gelman and Rubin convergence tests are performed to get formal confirmations of the algorithm convergence. The Geweke test tests the equality between the averages of the first 10% and of the last 50% of the chain. The results are reported in Table 3. The test statistic is distributed standard normal so we cannot reject convergence at 95%.

[INSERT TABLE 3 ABOUT HERE]

The Gelman and Rubin test allows us to check the global sensitivity of the final output to the starting values. Table 4 gives the starting values we have tried. These starting values have no

impact on the simulated chains. In other words, the estimated values do not depend on the initial conditions of the algorithm. Details on the algorithm are provided in Appendix C.

[INSERT TABLE 4 ABOUT HERE]

Tables 5 and 6 contain the estimated values of the parameters of our Bayesian model for the years 2005-2008. For each estimated mean of the parameters, we present a 95% confidence interval. Note that for each parameter of the model it is the 2.5% and 97.5% percentiles of the simulated chain that are used as the upper and lower bound of the confidence interval.

[INSERT TABLE 5 ABOUT HERE]

[INSERT TABLE 6 ABOUT HERE]

Table 7 gives an overall view of the technical efficiency scores as calculated using the Bayesian model. As for the estimated parameters, the mean of the simulated efficiency parameters is considered as the technical efficiency score of a typical farm.

[INSERT TABLE 7 ABOUT HERE]

The technical efficiency scores are fairly stable over the years spanned by our sample with no noticeable trend. There is a small increase in 2007 followed by a small decrease on 2008, but nothing that stands out. Roughly, given the inputs used by the farms, the production of Eure-et-Loir farms can be increased, on average, by a factor of 20%. These results are consistent with the literature on farm production.

Now that we have all the estimated parameters of our model, we can turn to the marginal product of the pesticides. The pesticide marginal product on the aggregate production is given by Equation (14). We present the results for the farm at the median marginal productivity. This choice is natural because the median is robust to extreme values, and in addition the median is not a virtual unit, it corresponds a real farm in the sample. We also report, for an illustrative purpose the result for the average over all farms. The results are presented in Table 8.

[INSERT TABLE 8 ABOUT HERE]

There are two possible ways to look at the results. The first one is the standard cost-benefit analysis. The problem that we have here, as we mentioned in the introduction, is that we do not

have the social cost to adjust the market price of pesticides to account for their true cost. However, the estimated marginal product can be compared to the actual market price of pesticides to determine how farmers behave with respect to market signals. When farmers are optimizing and behaving rationally, the value of the marginal product of pesticides must be equal or at least close to the price of the pesticides. Because we have a multi-output production process we can look at the value of the marginal product for the aggregate output and for each individual output. A characterization of the farms optimizing behavior would compare pesticide's marginal products to the real price (pesticide price divided by the output price). The pesticide price is obtained from the *INSEE* website. The computation of the aggregate output price index uses an index of the cereal mix and the industrial other crops for each farm for each year in our sample. Then, we use the estimated aggregator to compute the composite index for the median farm. (The computational details are presented in the Appendix D.) The results are reported in Table 9.

[INSERT TABLE 9 ABOUT HERE]

Table 10 presents a comparison of the pesticide real price of pesticides to the marginal effect of pesticides on the aggregate production for the median farm for each year in our sample. Recall that we have modeled the pesticides as a damage reducing input, so this marginal effect is interpreted as the number of aggregate production units protected or not damaged when using one more unit of pesticide. This is not per se a marginal product, because we do not filter out the aggregator effect, we will do that next by considering the specific crop effects. There is one very interesting features of pesticides use: as the real price of pesticides decreased, so did the marginal product. In other words, there is some rationality in the use of pesticides; cheaper pesticides are, larger the quantity of pesticides used (decreasing marginal effect). It is difficult to say more on this relationship however, because it just relates the aggregated output and the overall marginal use of pesticides. Therefore, we have to look at the relationship between the specific output and pesticides. Nonetheless, we have highlighted that the price signal works and induce a behavior.

[INSERT TABLE 10 ABOUT HERE]

Now we address the asymmetric effect of pesticides on the crops, that is we exploit the multi-output features of our model. We would like to know how the farm's choice of crop mix is related to pesticide uses. That is, we would like to know how specific crops relate to pesticide use and how it responds to price incentives. To do this, we combine the definition of the output

specific marginal product, Equation (16), with the definition of the aggregator. The results are presented in Table 11.

The results are clear on one point; the pesticide marginal product for the cereals is larger than it is for the other crops. It also seems that cereal production does not respond perfectly to price signals when compared to the response of other crops. This might mean that pesticide control incentives (policies) may have had a larger effect on the cereal production process than on the other crops. In other words, the pesticide use is not neutral with respect to the type of crops and the restrictions have been probably more demanding for cereals or it was easier to apply them to cereal than to other crops in Eure-et-Loir. The results also point out that every year, and for both outputs, farmers are not using as much pesticides as it would be optimal to do so from a pure profit maximizing behavior.[3] This "sub-optimal" behavior can probably be attributed to the effect of the environmental regulation and practices. This suggests that the declared policy objective to reduce pesticide use may have a direct impact on the farmers in Eure-et-Loir through various programs ranging from education on environmental practices to specific restrictions. That is, the government might be using other tools than price to induce farmers to use less pesticide and this shows up as larger than expected marginal products than those we would expect when farms are pure profit maximizers. In other words, the Authorities' pesticide management has an impact on farmers' practices as they use less pesticide than they would otherwise.

INSERT TABLE 11 ABOUT HERE]

At this stage we can look for a characterization of farms' pesticide use and in particular we can explore how it is related to farm size. The first step is to look at the marginal product as a function of the size of the land (UAA). For each quartile of the size we choose the median of the marginal product. Table 12 presents these results.

[INSERT TABLE 12 ABOUT HERE]

[3] We compare pesticide marginal product of each specific output to real pesticide price (pesticide price divided by the specific output price).

We observe, for all years, that the bigger the farm is, the larger the pesticide marginal product is. This positive relationship between the marginal product of pesticides and the size of the farms is observed for both outputs. The largest farms tend to drift away from a profit maximizing quantity of pesticide. In other words, the large farms seem to be less bound to price incentives. So the question is what can explain such a different behavior between small and large farms? One possible explanation is an asymmetric impact of the regulation, the impact being more strongly felt by large farms than small ones. This might be due to the fact that large farms have an easier access to common agricultural policies (CAP) information and programs than small farms, so they are more likely to respond well to policies and adopt low-pesticides production techniques. Large farms better know the available compensation mechanism and have often more qualified workers and up-to-date technology that allow them to experiment with new production practices.

Our results suggest that subscription to the Agro-environmental policies (AEP) would be stronger for large farms than small ones. The objective of the AEP is to shift the agricultural production processes toward greener by favoring various protocols like setting up grassy-band along rivers, banning the use of fertilizers on some natural grasslands, restricting the use of pesticides, inciting to shift to organic agricultural production processes, etc. The involvement of farmers in these AEP is voluntary, however. The compensatory payment depends on the area on which the AEP is experimented and are likely to offset the additional cost that come from the change in agricultural practice. The large farms are strongly incited to introduce these types of experimentation on a part of their large agricultural area, a luxury that small farms may not have, as suggested by our result on the marginal product of pesticide. In addition, it is a known fact that large farms have more qualified workers and the necessary technology to adhere to these programs, making it less costly to introduce them than for small units. Our results are not proof that large farms internalize better the "greener" processes, but they definitely suggest an asymmetric behavior and large farms respond more to incentives other than pesticide price.

6. Conclusion

In this paper we have estimated the pesticide marginal product of Eure-et-Loir farms during the period 2005-2008. The treatment of pesticides in production model is as close to the reality as it

is possible in this particular case. We have taken into account the multi-output nature of farm production. This can clearly impact on the use of pesticides on crops because there is no reason to believe that farmers will be neutral with respect to what they grow when they use of pesticides. An appropriate model of the pesticides in the production process is crucial in order to capture the real effect of this input. We have modelled pesticides as a damage reducing input *à la* Carpentier and Weaver [1997] to capture the specific effect of the pesticide on each standard input. In fact, we have made the distinction between standard inputs and effective inputs and we have associated to each standard input a specific damage reducing function. One of the features of our model is that overdosage is possible as the function is nonlinear in the pesticides. We have adapted this framework to a translog production function and identified all the parameters. The estimation of such a model (multi-output - effective input) is rarely used in practice because of the complexity of the estimation process. We have implemented Fernandez *et al.* [2000] Bayesian estimation procedure for stochastic frontiers to obtain estimates of all the parameters.

Our results show that the marginal product of pesticides is systematically larger than the cost of using them in Eure-et-Loir and we attribute this effect to the pesticide control policy. We have also identified that cereals are probably more flexible than the other crop when pesticide management comes around as we have identified that the pesticide marginal product for these crops tend to be larger than they are for the other crops. It also seems that large farms adopt a more environmental friendly behavior than small units.

References

Aigner, D., C.A.K. Lovell and P. Schmidt (1977). Formulation and Estimation of Stochastic Frontier Production Function Models, *Journal of Econometrics*, 6:1, 21-37.

Barnett, W. A. (1985). The minflex Laurent translog functional form, *Journal of Econometrics*, 30:1, 33-44.

Böcker, T., Britz, W., & Finger, R. (2018). Modelling the effects of a glyphosate ban on weed management in silage maize production. Ecological Economics, 145, 182-193.

Carlson, G. A. (1977). Long-Run Productivity of Pesticides. *American Journal of Agricultural Economics*, 59:543-548.

Carpentier, A. and Weaver, R. D. (1997). Damage Control Productivity: Why Econometrics Matters. *American Journal of Agricultural Economics*, 79 (No. 1):47-61.

Carson, R. (1962). *Silent Spring*. Greenwich, CT: Fawcett Publication.

Fernandez, C., Koop, G., and Steel, M. F. (2000). A Bayesian analysis of multiple-output production frontiers. *Journal of Econometrics*, 98:47-79.

Headley, J. C. (1968). Estimating the Productivity of agricultural Pesticides. *American Journal of Agricultural Economics*, 50 (No. 1):13-23.

Karagiannis, G., & Tzouvelekas, V. (2011). The damage-control effect of pesticides on total factor productivity growth. *European review of agricultural economics*, *39*(3), 417-437.

Koop, G., Osiewalski, J., and Steel, M. F. (1997). Bayesian efficiency analysis through individual effects: Hospital cost frontiers. *Journal of Econometrics*, 76:77-105.

Lichtenberg, E. and Zilberman, D. (1986). The Econometrics of Damage Control: Why Specification Matters. *American Journal of Agricultural Economics*, 68 (No. 2):261-273.

Lovell, C. K., Travers, P., Richardson, S., Wood, L. (1994). Resources and functionings: a new view of inequality in Australia. In *Models and measurement of welfare and inequality*, pp. 787-807. Springer, Berlin, Heidelberg.

Meeusen, W. and van den Broeck, J. (1977). Efficiency estimation from Cobb-Douglas production function with composed error. *International Economic Review*, 18:345-444.

O'Donnell, C. J. (2012). Econometric estimation of distance functions and associated measures of productivity and efficiency change. *Journal of Productivity Analysis*, pages 1-14.

Roberts, G., Gelman, A., and Gilks, W. (1997). Weak convergence and optimal scaling of random walk Metropolis algorithms. The Annals of Statistics, 7:110-120.

Roberts, G., Gelman, A., and Gilks, W. (1997). Weak convergence and optimal scaling of random walk Metropolis algorithms. *The Annals of Statistics*, 7:110-120.

Rudd, R. L. (1964). *Pesticides and the Living Landscape*. University of Wisconsin Press.

Zhengfei, G., Lansink, A. O., Wossink, A., and Huirne, R. (2005). Damage control inputs: a comparison of conventional and organic farming systems. *European Review of Agricultural Economics*, 32 (No. 2):167-189.

Appendices

A. The aggregate output and the likelihood.

In this appendix we give the details on how the specific output effects are related to the aggregate output to include them in the likelihood. Instead of dealing directly with the individual outputs, we can as well deal with their respective weights in the aggregate, this will convey just about the same information for our needs. The individual contribution of farm d to output j is given by the output elasticity defined as:

$$\eta_{d,j} = \frac{\partial \ln(A(\mathbf{y}_d))}{\partial \ln(y_{d,j})} = \frac{\alpha_j^q y_{d,j}^q}{\sum_{l=1}^{n} \alpha_l^q y_{d,l}^q} \text{ with } j = 1,\ldots,n \text{ and } d = 1,\ldots,D. \qquad (A1)$$

We group all these elasticities into a vector, $\boldsymbol{\eta}_d = (\eta_{d,1}, \eta_{d,2}, \ldots, \eta_{d,n})^\top$. Since these weights sum up to one, the $(n-1)$ weights are sufficient to complete the construction of the likelihood for the n observed outputs. In fact, given $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_n)^\top$ and q, there exists a correspondence between the observed output vector $\mathbf{y}_d \in \mathbb{R}_+^n$ $(\mathbf{y}_d = (y_{d,1}, \ldots, y_{d,n})^\top)$, and the $n-$vector $(\delta_d, \eta_{d,2}, \ldots, \eta_{d,n})^\top$. We need the distribution of the shares and then to establish the relationship between these variables and the distribution of the individual outputs, $y_{d,j}$ for $j = 1,\ldots,n$. That is, given the density of the shares and the aggregate, $f(\delta_d)$, the density of the outputs is given by:

$$\tilde{f}\left(y_{d,1}, y_{d,2}, \ldots, y_{d,n}\right) = f\left(\delta_d, \eta_{d,2}, \ldots, \eta_{d,n}\right) \cdot |\mathbf{J}|\,, \qquad (A2)$$

where $|\mathbf{J}|$ is the absolute value of the Jacobian of the transformation from $\left(\delta_d, \eta_{d,2}, \ldots, \eta_{d,n}\right)$ to $\left(y_{d,1}, y_{d,2}, \ldots, y_{d,n}\right)$. Using the definition of $\delta_d(\mathbf{x}_d, z_d)$ and $\boldsymbol{\eta}_d$ the Jacobian is given by:

$$\mathbf{J} = \mathbf{J_{n-1}} = q^{n-1} \times \frac{\eta_{d,1}}{y_{d,1}} \times \frac{\eta_{d,2}}{y_{d,2}} \times \frac{\eta_{d,3}}{y_{d,3}} \times \ldots \times \frac{\eta_{d,n}}{y_{d,n}}$$
$$= q^{n-1} \prod_{j=1}^{n} \frac{\eta_{d,j}}{y_{d,j}}\,. \qquad (A3)$$

Since $0 \leq \eta_{d,j} \leq 1$ and $\sum_{j=1}^{n} \eta_{d,j} = 1$, the Dirichlet distribution is the ideal choice. That is, we assume that all shares are jointly Dirichlet distributed:

$$p(\boldsymbol{\eta}_d \mid s) = f_{Dir.}^{n-1}(\boldsymbol{\eta}_d \mid \mathbf{s}), \qquad (A4)$$

where $f_{Dir.}^{n-1}(\boldsymbol{\eta}_d \mid \mathbf{s})$ denotes a Dirichlet probability density function of dimension $(n-1)$ with unknown parameters $\mathbf{s} = (s_1, \ldots, s_n)^\top \in \mathbb{R}_+^n$ to be estimated. Note that $f(\boldsymbol{\eta}_d \mid \delta_d)$ is nothing but the density of $\boldsymbol{\eta}_d$ given $\boldsymbol{\alpha}$ and q, *i.e.* $f(\boldsymbol{\eta}_d)$. Then we have $(\boldsymbol{\eta}_d \mid \delta_d) = f(\boldsymbol{\eta}_d) = f_{Dir.}^{n-1}(\boldsymbol{\eta}_d \mid \mathbf{s})$, so that for observation d, the density (or likelihood) is given by:

$$p(\mathbf{y}_d) = f_N(\mathbf{V}_d\boldsymbol{\beta}_d - u_d, \sigma^2) \times f_{Dir.}^{n-1}(\boldsymbol{\eta}_d \mid \mathbf{s}) \times q^{n-1} \prod_{j=1}^{n} \frac{\eta_{d,j}}{y_{d,j}}. \tag{A5}$$

The likelihood in equation (23) is obtained by multiplying all individual distribution under the assumption that all farms are identically and independently distributed.

B. The priors

The prior distribution of the frontier parameters is assumed to be a $k-$ dimension normal distribution, with mean $\mathbf{b}_0$ and covariance matrix $\mathbf{H}_0^{-1}$:

$$p(\boldsymbol{\beta}) = f_N^k(\boldsymbol{\beta} \mid \mathbf{b}_0, \mathbf{H}_0^{-1}) \times 1_{\Im}(\boldsymbol{\beta}), \tag{A6}$$

where $1(\cdot)$ is an indicator function that allows us to restrict the parameter space to a subset of $\Im \subseteq \mathbb{R}^k$. For a non-informative prior, we choose $\mathbf{b}_0 = 0_k$ and $\mathbf{H}_0 = 10^{-4} \times I_k$. These parameters produce a virtually flat surface centered on zero. For convenience, let $h = \sigma^{-2}$ and let the prior for h be Gamma. That is:

$$p(h) = f_G(h \mid n_0, a_0). \tag{A7}$$

In order to obtain a non-informative prior, we set $n_0 = 1$ (this returns an exponential distribution) and $a_0 = 10^{-6}$. The posterior is again virtually flat over $[0, +\infty[$ and is never larger than 10^{-6}. These two hyper parameters ensure that the posterior distribution is almost not affected by the choice of the distribution and as a consequence the data will guide the results.

Now, since all α_j are in the unit interval and add up to one, a Dirichlet distribution appears to be the natural choice for the prior. Thus, we assume that $\boldsymbol{\alpha}$ follows such a distribution with parameter vector $\mathbf{a}$. That is:

$$p(\boldsymbol{\alpha}) = f_{Dir.}^{n-1}(\boldsymbol{\alpha} \mid \mathbf{a}), \tag{A8}$$

where the hyper parameters are all positive, $\mathbf{a} = (a_1, a_2, \ldots\ldots, a_n)^\top \in \mathbb{R}^n_+$. Now, to get a non-informative prior, we impose that $a_j = 1$ for all *j*, to obtain a uniform distribution on a unit cube of dimension n.

Now, for the elasticity of substitution of the aggregator function, we need the parameter to be larger than one for consistency. Thus, to obtain $q > 1$ we assume that it is exponentially distributed on]0,1[. That is:

$$p(q) \propto f_G(\, q \mid 1, \kappa\,) \times 1_{(1,\infty)}(q). \qquad (A9)$$

To make this prior as uninformative as possible, we set $\kappa = 10^{-6}$. We assume that all individual components of the vector **s** are independent and Gamma distributed. Therefore, the prior is given by:

$$p(\mathbf{s}) = \prod_{j=1}^{n} p\left(s_j\right) = \prod_{j=1}^{n} \frac{(c_j)^{b_j}}{\Gamma\left(b_j\right)} \times s_j^{b_j - 1} \times \exp\{-c_j \times s_j\}. \qquad (A10)$$

To make this prior non informative, we set $b_j = 1$ and $c_j = 10^{-6}$ for all $j = 1, \ldots, n$. Finally, we need to address the distribution of the efficiency component. We assume that the efficiency terms are all identically and independently exponentially distributed.[4] That is:

$$p(\,\mathbf{u} \mid \lambda\,) = \prod_{d=1}^{D} f_G\,(\,u_d \mid 1, \lambda\,) = \lambda^D \exp\{-\lambda \times \mathbf{u}^\top \iota_D\}. \qquad (A11)$$

To model λ we set a prior on the prior's parameter. Koop *et al.* [1997] recommend using a Gamma distribution for *SFA* estimation:

$$p(\lambda) = f_G(\lambda \mid \lambda_1, \lambda_2)) = -\ln(\tau^*)\exp\{-\lambda \times -\log(\tau^*)\}\,. \qquad (A12)$$

To have a non-informative prior we set $\lambda_1 = 1$ and $\lambda_2 = -\ln(\tau^*)$ where τ^* is roughly an average of the efficiency score. By setting $\lambda_1 = 1$ we get an exponential prior. Since τ^* is not observed ex ante, we set $\tau^* = 0.875$.

[4] Note that any distribution that generates an efficiency score in the unit interval will do the trick. Truncated normal or half normal distributions are alternate options one may consider.

C. The posterior distributions

It is important to remember that Bayesian inference rests on the fact that the smaller problems are always adjusted so that the inference is consistent. In other words, when we focus on a group of parameters, everything that is not related to this variable is included in a multiplicative constant (adequately chosen) so that the problem is consistent. That is, we can always adjust the constants in an arbitrary fashion, as long as in the end the prior is a well-defined density (it integrates to one).

The posterior distribution for $\boldsymbol{\beta}$ is obtained by multiplying the prior (Equation (A7)) by the likelihood function (Equation (23)). After some manipulations, we obtain the following distribution for $\boldsymbol{\beta}$.

$$p(\boldsymbol{\beta} \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s}) \propto (2\pi)^{-\frac{k}{2}} \times (\det[[h^{-1}(\mathbf{V}^\top\mathbf{V})^{-1}]^{-1} + \mathbf{H}_0])^{\frac{1}{2}} \times$$
$$\exp\left\{-\frac{1}{2}\left(\boldsymbol{\beta} - \left(\left[h^{-1}\left((\mathbf{V}^\top\mathbf{V})\right)^{-1}\right]^{-1} b + \mathbf{H}_0\mathbf{b}_0\right)\left(\left[h^{-1}\left((\mathbf{V}^\top\mathbf{V})\right)^{-1}\right]^{-1} + \mathbf{H}_0\right)^{-1}\right)^\top \left(\left[h^{-1}\left((\mathbf{V}^\top\mathbf{V})\right)^{-1}\right]^{-1} + \mathbf{H}_0\right) \times \left(\boldsymbol{\beta} - \left(\left[h^{-1}\left((\mathbf{V}^\top\mathbf{V})\right)^{-1}\right]^{-1}\mathbf{b} + \mathbf{H}_0\mathbf{b}_0\right)\left(\left[h^{-1}\left((\mathbf{V}^\top\mathbf{V})\right)^{-1}\right]^{-1} + \mathbf{H}_0\right)^{-1}\right)\right\}, \qquad (A13)$$

which is a k −dimensional normal distribution with b_* and covariance matrix H_*. That is:

$$b_* = ([h^{-1}(\mathbf{V}^\top\mathbf{V})^{-1}]^{-1} + \mathbf{H}_0)^{-1}([h^{-1}(\mathbf{V}^\top\mathbf{V})^{-1}]^{-1}\mathbf{b} + \mathbf{H}_0\mathbf{b}_0)$$
$$= ([h^{-1}(\mathbf{V}^\top\mathbf{V})^{-1}]^{-1} + \mathbf{H}_0)^{-1}(h\mathbf{V}^\top\delta' + \mathbf{H}_0\mathbf{b}_0)\,, \qquad (A14)$$

and

$$H_* = ([h^{-1}(\mathbf{V}^\top\mathbf{V})^{-1}]^{-1} + \mathbf{H}_0)^{-1}, \qquad (A15)$$

where $\mathbf{b} = (\mathbf{V}^\top\mathbf{V})^{-1}\mathbf{V}^\mathbf{T}\mathbf{lnA}(\mathbf{Y})'$ with $\mathbf{lnA}(\mathbf{Y})' = \mathbf{lnA}(\mathbf{Y}) + \mathbf{u}$. $\mathbf{b}_0$ and $\mathbf{H}_0$ are the hyperparameters set above.

The posterior distribution of the variance, σ^2, is given by the product of the prior (Equation (A7)) by the likelihood (Equation (24)). Recalling that $h = \sigma^{-2}$, we have:

$$p(h) = \frac{(a_0)^{n_0}}{\Gamma(n_0)} \times h^{n_0-1} \times \exp\{-a_0 \times h\}\,, \qquad (A16)$$

and

$$p(\mathbf{y} \mid \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s})$$
$$= (2\pi)^{-\frac{D}{2}}(h)^{\frac{D}{2}}\exp\left\{-\frac{h}{2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right\} \times T_1 \,. \qquad (A17)$$

where T_1 represents all the terms of the density that do not depend on h. After some manipulations we get the following posterior distribution:

$$p(\, h \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, \boldsymbol{\alpha}, q, \mathbf{s}\,) \propto \frac{\left(a_0 + \frac{1}{2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right)^{n_0+\frac{D}{2}}}{\Gamma\left(n_0 + \frac{D}{2}\right)}$$
$$\times\, h^{n_0+\frac{D}{2}-1}$$
$$\times \exp\left\{-h\left(a_0 + \frac{1}{2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right)\right\} \qquad (A18)$$

This distribution is of the Gamma distribution form, so we write it as:

$$p(\, h \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, \boldsymbol{\alpha}, q, \mathbf{s}\,) \propto f_G\left(n_0 + \frac{D}{2}, a_0 + \frac{1}{2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right). \qquad (A19)$$

We can deduce the posterior distribution of the weights of the *CES* aggregator by proceeding as above. Multiplying the Dirichlet prior distribution by the likelihood and after some manipulations, we get:

$$p(\, \boldsymbol{\alpha} \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, q, \mathbf{s}\,) = p(\, \mathbf{Y} \mid \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s}\,) \times p(\boldsymbol{\alpha})$$
$$= Cst \times \exp\left\{-\frac{h}{2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right\} \times$$
$$\prod_{d=1}^{D}\left(\frac{\Gamma\left(\sum_j s_j\right)}{\prod_j\ \Gamma(s_j)}\right) \times \prod_{d=1}^{D}\prod_{j=1}^{n} \eta_{d,j}^{s_j} \times \frac{\Gamma\left(\sum_j a_j\right)}{\prod_j\ \Gamma(a_j)} \times \prod_{j=1}^{p}\ \alpha_j^{a_j-1} \qquad (A20)$$

where Cst represents all the terms of the density that do not depend on $\boldsymbol{\alpha}$. Thus, we have the following posterior distribution for $\boldsymbol{\alpha}$:

$$p(\, \boldsymbol{\alpha} \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, q, \mathbf{s}\,) \propto \prod_{j=1}^{n}\ \alpha_j^{a_j+s_j qD-1} \times \prod_{d=1}^{I}\prod_{j=1}^{n}\left(\sum_j^n \alpha_j^q\, y_{d,j}^q\right)^{-\sum_j^n s_j}$$
$$\times \exp\left\{-\frac{1}{2\sigma^2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^{\top}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right\}. \qquad (A21)$$

Note that all the terms containing **lnA** are preserved because they all depend on the parameter vector **α**. This is not a known distribution and the simulation method would require some adjustments. This is explained below. For the elasticity of substitution parameter of the *CES* aggregator, q, we proceed as before. Using the likelihood and q's prior, the posterior distribution is given by:

$$p(\,q \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, \mathbf{s}\,) = (2\pi)^{-\frac{D}{2}}(h)^{\frac{D}{2}} \times \exp\left\{-\frac{1}{2\sigma^2}(\,\mathbf{lnA}(\mathbf{Y}) - \mathbf{V}\boldsymbol{\beta} + \mathbf{u})^{\top}(\mathbf{lnA}(\mathbf{Y}) - \mathbf{V}\boldsymbol{\beta} + \mathbf{u})\right\} \times$$

$$\prod_{d=1}^{D}\left(\frac{\Gamma(\sum_j s_j)}{\prod_j \Gamma(s_j)}\right) \times\; q^{D(n-1)} \prod_{d=1}^{D}\prod_{j=1}^{p} \frac{\eta_{d,j}^{s_j}}{y_{d,j}} \times \frac{\kappa}{\Gamma(1)} \times \exp\{-\kappa \times q\} \times 1_{(1,\infty)}(q). \qquad (A22)$$

Rearranging terms, we find that the posterior distribution of q is proportional to:

$$p(\,q \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, \mathbf{s}\,) \propto 1_{(1,\infty)}(q) \times q^{D(n-1)} \times \exp\{-\kappa \times q\} \times$$
$$\exp\left\{-\frac{1}{2\sigma^2}(\mathbf{lnA}(\mathbf{Y}) - \mathbf{V}\boldsymbol{\beta} + \mathbf{u})^{\top}(\mathbf{lnA}(\mathbf{Y}) - \mathbf{V}\boldsymbol{\beta} + \mathbf{u}) - \textstyle\sum_{d,j} s_j \times \ln\left(\frac{\sum_j^n \alpha_j^q y_{d,j}^q}{\alpha_j^q y_{d,j}^q}\right)\right\}. \qquad (A23)$$

As above, we keep all the terms involving **lnA** and again, the posterior distribution is not a known form we can simulate directly.

We also need the parameter vector of the Dirichlet distribution for **η**, *i.e.* **s**. We have assumed that all the components of the vector are independently Gamma distributed and again after some manipulation on the posterior we get:

$$p(\,s_j \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, s_{-j}\,) = f_N^D(\mathbf{V}\boldsymbol{\beta} - \mathbf{u}, \sigma^2 I_D) \times \prod_{d=1}^{D}\left(\frac{\Gamma(\sum_j s_j)}{\prod_j \Gamma(s_j)}\right) \times q^{D(n-1)} \times$$

$$\prod_{d=1}^{D}\prod_{j=1}^{n} \frac{\eta_{d,j}^{s_j}}{y_{d,j}} \times \frac{(c_j)^{b_j}}{\Gamma(b_j)} \times s_j^{b_j-1} \times \exp\{-c_j \times s_j\}, \qquad (A24)$$

where s_{-j} is vector **s** without the j^{th} component. Rearranging terms, gives the following posterior distribution:

$$p(\,s_j \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, s_{-j}\,)$$
$$= \frac{\Gamma(\sum_j s_j)^D}{\Gamma(s_j)^D} \times s_j^{b^j-1} \times \exp\left[-s_j\left(c_j + \sum_i log\left(\frac{\sum_j^n \alpha_j^q\, y_{d,j}^q}{\alpha_j^q y_{d,j}^q}\right)\right)\right]. \qquad (A25)$$

The posterior distribution of the parameter λ is given by the multiplication of its prior with the density distribution of the efficiency term (u_d). Consequently, the posterior distribution is given by:

$$p(\lambda \mid \mathbf{u}) = p(\mathbf{u} \mid \lambda) \times p(\lambda)$$
$$= \lambda^D \exp\{-\lambda \times \mathbf{u}^\top \iota_D\} \times -\ln(\tau^*)\exp\{-\lambda \times -\ln(\tau^*)\}. \qquad (A26)$$

Rearranging terms gives:

$$p(\lambda \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s})$$
$$\propto \frac{(\mathbf{u}^\top \iota_D - \ln(\tau^*))^{D+1}}{\Gamma(D+1)} \times \lambda^D \times \exp\{-\lambda(\mathbf{u}^\top \iota_D - \ln(\tau^*))\}. \qquad (A27)$$

This is a Gamma distribution with parameters $a_* = D + 1$ and $b_* = \mathbf{u}^\top \iota_D - \ln(\tau^*)$. That is:

$$p(\lambda \mid \mathbf{Y}, \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s}) \propto f_G\left(D + 1, \mathbf{u}^\top \iota_D - \ln(\tau^*)\right). \qquad (A28)$$

The last posterior distribution is for the efficiency term, $\mathbf{u}$. This is again obtained by multiplying its prior with the likelihood.

$$p(\mathbf{u} \mid \mathbf{Y}, \boldsymbol{\beta}, \lambda, h, \boldsymbol{\alpha}, q, \mathbf{s})$$
$$= p(\mathbf{y} \mid \boldsymbol{\beta}, \mathbf{u}, h, \boldsymbol{\alpha}, q, \mathbf{s}) \times p(\mathbf{u} \mid \lambda)$$

$$= (2\pi)^{-\frac{D}{2}}(h)^{\frac{D}{2}}\exp\left\{-\frac{1}{2\sigma^2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^\top(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})\right\} \times \lambda^D \exp\{-\lambda \times \mathbf{u}^\top \iota_D\}$$
$$\times T_1. \quad (A29)$$

So that:

$$p(\mathbf{u} \mid \mathbf{Y}, \boldsymbol{\beta}, \lambda, h, \boldsymbol{\alpha}, q, \mathbf{s}) = (2\pi)^{-\frac{D}{2}}(h)^{\frac{D}{2}} \times \lambda^D$$
$$\times \exp\left\{-\frac{1}{2\sigma^2}(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u})^\top(\mathbf{lnA(Y)} - \mathbf{V\beta} + \mathbf{u}) - \lambda \times \mathbf{u}^\top \iota_D\right\} \times T_1. \qquad (A30)$$

Multiplying by a properly chosen constant that does not depend on $\mathbf{u}$, and we conclude that the posterior distribution is normal with mean $T_0 = \mathbf{V\beta} - \mathbf{lnA} - \lambda \iota_D h^{-1}$ and variance $P_0 = h^{-1} I_D$. That is:

$$p(\mathbf{u} \mid \mathbf{Y}, \boldsymbol{\beta}, \lambda, h, \boldsymbol{\alpha}, q, \mathbf{s})$$
$$\propto f_N^D(\mathbf{u} \mid \mathbf{V}\boldsymbol{\beta} - \mathbf{lnA}(\mathbf{Y}) - \lambda \times h^{-1} \times \iota_D, h^{-1} \times I_D). \qquad (A31)$$

All these posterior distribution will be sequentially simulated using Gibbs algorithm. The posterior distributions of $\boldsymbol{\beta}$, h, λ and $\mathbf{u}$ have well known forms and can be easily simulated. So for each iteration of the algorithm we simulate a $k-$vector of parameters $\boldsymbol{\beta}$ from a normal distribution with the hyper-paramaters set as above, then h and λ are obtained from unidimensional Gamma distributions and $\mathbf{u}$ is obtained from a normal distribution. It is not as easy for the posterior distributions of $\boldsymbol{\alpha}$, $\mathbf{s}$ and q, as the distributions do not have standard forms. There exists, however, multiple methods to simulate a non-standard distribution. As suggested by Fernandez *et al.* [2000], we use the Metropolis random walk algorithm. This algorithm allows us to simulate a candidate value from an arbitrary distribution (transition distribution) and to include this candidate value into the iterative process with a positive probability. If the value is rejected, we keep the current value in the iterative process. The proportion of generated candidate accepted in the Metropolis random walk algorithm can increase (decrease) by reducing (increasing) the weights given to the variance-covariance of the transition distribution. Roberts *et al.* [1997] have shown that when the transition density and the posterior density are normal, the optimal acceptation rate is approximately between 0.25 and 0.45 (this is the rate that minimizes the auto-correlation between the generated values). In spite of the fact that we are using normal transition density, the posterior is not normal, so we are not in the framework described above. However, we keep in mind these figures, which corresponds to the standard practices for *SFA* estimation (O'Donnell [2012] have used a range between 0.2 and 0.6, while Fernandez *et al.* [2000] use an acceptance rate between 0.18 and 0.545). With the normal transition and the random walk Metropolis algorithm, the mean of the normal transition density is given by the value of the simulation at the preceding step and the variance allows us to control for the convergence speed of the algorithm. We calibrate this parameter such that the acceptation probability is reasonable.

D. Deflation of nominal outputs

Because we use data for various years, we have to deflate the production to get comparable quantities over time. Our methodology to deflate these aggregate outputs makes use of the *UAA*

and crop price indexes. We compute a composite deflator for each of our two aggregate outputs. The procedure to construct the price deflators is explained in this Appendix.

Cereal crops price index

The variable "cereal" crops is an aggregate of wheat, durum wheat, spring barley, winter barley, irrigated corn, corn, oat, spring wheat and other cereals. The French National Institute of Statistics and Economic Studies (*INSEE*) offers a perfect match for the price of wheat, durum wheat, spring barley, corn and oat. However, for winter barley, irrigated corn, durum wheat and other cereals, we do not have this perfect match. So we use instead the price index of spring barley, the price index of corn, the price index of wheat and the price index of cereal, respectively. This is justified by the fact that these cereal varieties are relatively close.

A composite index is constructed using these indexes using the following formula:

$$I_{cer.} = \sum_{i=1}^{nc} w_i \, p_i, \qquad (A32)$$

where $w_i = UAA_i / UAA_{cer.}$, p_i is the price index of cereal crop i, UAA_i is the Utilized Agricultural Area of cereal crop i and $UAA_{Cer.}$ is the Utilized Agricultural Area for all cereal crops. When $UAA_{cer.}$ is zero, the variable "cereal" crops is also zero so no composite index is calculated in that case. Finally as mentioned above, there are nine cereals, so nc = 9. This composite index is the deflator of the variable "cereal" crops. The index values are reported in Table D1.

[INSERT TABLE D1] ABOUT HERE]

Industrial-other crops price index

The variable "industrial-other" crops is made of protein pea, beet, potato, rape, sunflower, flax, poppy, lucerne, other industrial crops, fodder, fruits, vegetables, and horticulture. The French National Institute of Statistics and Economic Studies ($INSEE$) publishes the price indexes for protein pea, beet, potato, rape, sunflower, poppy, flax, lucerne, fodder, fruits, vegetables, and horticulture. We do not have a price index for "other industrial crops". However, the contribution if this output is very small (the share of the UAA for these produces represents

1.2349% of the UAA). Consequently, we have simply decided to ignore it in the computation of the new other crops price index. To compute the composite index we use the formula for the composite index of cereal crops. This composite index is the deflator of the variable "New other" crops. The index values are reported in Table D2.

[INSERT TABLE D2 ABOUT HERE]

Inputs deflators

The construction of the input deflators goes as follows. The variable "intermediate consumption" is deflated using its corresponding price index, obtained from French National Institute of Statistics and Economic Studies (*INSEE*). This price index captures the price of purchasing "intermediate consumption" (e.g., energy, seeds, fertilizers). The variable "depreciation" is deflated using the price index of fixe capital consumption (harvesting equipment, tractors, farm buildings, etc.). This index is also from the French National Institute of Statistics and Economic Studies (*INSEE*). The variable "Pesticide" is deflated by the price index of crop protection products. These indexes are reported in Table D3.

[INSERT TABLE D3 ABOUT HERE]

Tables et Figures

Table 1. *Descriptive statistics*

		Land	Land	Dep.	Int. Cons.	Pest.	Cereal	Ind-Oth
	Min.	0.5000	14.790	526.10	11,668.00	2,814.00	5,366.00	18.00
	Qu. 1	1.0000	87.970	13,670.68	32063.00	15,041.00	44,565.00	18,750.00
	Qu. 2	1.0000	118.450	22,370.48	44,122.00	20,629.00	60,960.00	29,532.00
2005	Mean	1.2450	125.483	24,954.99	50,639.62	22,383.00	65,641.33	41,608.44
	Qu. 3	1.3000	154.500	34,266.06	63,075.00	28,502.00	81,700.00	52,020.00
	Max.	4.0000	360.850	72,973.89	196,536.00	63,085.00	209,897.00	251,397.00
	Std.	0.4739	50.576	15,092.38	26,125.01	10,028.87	29,478.91	35,355.95
	Min.	0.5000	14.790	885.74	12,489.29	0.00	4,060.27	103.00
	Qu. 1	1.0000	90.642	13,291.01	31,577.33	14,408.68	45,480.44	14,608.66
	Qu. 2	1.0000	119.900	22,699.21	44,298.63	19,479.04	63,540.30	24,946.98
2006	Mean	1.2396	127.344	24,819.94	50,371.32	21,244.82	68,632.46	36,682.38
	Qu. 3	1.2500	155.232	34,123.04	62,274.31	26,846.05	84,935.78	46,278.67
	Max.	4.0000	368.210	70,041.01	179,648.83	59,423.15	198,126.91	242,566.68
	Std.	0.4618	51.168	15,009.07	25,923.88	9,489.41	31,779.86	33,843.50
	Min.	0.4600	14.790	662.26	11,874.08	0.00	4,461.36	419.00
	Qu. 1	1.0000	92.000	13,370.75	30,605.69	15,063.87	4,7744.39	16,227.83
	Qu. 2	1.0000	121.060	22,136.79	43,129.59	20,478.04	66,627.23	27,188.18
2007	Mean	1.2400	128.992	25,440.80	49,226.16	22,263.16	71,918.23	38,243.30
	Qu. 3	1.2000	158.130	35,482.07	59,863.05	28,093.81	91,000.72	47,161.33
	Max.	4.0000	393.970	74,123.58	296,767.46	66,040.91	201,235.75	24,0025.77
	Std.	0.4687	52.331	15,888.37	27,215.24	10,216.34	33,059.07	34,956.50
	Min.	0.4600	32.390	723.30	11,337.13	0.00	5,917.96	486.00
	Qu. 1	1.0000	91.060	13,178.38	33,754.68	16,226.57	42,550.27	14,924.64
	Qu. 2	1.0000	120.735	21,738.87	45,966.61	22,028.01	58,318.01	25,644.21
2008	Mean	1.2031	127.609	24,416.24	50,793.82	24,065.18	64,021.56	34,945.46
	Qu. 3	1.0000	154.630	33,764.45	63,105.65	30,938.88	78,934.40	42,328.27
	Max.	3.0000	310.940	72,142.34	176,463.35	74,014.49	195,079.40	256,219.81
	Std.	0.4306	50.667	15,161.80	24,239.40	10,766.08	31,502.07	32,203.45

Table 2. *Acceptation rates and standard errors of* $\boldsymbol{\alpha}$*, q, s1 and s2*

		2005	2006	2007	2008
Acceptation rate	$\boldsymbol{\alpha}$	0.4076	0.3807	0.3335	0.4896
	q	0.3673	0.2646	0.311	0.3475
	s1	0.4022	0.355	0.3404	0.4171
	s2	0.3625	0.3791	0.4369	0.3574
Standard errors	$\boldsymbol{\alpha}$	0.015	0.015	0.015	0.015
	q	0.02	0.03	0.02	0.01
	s1	0.23	0.23	0.23	0.23
	s2	0.25	0.25	0.25	0.25

Table 3. *Results of the Geweke convergence test*

	2005	2006	2007	2008
	-0.5563	0.3856	-0.6454	0.5475
γ1	1.0061	-0.4768	1.2216	-1.1592
γ2	-0.4479	-1.2425	0.0832	-0.4789
γ3	0.839	0.3184	0.9319	-1.4309
γ4	0.3852	-0.4518	0.1507	1.656
π11	0.2563	-0.0458	-0.6061	0.4902
π22	0.1631	-0.0121	-0.554	-1.0877
π33	0.244	0.3094	-0.005	-0.5902
π44	1.5578	-0.905	0.8227	0.5372
π12	-0.4236	0.8631	0.1215	1.4595
π13	0.2465	0.1689	0.131	0.1894
π14	1.5995	-1.9468	0.0991	-1.4513
π23	1.2804	-1.4962	1.0883	-0.9943
π24	-0.5241	0.8887	-1.7479	0.2418
π34	0.6457	0.9996	1.6786	-0.6471
η1	-0.3895	-0.5278	0.1148	-0.5499
η2	1.3925	-0,9752	-1,734	-1.2237
η3	0.2018	0.4697	1.498	-0,5729
η4	1.2577	-0.613	1.7677	0.2944
δ1	-1.1871	0.3227	-1.0546	0.6324
δ2	0.035	1.5574	0.2585	-1.3663
δ3	1.0013	0.6697	-0.0245	0.6931
δ4	-0.7162	-1.2191	1.2018	-0.1732
ϕ1	-0.7115	-0.1171	-1.3683	1.1557

ϕ2	1.0997	0.1674	1.52	-0.9042
ϕ3	0.2135	-0.527	-0.6236	-0.6182
ϕ4	-1.1645	-0.1367	-0.767	1.4969
σ	0.1765	-0.1486	0.5716	-0.2513
λ	0.8812	-0.4743	0.1357	-1.2917
q	0.5567	-0.7922	-1.6529	0.6878
α1	1.6171	0.0745	-0.5768	0.0811
α2	-0.5389	-0.0745	0.5768	-0.0811
s1	-0.4665	0.3339	-0.5929	0.1475
s2	0.4766	0.0945	0.5483	-0.0211

The test statistic follows a standard normal distribution. There is evidence of convergence (at 95%) when the calculated statistics belongs to [-1,96, 1,96].

Table 4*. Parameter starting values*

	Starting values 1	Starting values 2	Starting values 3
β	0.5 for all elements	1 for all elements	1.5 for all elements
σ	0.5	0.8	0.7
λ	10	15	20
u	1 for all elements	0.8 for all elements	0.7 for all elements
α	0.5 for all elements	0.33 et 0.66	0.75 et 0.25
q	2	3	4
s	1 for all elements	2 for all elements	2.5 for all elements

Table 5*. Parameter estimated values (2005-2006)*

	2005			2006		
	Mean	Quart. 2.5	Quart. 97.5	Mean	Quart. 2.5	Quart. 97.5
cte	10.37252	10.20557	10.53984404	10.16896	9.999989	10.33327442
γ1	0.4980405	0.3078862	0.671712843	0.4740331	0.2266007	0.702916712
γ2	0.008149418	-0.01120299	0.027804788	0.00750652	-0.01677335	0.031888318
γ3	-0.001092679	-0.01039244	0.00819192	0.002176794	-0.009499816	0.014101083
γ4	-0.001349589	-0.01006352	0.007273573	-0.002130187	-0.01308896	0.008676265
π11	0.08853796	0.003165276	0.25051828	0.1628897	0.008058007	0.413029558
π22	0.005470419	0.00171675	0.009413565	0.00408787	0.00037694	0.008677166
π33	0.000371603	1.67805E-05	0.001008155	0.000747272	6.68908E-05	0.001665737
π44	0.00018904	6.32702E-06	0.000586644	0.000259847	8.71429E-06	0.000790797
π12	0.007217117	0.000187007	0.025352124	0.008982718	0.000245138	0.031855272
π13	0.003508334	9.6682E-05	0.012187719	0.004598673	0.000131844	0.01587034

π14	0.00335176	9.58803E-05	0.011432031	0.004232707	0.000114802	0.014529145
π23	0.000352817	9.55987E-06	0.001155804	0.000407717	1.05541E-05	0.00137657
π24	0.000279678	7.88047E-06	0.00093312	0.000321295	8.64401E-06	0.001088868
π34	0.000130108	3.87272E-06	0.000434543	0.000190157	5.22052E-06	0.000628641
ψ1	0.01760067	0.000598656	0.053853581	0.0223857	0.000699414	0.070865838
ψ2	0.01544286	0.008632272	0.022313269	0.01417511	0.005307952	0.022828577
ψ3	0.001818763	9.11138E-05	0.00468398	0.002844195	0.000184473	0.006867365
ψ4	0.001031738	3.63824E-05	0.003034414	0.001432367	5.12787E-05	0.004152524
w1	-0.04305101	-0.05686121	-0.029993056	-0.05930241	-0.07905371	-0.040506481
w2	0.004941528	0.00292663	0.006968493	0.005539683	0.002717738	0.008372986
w3	0.000245166	-0.000867772	0.001349027	0.00047112	-0.001047336	0.002034965
w4	3.14497E-05	-0.000976153	0.00105784	3.24742E-05	-0.001405177	0.001480557
ϕ1	-0.09948864	-0.1386009	-0.062620948	-0.1160487	-0.165707	-0.068375919
ϕ2	0.02318314	0.01179137	0.035010069	0.02839322	0.01233613	0.044752898
ϕ3	0.000206595	-0.002663267	0.003074516	0.000534253	-0.00387031	0.004956209
ϕ4	-0.000320574	-0.000844757	0.000189978	-0.000449142	-0.001330484	0.000413223
σ	0.3255726	0.2720959	0.377785054	0.4073819	0.3557411	0.459362552
λ	4.013736	3.327364	4.931499505	4.025731	3.226282	5.114601016
q	1.007243	1.000184	1.026459915	1.006745	1.00017	1.025222478
α1	0.3733011	0.2839632	0.476988061	0.2604554	0.1983607	0.334086789
α2	0.6266989	0.5230119	0.716036814	0.7395446	0.6659132	0.801639253
s1	4.353528	3.549725	5.441964561	3.195245	2.714313	3.787863766
s2	3.833002	3.137094	4.656513502	3.727351	3.071032	4.496118437

***Table 6**. Parameter estimated values (2007-2008)*

	2007			2008		
	Mean	Quart. 2.5	Quart. 97.5	Mean	Quart. 2.5	Quart. 97.5
cte	10.14014	9.972481	10.30237788	10.34732	101.4054	10.55057447
γ1	0.4708065	0.2525862	0.674322901	0.5722866	0.3031464	0.828462659
γ2	0.006123973	-0.01654819	0.029139484	0.007404153	-0.01673279	0.032745199
γ3	0.001411512	-0.009658335	0.0127156	-0.001097232	-0.01285639	0.010814046
γ4	-0.001746682	-0.01210329	0.008749365	-0.001904778	-0.01287983	0.009062091
π11	0.1212265	0.005394652	0.326413512	0.1599907	0.007732211	0.41774053
π22	0.004051314	0.000385153	0.008466765	0.005437384	0.001100437	0.010333741
π33	0.000657764	5.35337E-05	0.001508455	0.000433055	1.73217E-05	0.001211712
π44	0.000253501	8.94437E-06	0.000764835	0.000207046	6.04524E-06	0.00066093
π12	0.008652488	0.000240972	0.030248832	0.01154446	0.000315801	0.039864972
π13	0.004648162	0.000120799	0.015714684	0.005310333	0.000133026	0.018143499
π14	0.004312861	0.00012509	0.014624586	0.005027208	0.000145134	0.016903432
π23	0.000387496	1.11726E-05	0.001299313	0.000470345	1.34919E-05	0.001537936

π24	0.000311226	9.03098E-06	0.001067078	0.000349395	1.02702E-05	0.001181587
π34	0.000177166	5.13371E-06	0.000578193	0.000159808	4.34228E-06	0.000536658
ψ1	0.0179046	0.000612182	0.057163339	0.02196728	0.000690418	0.069915958
ψ2	0.01590547	0.007958979	0.023778338	0.01038437	0.002600325	0.018398101
ψ3	0.002602085	0.000160797	0.006314843	0.001862489	7.69982E-05	0.005096942
ψ4	0.001311614	5.11183E-05	0.003826155	0.001105584	3.65261E-05	0.003385237
w1	-0.05402173	-0.06985112	-0.038784435	-0.04669198	-0.06283956	-0.031360455
w2	0.005975934	0.003633699	0.008340028	0.00415426	0.001990433	0.006357543
w3	0.000448813	-0.0008447	0.001769793	0.000126284	-0.001064506	0.001337889
w4	6.1603E-05	-0.001137201	0.001263563	7.98685E-06	-0.001101154	0.001129136
ϕ1	-0.1226026	-0.1681808	-0.078460202	-0.08988481	-0.1350834	-0.04656021
ϕ2	0.0285233	0.0149936	0.042392018	0.02003482	0.007731813	0.032866598
ϕ3	0.00068739	-0.002704374	0.004025872	5.77747E-05	-0.002847153	0.002950595
ϕ4	-0.000331766	-0.000908558	0.000253519	-0.000283807	-0.000715782	0.000139152
σ	0.3883262	0.3369021	0.438754673	0.3451994	0.2926763	0.399303053
λ	4.208214	3.344677	5.424572372	3.609299	2.863625	4.653406794
q	1.005613	1.000137	1.020087736	1.019461	1.000498	1.072687122
α1	0.2178712	0.1590782	0.287231246	0.4102552	0.3137282	0.515745862
α2	0.7821288	0.7127688	0.840921793	0.5897448	0.4842541	0.686271844
s1	3.137084	2.680883	3.6852295	4.588913	3.57559	5.949704997
s2	4.609831	3.701227	5.746320869	3.048145	2.487865	3.705161055

Table 7. *Descriptive statistics - efficiency scores*

	2005	2006	2007	2008
Mean	0.79673	0.79737	0.80465	0.77811
Median	0.81269	0.80872	0.81584	0.80160
St. dev.	0.08866	0.07646	0.07558	0.09845
Quartile 1	0.75810	0.76170	0.77070	0.72961
Quartile 3	0.85711	0.84977	0.85588	0.84671
Maximum	0.92658	0.91987	0.92835	0.92946
Minimum	0.37732	0.32509	0.43497	0.30630

Table 8. *The marginal product of pesticides*

	2005	2006	2007	2008
Median	0.8195	0.7764	0.8786	0.6211
Mean	0.9282	0.9056	1.0444	0.6731

Table 9. *Composite index*

	Cereal	Ind-oth	α_1	α_2	q	Composite index
2005	1.00000	1.00000	0.3733	0.6267	1.00724	1.00000
2006	1.17757	1.17632	0.26046	0.73954	1.00675	1.17466
2007	1.88282	1.50000	0.21787	0.78213	1.00561	1.58649
2008	1.83545	1.88500	0.41026	0.58974	1.01946	1.88551

Table 10. *Marginal Product and Real Price*

	Marginal Product	Pesticides Price	Output Price	Real Price
2005	0.81952	1.000	1.00000	1.0000
2006	0.77643	1.002	1.17466	0.8530
2007	0.87861	1.002	1.58649	0.6316
2008	0.62108	1.035	1.88551	0.5489

Table 11. *The marginal products of pesticides of the two outputs*

	2005		2006		2007		2008	
	Cer.	Ind-Oth.	Cer.	Ind-Oth.	Cer.	Ind-Oth.	Cer.	Ind-Oth.
Real Price	1.0000	1.0000	0.8509	0.8518	0.5322	0.6680	0.5664	0.5491
Median	2.2047	1.3152	2.9960	1.0543	4.0530	1.1266	1.5280	1.0724
Mean	2.4995	1.4882	3.4985	1.2286	4.8226	1.3385	1.6589	1.1594

Cer. et Ind-Oth. respectively represent the block of cereal production and the industrial-Other production.

Table 12. *The pesticide marginal product and size*

		Cereal	Industrial-Other.	UAA
	Quartile 1	0.903	0.7218	69.54
	Quartile 2	1.6405	1.3134	104.545
2005	Quartile 3	2.3439	1.8736	134.17
	Quartile 4	3.5094	2.8023	184.6
	Quartile 1	1.0868	0.3848	70.985
	Quartile 2	2.0502	0.7204	106.975

2006	Quartile 3	3.3165	1.1678	134.87
	Quartile 4	4.7575	1.6713	188.26
	Quartile 1	1.5502	0.4324	70.76
	Quartile 2	2.966	0.8244	107.17
2007	Quartile 3	4.1023	1.1379	136.97
	Quartile 4	6.0114	1.6667	191.88
	Quartile 1	0.6817	0.4805	69.53
2008	Quartile 2	1.0322	0.7208	106.65
	Quartile 3	1.4415	1.0122	135.65
	Quartile 4	1.7364	1.212	190.85

Table D1. *Price indexes of cereal crops*

	2005	2006	2007	2008
Wheat	100.0000	120.0000	189.3400	190.0000
Durum wheat	100.0000	107.0000	174.0250	218.4400
Spring barley	100.0000	114.8200	193.0000	169.3100
Winter barley	100.0000	114.8200	193.0000	169.3100
Corn	100.0000	115.3500	164.6000	161.8800
Oat	100.0000	137.0250	197.1900	208.0000
Irrigated corn	100.0000	115.3500	164.6000	151.8800
Spring wheat	100.0000	120.0000	189.3400	190.0000
Other cereals	100.0000	117.5000	183.1000	179.3000

Table D2. *Price indexes of Industrial-other crops*

	2005	2006	2007	2008
Protein pea	100.0000	111.7000	181.2000	183.7000
Beet	100.0000	81.8000	74.1000	71.000
Potato	100.0000	159.3000	177.7000	121.6000
Rape	100.0000	119.4000	150.0000	188.5000
Sunflower	100.0000	96.6000	163.1000	160.9000
Flax	100.0000	111.5000	94.3000	56.6000
Poppy	100.0000	111.2000	118.1000	117.9000
Lucerne	100.0000	97.1300	110.1750	171.3600
Fodder	100.0000	98.3000	133.8000	192.2000

crops				
Fruits	100.0000	111.3000	118.2000	133.3000
Vegetables	100.0000	110.1000	110.7000	107.1000
Horticulture	100.0000	105.6000	102.6000	108.9000

Table D3. *Input price indexes*

	2005	2006	2007	2008
Price index of fixe capital consumption	99.6000	102.4000	106.0000	112.4000
Price index of intermediate consumption	100.0000	102.8000	108.8000	122.9000
Price index of the crop protection products	100.0000	102.2000	102.2000	103.5000

Figure 1. *Damage reduction function*