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Can the Energy Transition Be Smooth?

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Abstract

In a decentralised two-sector model (energy and final good sectors), we analyse the short and long term macroeconomic implications of an energy transition characterized by a progressive rise of renewables in the energy mix of the economy. We show that in the long run, there is a tight negative relationship between the energy return (EROEI ratio) offered by the energy production and the share of investment that must be allocated to the energy sector. Consequently, an energy transition characterized by a decreasing trend of the EROEI ratio leads to major macroeconomic changes, both in the allocation of final output between consumption and investment and in the allocation of capital between the energy and non-energy sectors. As a result, the transition may be characterized by a non-monotonic evolution of aggregate income and private consumption: after a peak, the economy experiences a contraction episode which necessarily begins before the end (of the extraction) of the non-renewables. We analyze how 1) the magnitude of this contraction and 2) the possibility of an ulterior recovery of income are affected by the initial stock of non-renewables, the potentials of technical progress in the energy and non-energy sectors and the substitutability between capital and energy.

Keywords: energy transition, renewable energy, non-renewable energy, EROEI, growth, savings rate.

JEL classification: Q32, Q43, Q57, O44.

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1 Introduction

Even if a transition from non-renewable to renewable energy sources seems inescapable in the long run, the primary energy supply to the World economy remains largely dominated by non-renewables and more than 80% of the World energy supply still rely on three fossil fuels (coal, oil, gas). According to British Petroleum (2014), the share of renewables (including hydroelectricity) only amount to 9.7% of the primary energy mix of the World economy. The International Energy Agency (IEA, 2014) expects it to rise to 18% by 2035. Such a projection does not mean that renewable energy supply will not grow rapidly in absolute terms: it simply reflects that the IEA expects World economic growth to remain intensively dependant on non-renewable energies over the (at least two) decades to come. One may however question the possibility of maintaining much longer a growth process fed by an intensive use of non-renewables. Some authors like Meadows *et alii* (2004) and Cappelan-Peréz *et alii* (2014) even doubt that the transition towards renewable energy will allow our rich societies to maintain the level of material welfare (i.e. the income levels) reached when the access to non-renewable energies was relatively cheap. This paper comes back on this issue in a macroeconomic growth model.

A detour by the energy science literature highlights why energy transition may challenge economic prosperity. In order to survive or grow, "any being or system" needs energy and "must gain substantially more energy than it uses in obtaining that energy" (Hall *et alii*, 2009). The energy surplus generated by an energy production process can be measured by its EROEI ratio (EROEI for Energy Returned On Energy Invested): it is the ratio of the quantity of energy delivered by this process to the quantity of energy used by the process (see a.o. Cleveland (2008), Hall *et alii*, op citum). This use of energy can be direct (e.g. the coal burned in a boiler) or indirect (e.g. the energy necessary for building the boiler). High quality resources have a high EROEI ratio: relatively to their energy density, they require little energy to be discovered, extracted, processed and delivered to the point of use. If the EROEI concept can be used at a very disaggregated level to describe the efficiency of a given process (e.g. an oil refinery), it is also meaningful at a more global level to describe the efficiency of the energy supply to the economy.

The development of contemporary industrial societies over the last two hundred years has been heavily dependent on fossil fuels that offered a very high EROEI ratio. This large energy surplus allowed our economies to allocate most of their labour and man-made ressources (capital) to other activities than energy production. High EROEI resources have thereby contributed to economic growth: as e.g. Cleveland (2008) notes, "because the production of goods and services is a work process [in the physical sense], economies with access to higher EROEI fuel sources have greater potential for economic expansion and/or diversification." However, a declining trend of the global EROEI seems unavoidable. On the one hand, non-renewable ressources of high quality are progressively depleting and the exploitation of the residual resources is accompanied by a fall in their EROEI, either because their energy density is lower and/or because their extraction gets -directly or indirectly- increasingly energy consuming. On the other hand, most renewable energies seem characterized by much lower EROEI ratios than conventional non-renewable energies (see e.g. Cleveland (2004), Murphy and Hall, (2010))[1]. Several authors suspect that this "declining EROEI will take a huge economic toll in the future" (Hall *et alii*,

[1]Hydropower is an exception but its development potential is limited.

2009, p.34). van den Bergh (2013, p.15) describes why in the following words: if renewable energy sources offer a lower EROEI, "an economy running entirely on renewable energy would then devote a disproportionally large share of activity and labor to provide intermediate services to the renewable energy sector: energy delivery, extraction of material resources, production of high-quality materials and equipment (solar PV panels, wind turbines), transport of materials and equipment, maintenance, and education of experts. Innovations in renewable energy technologies (whether wind, solar or biofuels) tend to increase the complexity and roundabout character of the supportive system, causing significant progress in associated EROEI values to be uncertain."

One understands that if the energy transition forces an economy to devote much more inputs to energy production, it might experience a scenario à la Meadows (2004) where investments in the energy sector would crowd investments out of the final good sector, which could imply a decrease in (or even collapse of) final production and consumption. In such a scenario, generations born at the dawn of -or during- this economic contraction would suffer a severe loss of material well-being and, once the transition would be achieved, the forthcoming generations would not be guaranteed to enjoy a level of material well-being as high as the one of their luckiest ancestors.

Papers interested in the EROEI measure (like a.o. Hall et al. (op citum), King and Hall (2011), Murphy et Hall (2010), Heun et de Wit (2012)) offer partial equilibrium analyses[2]. The economic growth literature, for its part, has largely ignored the EROEI concept until now. We propose here a dynamic growth model in which we analyse the macroeconomic consequences of the use of energy sources (either non-renewable or renewable) offering lower EROEIs. Thereby, our model contributes to bridging a gap between the macroeconomic growth- and energy science literatures, two literatures that are used to progress quite independantly[3].

If they ignore the EROEI concept, several recent contributions to economic growth theory have nevertheless analyzed the issue of the energy transition. Tahvonen and Salo (2001) have proposed an optimal growth model able to explain the progressive rise and subsequent fall in the use of the non-renewable energy sources[4]: in a first phase of its development (a kind of preindustrial era where the capital stock is low), the economy only uses renewables; in a second phase where the capital stock is high enough and the demand for energy higher, the extraction of non-renewables becomes economically possible and the share of non-renewables in energy production increases progressively, before decreasing again once the reduction in the resource stock increases the marginal cost of extraction; in a last phase, the non-renewables extraction has become too costly and only renewables are used. Contrary to ours, this paper does not analyze the determinants of a possible fall in final production or consumption after a period of growth (albeit some figures of

[2]The GEMBA model developed by Dale et al. (2012) is an exception which incorporates explicitly the EROEI measure into an aggregate model. However, it does not describe explicitly the economic agents' behaviours and is not closed in the sense that a component of aggregate demand remains exogenous.

[3]Fagnart-Germain (2016) is another contribution in this direction. Using a dynamic input-output model with two sectors, they analyse a.o. the macroeconomic implications of the quality of the energy resource used by the economy. However, they adopt a purely accounting approach and do not model neither the agents' behaviours nor market prices. It is thus not a dynamic general equilibrium model of the energy transition contrary to what we propose here.

[4]This possibility of a non-monotonic evolution in the nonrenewable use contrasts with the outcomes of the traditional models of Dasgupta and Heal (1974) and Stiglitz (1974) in which it is necessarily monotonically decreasing.

their articles show that such a negative output adjustment may happen in the absence of technical progress). Nor is this issue dealt with in Tsur and Zemel (2005), in Amigues *et alii* (2011) or in Bonneuil and Boucekkine (2016). Tsur and Zemel characterize the dynamics of an optimal growth model with R&D investments which reduce the cost of use of backstop technologies. Amigues *et alii* analyse the optimal use of a polluting non renewable energy and a clean renewable one in the presence of a ceiling on the athmospheric carbon concentration. Bonneuil and Boucekkine use viability theory to study the best transition to a clean renewable energy when an irreversible pollution threshold exists. The possibility of a downward adjustment in output and consumption during the energy transition is analyzed in Growiec and Schumacher (2008) and Jouvet and Schumacher (2012). Growiec and Schumacher analyze the consequences of an imperfect substituability between non-renewables and renewables in a growth model without man-made capital. In Jouvet and Schumacher, non-renewables and renewables are perfect substitutes but are never used simultaneously and the energy transition takes the extreme form of a one-period switch from a 100% non-renewable use to a 100% renewable use in energy production. If a downward adjustment of output takes place in their model, it occurs once the non-renewable ressource is exhausted and turns out to be particularly disastrous: indeed, unless a learning-by-doing effect reduces sufficiently the cost of renewable energy production, it will mean a neverending collapse of the economy towards a zero output equilibrium. In our model where the energy transition takes the form of a progressive rise in the share of renewable energy, a contraction of output will be shown to start strictly before the end of the extraction of fossil fuels and will take a less extreme form.

On top of introducing the EROEI concept, our model has three methodological differences with respect to the above quoted growth models. Firstly, the energy sector is explicitly distinguished from the rest of the economy (modelled as the final good sector)[5]. In accordance with the intuitions developped hereabove in van den Bergh's quotation, this distinction enables us to highlight that the transition towards a 100%-renewable energy supply is characterized by deep changes not only in the allocation of final output between consumption and investment but also in the allocation of capital between the energy- and non-energy sectors. Secondly, our model respects the postulates of ecological economics in the line of the macroeconomic models of a.o. Krysiak (2006), Fagnart and Germain (2011) and Germain (2012). In particular, we consider that technological progress is bounded by physical limits: it can reduce the energy intensiveness of human productions up to some point but the energy content of any such production is bounded from below by a strictly positive value. Consequently, perpetual economic growth (measured by the quantity of goods produced) is impossible in our framework, contrary to what may occur in the above mentionned models. Thirdly, we analyze the dynamics of a decentralized economy and not the behaviour of a hypothetical central planner as in the optimal growth tradition. Our aim is to identify the private agents' behaviours and/or technological/environmental conditions that favor or not a smooth energy transition.

The article is structured as follows. Section 2 presents the model[6], decribes the agents' behaviours and markets and constructs EROEI ratios at the level of the non-renewable

[5]For another growth model characterised by the same distinction, see D'Alessandro *et alii* (2010).

[6]A first version of the model was developed in Fagnart and Germain (2014) and relied on several simplifying assumptions: the absence of extraction cost at the level of the non-renewable energy sector, exogenous technical progress, a Leontief production function in the final good sector and a logartithmic utility fonction. We relax these assumptions in the present version of the model.

and renewable energy subsectors and at a global level. Section 3 analyses the stationary state reached once only renewable energy is used. It identifies the determinants of the long run EROEI and its relationship with 1) the share of capital invested in energy production and 2) the real energy price. In section 4, we study the model dynamics. Because our interest is mainly prospective and not historical, we concentrate on trajectories along which the energy supply progressively changes from a mix of non-renewables and renewables to the exclusive use of renewables. In spite of technical progress, the transition may be characterised by a contraction of final output and consumption. We show that if it happens, such a contraction necessarily starts before the completion of the transition, i.e. at a time when non-renewables are still used. The model dynamics is next analysed numerically. The energy transition in the baseline scenario is characterized by a peak of GDP and consumption, followed by a contraction episode of the economy and next by its convergence towards a stationary state. We then present a sensitivity analysis showing how the non-renewable resource stock, the substitutability between energy and capital, the potential of technological progress in the energy and final good sectors affect the possibility of a non monotonic trajectory of the economy and the magnitude of the contraction of income and consumption in such a case. Section 5 summarizes our main results.

2 The model

We consider an economy with three competitive markets: final good, capital and energy. The final good is used for consumption and investment. It also serves as numéraire. Its production requires physical capital and (renewable or non-renewable) energy. Capital is accumulated by households and rent to firms.

The total energy supply is a mix of non-renewable and renewable energies. Non-renewable energy (NRE hereafter) is supplied by competitive firms who run a NRE stock and face extraction costs. Renewable energy (RE hereafter) is supplied by competitive firms, which operate a free and constant primary RE flux (for example solar energy). The two energy types are assumed to be *final* (they can be used for productive purposes without any further transformation) and perfect subtitutes in the final production process. They are thus sold at the same price to final firms. Both types of energy production requires capital goods delivered by the final good sector. As we focus on the interactions between the energy and final sectors in a macroeconomic perspective, we adopt an approach in terms of net energy, i.e. we focus on the part of energy production that is available to the non-energy sector of the economy and disregard the energy consumption of the NRE and RE sectors.

In the final good sector, technical progress increases energy efficiency through time. In the energy sector, technical progress makes energy production less capital intensive *ceteris paribus*. Technical progress is however bounded by physical constraints. Energy efficiency in the final sector is bounded from above: the ratio between energy input and final output cannot tend towards zero, even asymptotically. Similarly, the capital intensiveness of energy production is bounded from below: the ratio between the capital allocated to the energy sector and the produced energy cannot tend towards zero.

2.1 The non-renewable energy sector

A representative competitive firm runs the NRE stock. Let us label $\mathcal{S} > 0$ the NRE endowment of the economy (i.e. the initial NRE stock), S_t the remaining stock at the beginning of period t and E_t the quantity of NRE extracted during period t. The dynamics of the NRE resource stock is given by

$$S_{t+1} = S_t - E_t \geq 0 \tag{1}$$

NRE extraction in period t requires a quantity of capital L_t that is increasing in the extracted quantity E_t and decreasing in the remaining stock S_t. Specifically, we assume

$$L_t = \rho_t \frac{E_t}{S_t - E_t}, \tag{2}$$

where ρ_t is a positive technological factor exogenous at the firm's level. Technology (2) reflects that at given ρ_t, extraction gets more costly (here more capital intensive) when the remaining resource stock is smaller.

If q_t is the price of energy in t and v_t is the rental price of capital in t, the optimisation problem of the representative NRE producer writes as follows:

$$\max_{\{E_t, S_{t+1}, L_t\}_{t=1,...,T}} \sum_{t=1}^{T} R_t \left[q_t E_t - v_t L_t\right] \tag{3}$$

under constraints (1) and (2), with $S_1 = \mathcal{S}$ and $R_t = \prod_{\tau=1}^{t} \left[1 + r_\tau\right]^{-1}$ where r_t is the firm's discount rate.

In this paper interested in the contemporary energy transition, we focus on trajectories characterised first by a phase where NRE and RE are simultaneously used and next by a phase where only RE is produced[7]. We thus look for a solution characterised by (i) $E_t > 0$ or $S_{t+1} > S_t$ for $t = 1, ..., T_e$ with T_e $(1 \leq T_e \leq T)$ the optimal time length during which the NRE stock is exploited and (ii) $E_t = 0$ or $S_{t+1} = S_t$ for $t = T_e + 1, ..., T$. We solve this problem in two steps. First, taking T_e as given, we determine the optimality conditions for the other decision variables: these variables are so expressed as functions of T_e. We next determine the optimal value of T_e.

The optimality conditions of (3) lead the representative firm to manage the NRE stock according to the following rule (see Appendix 7.1):

$$q_t - \frac{v_t \rho_t S_t}{S_{t+1}^2} = \frac{\mathbf{1}_{t \leq T_e - 1}}{1 + r_{t+1}} \left[q_{t+1} - \frac{v_{t+1} \rho_{t+1}}{S_{t+2}}\right], \; t \in \{1, ..., T_e\}, \tag{4}$$

where $\mathbf{1}_{t \leq T_e - 1} = 1$ if $t \leq T_e - 1$ and 0 if not. Given S_1, this equation enables to compute $S_2, ..., S_{T_e}$ as functions of the variables exogenous at the firm level. Equation (4) means that the resource is managed according to the Hotelling rule with extraction costs: discounted marginal benefits of extraction are equalised across periods during the whole extraction process.

[7] In Tahvonen and Salo (2001), the extraction cost of the first NRE unit is larger than the operating cost of the first RE unit. Hence, the trajectory of their model is characterized by 3 phases: a first one where only RE is used, a second one where the two energy sources are exploited, a last phase where NRE is economically depleted (i.e. too costly to extract) and only RE is used.

In a second step, we determine T_e. Let $\{S_t^*\}_{1\leq t\leq T}$, $\{L_{t+1}^*\}_{1\leq t\leq T}$ and $\{E_t^*\}_{1\leq t\leq T_e}$ be the vectors that are solutions to problem (3) at given T_e. These vectors depend on T_e. The optimal T_e is the solution to the following problem:

$$\max_{T_e\in\{1,\ldots,T\}} \sum_{t=1}^{T} R_t \left[q_t E_t^* - v_t L_t^*\right]. \tag{5}$$

Because T_e is discrete, it cannot be determined by using differential calculus and numerical methods are necessary.

2.2 Renewable energy sector

The economy enjoys a constant flow $\mathcal{F}$ of renewable energy (say for example the radiant energy of the sun). We consider a perfectly competitive RE sector with N identical price-taking producers. In order to capture (part of) the flow $\mathcal{F}$, each RE producer needs capital goods, which it rents from households.

Let f_t be the energy supply of a RE firm and g_t its capital stock. RE production is described by the following technological relationship

$$f_t = B_t g_t^{\gamma}, \qquad \text{with} \quad 0 < \gamma < 1. \tag{6}$$

$B_t > 0$ is a productivity term. Technology (6) implies that returns-to-scale are decreasing: the capital intensiveness of a RE firm is increasing in its production level.

In each period t ($t = 1, ..., T$), a RE producer chooses a RE supply f_t and a capital stock g_t by solving

$$\max_{f_t, g_t} \; q_t f_t - v_t g_t \tag{7}$$

under the technological constraint (6).

At the RE sector level, the total energy supply and capital demand are given by $F_t = N\, f_t$ and $G_t = N\, g_t$. Appendix 7.2 shows that the solutions to (7) lead to the following aggregate relationship between F_t and G_t.

$$G_t = \gamma \frac{q_t}{v_t} F_t. \tag{8}$$

(8) implies that γ is the share of the capital cost ($v_t G_t$) in the RE sector income ($q_t F_t$).

We assume that B_t is affected by a negative aggregate externality linked to the aggregate RE production level F_t: $B_t = B(F_t)$ with $B'(F_t) < 0$. This means that the capital requirement of a RE firm is increasing in the rate of use of the RE flow, $F_t/\mathcal{F}$ (or equivalently in F_t if $\mathcal{F}$ is constant)[8]. We assume the following functional form for $B(F_t)$:

$$B(F_t) = \frac{1}{d_t}\left[1 - \frac{F_t}{\mathcal{F}}\right], \tag{9}$$

[8]Operating solar or wind energy requires an access to different sites where this energy can be captured. Some sites are less favourable than others and need a higher windmill or more solar pannels to obtain the same quantity of energy than the best sites. In other words, as in a Ricardian resource model, decreasing returns-to-scale follow from the fact that increasing production requires to operate less and less favourable sites. See also Fagnart-Germain (2011) for a more formal justification of the assumption of decreasing returns-to-scale in the use of a renewable resource.

where $d_t > 0$ is a positive function decreasing over time as a result of technical progress (exogenous at the firm level).

Under (9), it is easily shown (see Appendix 7.2) that the aggregate technology of the RE sector is given by

$$G_t = \left[\frac{b_t F_t}{1 - \frac{F_t}{\mathcal{F}}} \right]^{\frac{1}{\gamma}} \quad \text{where} \quad b_t = N^{\gamma-1} d_t. \tag{10}$$

G_t and F_t are increasing in the energy price and decreasing in the rental price of capital.

The technologies described by (2) and (10) share a same fundamental property: the operating cost of energy production explodes when the energy production level reaches the resouce limits (i.e. respectively when $E_t \to S_t$ in the NRE case and when $F_t \to \mathcal{F}$ in the RE case).

2.3 The final good sector

Final good production Y_t requires capital and energy. The production technology is of the CES type with constant returns-to-scale: in $t \in \{1, ..., T\}$,

$$Y_t = \left[a[A_t X_t]^{\frac{\sigma-1}{\sigma}} + [1-a][\zeta H_t]^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \quad \text{with} \quad 0 < a < 1, \sigma > 0, \tag{11}$$

where X_t and H_t are respectively the energy flow and the capital stock allocated to final production; $A_t > 0$ and $\zeta > 0$ are productivity factors; σ is the elasticity of substitution between capital and energy.

With the final good price choosen as numéraire, the profit maximization problem of the final firm can be written as

$$\max_{X_t, H_t} \left[a[A_t X_t]^{\frac{\sigma-1}{\sigma}} + [1-a][\zeta H_t]^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - q_t X_t - v_t H_t,$$

where the capital stock H_t is rent from households at the beginning of the period.

The first order conditions for X_t and H_t are respectively given by (see Appendix 7.3):

$$aA_t \left[\frac{Y_t}{A_t X_t} \right]^{\frac{1}{\sigma}} = q_t \tag{12}$$

$$[1-a]\,\zeta \left[\frac{Y_t}{\zeta H_t} \right]^{\frac{1}{\sigma}} = v_t. \tag{13}$$

The ratio between these two first order conditions shows that the capital-energy ratio H_t/X_t is a an increasing function of the relative factor price q_t/v_t:

$$\frac{H_t}{X_t} = \left[\frac{A_t}{\zeta} \right]^{1-\sigma} \left[\frac{1-a}{a} \frac{q_t}{v_t} \right]^{\sigma}. \tag{14}$$

Under constant returns-to-scale, perfect competition in the final good sector implies a nil profit: $Y_t = q_t X_t + v_t H_t$. Given (12) and (13), it leads to the following relationship between the price of energy and the rental price of capital:

$$a^{\sigma} \left[\frac{q_t}{A_t} \right]^{1-\sigma} + [1-a]^{\sigma} \left[\frac{v_t}{\zeta} \right]^{1-\sigma} = 1, \ t \in \{1, ..., T\}. \tag{15}$$

The term in q_t/A_t on the left-hand side is the real unit cost of energy and the term in v_t/ζ is the real unit cost of capital. Under perfect competition and constant returns-to-scale, these two real unit costs add up to one.

2.4 Technological progress

Productivity factors A_t, b_t and ρ_t evolve through time as a result of an endogenous technological progress. Technological progress is however bounded by physical laws. Energy efficiency, A_t, is bounded from above by a strictly positive $A_* < \infty$: this means that it will never be possible to produce a given quantity of final good with an infinitely small quantity of energy, even asympotically. Similarly, the productivity of capital in energy production (resp. $1/b_t$ in the RE sector) is bounded from above: in the RE (resp. NRE) sector, b_t (resp. ρ_t) is bounded from below by a strictly positive value $b_* > 0$ (resp. $\rho_* > 0$). This means that the production of an energy unit will always require a non infinitesimal quantity of capital.

The evolutions of A_t, b_t and ρ_t towards their respective asymptotic values are endogenous and follow from a *learning-by-doing* process, which is sector specific. In the final good sector, energy efficency A_t is an increasing function of the cumulative energy consumption of the sector $\sum_{i=1}^{t-1} X_i$: i.e.,

$$A_t = A\left(\sum_{i=1}^{t-1} X_i\right) \tag{16}$$

where A is an increasing function which tends asymptotically towards $A^* < \infty$. Similarly, the capital intensiveness of the RE sector b_t is a decreasing function of the cumulative renewable energy production $\sum_{i=1}^{t-1} F_i$; in the NRE sector, the capital intensiveness of the extraction process ρ_t is a decreasing function of the cumulative extraction $\sum_{i=1}^{t-1} E_i$. Formally,

$$b_t = b\left(\sum_{i=1}^{t-1} F_i\right) \qquad \text{and} \qquad \rho_t = \rho\left(\sum_{i=1}^{t-1} E_i\right), \tag{17}$$

where functions $b(\cdot)$ and $\rho(\cdot)$ are decreasing in their argument and tend asymptotatically towards their respective lower bounds, b^* and ρ^*.

2.5 Households

We consider a representative agent with a very long time horizon. During a given period t, she chooses a consumption level C_t and makes an investment decision K_{t+1}. She receives the macroeconomic income under the form of capital rent and profits Ω_t. Her budget contraint of period t can be written as

$$C_t + \frac{K_{t+1}}{\varphi} = v_t K_t + \Omega_t, \tag{18}$$

where K_t is the capital stock accumulated in $t-1$, v_t is the rental price of capital. K_{t+1}/φ is the investment level in period t, $\varphi > 0$ reflecting the productivity of the transformation of investment goods into productive capital. For analytical tractability, we assume a unitary depreciation rate, which implies that the length of a time period corresponds to the average life-time of productive capital.

The household's preferences are represented by a time-separable isoelastic utility function with a discount factor $0 < \beta < 1$. The intertemporal decision problem writes as:

$$\max_{\{C_t, K_{t+1}\}_{1 \le t \le T}} \quad U = \sum_{t=1}^{T} \beta^t \frac{C_t^{1-\alpha} - 1}{1-\alpha}, \alpha > 0 \tag{19}$$

under constraint (18) and with K_1 and Ω_1 given. T is the exogenous time horizon (possibly infinite). The first-order conditions for an interior solution lead to:

$$\left[\frac{C_{t+1}}{C_t}\right]^{\alpha} = \beta \, \varphi \, v_{t+1}, \; t \in \{1, ..., T-1\} \tag{20}$$

and to the terminal condition $K_{T+1} = 0$.

Equation (20) describes the well-known consumption smoothing behaviour. In the present model, the term $\varphi \, v_{t+1}$ corresponds to one plus the real interest rate. We assume that NRE firms are managed in the interest of their shareholders. These firms thus discount future profits in a way consistent with households' preferences: they use a discount rate consistent with (20) and thus given by

$$1 + r_{t+1} = \varphi \, v_{t+1}, \; t \in \{1, ..., T-1\} \, . \tag{21}$$

2.6 Market equilibrium conditions

The market equilibrium conditions can be written as follows:

- On the final good market, output is allocated either to consumption or to investment (which determines the capital stock of the following period):

$$Y_t = \begin{cases} C_t + \frac{K_{t+1}}{\varphi}, \forall \, t \in \{1, ..., T-1\} \\ C_t, \qquad \text{for } t = T. \end{cases}$$

- On the capital market, the capital stock supplied by households is demanded either by energy producers or by final good firms:

$$K_t = G_t + H_t + L_t, \; \forall t \in \{1, ..., T\} \, .$$

- On the energy market, the demand of final good firms matches the supply of RE and NRE firms:

$$X_t = \begin{cases} E_t + F_t, \; \forall t \in \{1, ..., T_e\} \\ F_t, \qquad \forall t \in \{T_{e+1}, ..., T\} \end{cases}$$

2.7 The dynamic system

The trajectory of the economy consists of two phases : (i) a first phase lasts the first T_e periods and is characterised by strictly positive RE and NRE productions; ; (ii) a second phase starts from $T_e + 1$ and is characterised by the absence of any NRE use ($E_t = 0, \; \forall t > T_e$).

2.7.1 The model equations

The macroeconomic equilibrium can be summarized by the following equations describing the agents' behaviours, the production technologies and the equilibrum conditions:

$$\left[\frac{C_{t+1}}{C_t}\right]^{\alpha} = \beta\varphi v_{t+1},\ t\in\{1,...,T-1\} \tag{22}$$

$$Y_t = \begin{cases} C_t + \frac{K_{t+1}}{\varphi},\ t\in\{1,...,T-1\} \\ C_T,\ t=T \end{cases} \tag{23}$$

$$X_t = \begin{cases} E_t + F_t,\ t\in\{1,...,T_e\} \\ F_t,\ t\in\{T_e+1,...,T\} \end{cases} \tag{24}$$

$$K_t = \begin{cases} G_t + H_t + L_t,\ t\in\{1,...,T_e\} \\ G_t + H_t,\ t\in\{T_e+1,...,T\} \end{cases} \tag{25}$$

$$G_t = \gamma\frac{q_t}{v_t}F_t,\ t\in\{1,...,T\} \tag{26}$$

$$G_t = \left[\frac{b_t F_t}{1-\frac{F_t}{\mathcal{F}}}\right]^{\frac{1}{\gamma}},\ t\in\{1,...,T\} \tag{27}$$

$$E_t = S_t - S_{t+1},\ t\in\{1,...,T_e\} \tag{28}$$

$$L_t = \rho_t\frac{E_t}{S_t - E_t},\ t\in\{1,...,T_e\} \tag{29}$$

$$q_t = aA_t\left[\frac{Y_t}{A_t X_t}\right]^{\frac{1}{\sigma}},\ t\in\{1,...,T\} \tag{30}$$

$$v_t = [1-a]\,\zeta\left[\frac{Y_t}{\zeta H_t}\right]^{\frac{1}{\sigma}},\ t\in\{1,...,T\} \tag{31}$$

$$1 = a^{\sigma}\left[\frac{q_t}{A_t}\right]^{1-\sigma} + [1-a]^{\sigma}\left[\frac{v_t}{\zeta}\right]^{1-\sigma},\ t\in\{1,...,T\} \tag{32}$$

$$q_t - v_t\rho_t\frac{S_t}{S_{t+1}^2} = \frac{\mathbf{1}_{t\le T_e-1}}{1+r_{t+1}}\left[q_{t+1} - v_{t+1}\rho_t\frac{1}{S_{t+2}}\right],\ t\in\{1,...,T_e\} \tag{33}$$

$$1 + r_{t+1} = \varphi v_{t+1},\ t\in\{1,...,T-1\} \tag{34}$$

To these equations, we must add the three equations governing the evolutions of A_t, b_t and ρ_t (equations (16-17)) and add a(n implicit) condition on T_e following from (5).

$K_1, S_1, A_1, b_1, \rho_1\mathcal{F}, \alpha, \beta, \gamma, \sigma, \varphi, a$ are exogenously given[9]. In the sequel, we substitute $1 + r_{t+1}$ by φv_{t+1} in equation (33), which allows us to omit (34).

The savings rate of the economy is given by:

$$s_t = \frac{K_{t+1}}{\varphi Y_t} \tag{35}$$

[9]One can verify that the system consisting of (16-17) and (22)-(34) contains as many equations as unknowns. Indeed, the unknowns are $C_t, Y_t, X_t, F_t, G_t, H_t, v_t, q_t\ \forall t \in \{1,...,T\}$, $K_t, r_t, A_t, b_t, \rho_t\ \forall t \in \{2,...,T\}$, $E_t, S_{t+1}, L_t\ t\in\{1,...,T_e\}$. This amounts to $8T + 5(T-1) + 3T_e = 13T + 3T_e - 5$ unknowns, which is precisely the number of equations of the system.

and, using (25), it is useful to decompose it the following way:

$$s_t = s_t^h + s_t^\ell + s_t^g \qquad \text{with} \quad s_t^h = \frac{H_{t+1}}{\varphi Y_t}, \quad s_t^\ell = \frac{L_{t+1}}{\varphi Y_t} \quad \text{and} \quad s_t^g = \frac{G_{t+1}}{\varphi Y_t}. \tag{36}$$

s_t^h is the fraction of period t output invested in the final production sector in $t+1$; s_t^ℓ (resp. s_t^g) is the fraction of period t output invested in the NRE sector (resp. in the RE sector) in $t+1$.

2.7.2 Evolution of the EROEI ratio during the transition

Recall that the EROEI is the ratio between the energy delivered by an energy production process and the energy consumed by this process. The operationalization of this definition raises different questions that are carefully analysed by Brandt and Dale (2011) and Brandt et al. (2013). These authors define four Energy Return Ratios (ERR), which differ from each other in (a) whether the numerator is defined in terms of gross or net energy and in (b) whether the denominator includes the energy content of all inputs (including the auto-consumption of energy) or only the one of the external inputs (then excluding the auto-consumption of energy). Since we have focused on net energy in our description of the NRE and RE sectors, we compute our EROEI ratios accordingly: the EROEI numerator is defined in terms of net energy, i.e. the energy that is truly available for the non-energy sector of the economy; the EROEI denominator only measures the energy content of the capital goods used in energy production. This way of operationalizing the EROEI concept allows us to establish simple and elegant relationships between the corresponding EROEI value and other important macroeconomic variables (like the share of savings invested in the energy sectors as shown hereafter).

In the NRE (resp. RE) sector, L_t (resp. G_t) units of capital goods are necessary in period t to produce E_t (resp. F_t) units of net energy. The building of these L_t (resp. G_t) units of capital goods has required an investment L_t/φ (resp. G_t/φ) during $t-1$; this investment corresponds to a fraction s_{t-1}^ℓ (resp. s_{t-1}^g) of period t output Y_t. As X_{t-1} units of energy have been necessary to produce Y_{t-1}, the quantity of energy absorbed by the building of L_t (resp. G_t) is $s_{t-1}^\ell X_{t-1}$ (resp. $s_{t-1}^g X_{t-1}$). Hence, the EROEI of the NRE and RE subsectors (respectively ε_t^{NRE} and ε_t^{RE}) are given by the following ratios:

$$\varepsilon_t^{NRE} = \frac{E_t}{s_{t-1}^\ell X_{t-1}} = \varphi \frac{Y_{t-1}}{X_{t-1}} \frac{E_t}{L_t}, \tag{37}$$

$$\varepsilon_t^{RE} = \frac{F_t}{s_{t-1}^g X_{t-1}} = \varphi \frac{Y_{t-1}}{X_{t-1}} \frac{F_t}{G_t}, \tag{38}$$

where the second equality in (37) (resp. (38)) follows from the definition of s_{t-1}^ℓ (resp. s_{t-1}^g) in (36). For each energy subsector, the EROEI is the product of the average productivity of capital in this subsector (E_t/L_t or F_t/G_t) and two terms that jointly determine the inverse of the energy content of one unit of capital invested in energy production. A higher product $\varphi Y_{t-1}/X_{t-1}$ means that the energy content of one unit of capital used in t is lower, which contributes to a higher EROEI. A higher productivity of capital in energy production does the same.

At the level of the aggregate energy production, producing $X_t = E_t + F_t$ has indirectly consumed a quantity of energy given by $[s_{t-1}^\ell + s_{t-1}^g]X_{t-1}$. The macroeconomic EROEI of period t, ε_t, is thus equal to

$$\varepsilon_t = \frac{X_t}{[s^{\ell}_{t-1} + s^{g}_{t-1}]X_{t-1}} = \varphi \frac{Y_{t-1}}{X_{t-1}} \frac{X_t}{L_t + G_t}, \tag{39}$$

where the second equality follows from the definition of s^{ℓ}_{t-1} and s^{g}_{t-1}. The interpretation of the last expression is very much the same as the one of the expressions obtained at the level of each energy subsector.

We can rewrite the first equality in (39) as $X_t/X_{t-1} = \varepsilon_t[s^{\ell}_{t-1} + s^{g}_{t-1}]$, which leads straightforwardly to the following proposition:

Proposition 1 *In order to reach a given rate of energy growth, the fraction of final output that must be allocated to investment in the energy sector is inversely related to the EROEI level of the energy production process.*

The relationship $X_t/X_{t-1} = \varepsilon_t[s^{\ell}_{t-1} + s^{g}_{t-1}]$ defines the set of combinations of ε_t and $[s^{\ell}_{t-1} + s^{g}_{t-1}]$ that allow the economy to achieve a given growth rate of energy consumption (which itself induces a given growth rate of output *ceteris paribus*). In the positive orthant of the space $(\varepsilon_t, [s^{\ell}_{t-1} + s^{g}_{t-1}])$, these combinations form a hyperbola and define an iso-growth curve comparable to an isoquant in production theory. This also implies that if the energy transition is accompanied by a fall in the EROEI ratio, maintaining a given energy growth rate (and a given output rate growth *ceteris paribus*) will require a compensatory increase in the share of final output that must be invested in energy production.

3 Stationary state

In this section, we assume that $T \to +\infty$ so that (16-17) imply that b_t and A_t are equal to their respective asymptotic values b_* and A_*. NRE extraction has stopped and production activities only rely on RE. Thus, the energy market clears as $X_* = F_*$.

Given our technological assumptions, the stationary state is characterized by constant values $C_*, Y_*, F_*, G_*, H_*, K_*, v_*, q_*$ of variables $C_t, Y_t, F_t, G_t, H_t, K_t, v_t, q_t$. (22) gives the stationary value of the rental price of capital v_* and (32) determines the real unit cost of energy (herebelow u_* for notational convenience) as a function of v_*:

$$v_* = \frac{1}{\varphi\beta} \tag{40}$$

$$u_* =_{def} a^{\sigma} \left[\frac{q_*}{A_*}\right]^{1-\sigma} = 1 - [1-a]^{\sigma} \left[\frac{v_*}{\zeta}\right]^{1-\sigma} = 1 - [1-a]^{\sigma} [\zeta\varphi\beta]^{\sigma-1}, \tag{41}$$

where the last equality follows from (40).

In a stationary state, (30) and (31) can next be used to express the capital-output (resp. energy-output) ratio of the final good sector as a function of v_* (resp. q^*):

$$\frac{H_*}{Y_*} = \zeta^{\sigma-1} \left[\frac{1-a}{v_*}\right]^{\sigma} \tag{42}$$

$$\frac{F_*}{Y_*} = A_*^{\sigma-1} \left[\frac{a}{q_*}\right]^{\sigma}. \tag{43}$$

(26) implies that the capital intensiveness of energy production is proportional to q^*/v^*:

$$\frac{G_*}{F_*} = \gamma \frac{q_*}{v_*}, \tag{44}$$

where the renewable energy supply F_* and G_* are technologically linked by (see (27))

$$\left[\frac{G_*}{F_*}\right]^{\gamma} = b_* F_*^{1-\gamma} \left[1 - \frac{F_*}{\mathcal{F}}\right]^{-1}. \tag{45}$$

Equilibrium conditions of the capital and final good markets are respectively written as:

$$K_* = G_* + H_* \tag{46}$$

$$C_* = \left[1 - \frac{K_*}{\varphi Y_*}\right] Y_*. \tag{47}$$

From now on, we assume that energy and capital are complementary factors, i.e. $\sigma < 1$, which is consistent with empirical evidence (see later discussion).

Lemma 1 *:*

1. *Existence condition: The stationary state equilibrium exists only if households are sufficiently long-termist and if investment goods are productive enough, i.e. if*

$$\zeta \varphi \beta > [1 - a]^{\frac{\sigma}{1-\sigma}}. \tag{48}$$

2. *Uniqueness: If it exists, the stationary state equilibrium is necessarily unique.*

Proof: See Appendix 7.4 ■

Intuitively, the existence of a stationary state requires a sufficiently low discount rate (or a sufficiently high β). Moreover, investment must be productive enough: one unit of investment good creates φ physical units of capital or $\zeta\varphi$ efficient units of capital. If $\zeta\varphi$ was too small, the economy would be unable to maintain its capital stock at a constant level (it would unavoidably decrease through time), even in the extreme case where output would be exclusively allocated to investment.

The existence condition (48) gets more restrictive when the elasticity of substitution between capital and energy in final production is lower: the lower bound at the right-side of (48) is decreasing in σ, from a value of 1 when $\sigma \to 0$ (Leontief technology) to a value of 0 when $\sigma \to 1$ (Cobb-Douglas technology).

Proposition 2 *About the long run EROEI and its economic implications*

1. *The long run EROEI is given by:*

$$\varepsilon_* = \frac{1}{\beta\gamma}\left[1 - \frac{[1-a]^{\sigma}}{[\zeta\varphi\beta]^{1-\sigma}}\right]^{-1} = \frac{1}{\beta\gamma u^*}. \tag{49}$$

2. *The share of output allocated to energy production is inversely proportional to the long run EROEI:*

$$s_*^g = \frac{1}{\varepsilon_*}. \tag{50}$$

3. *The stationary state exists only if the EROEI ratio is sufficiently larger than 1:*

$$\varepsilon_* > \left[1 - \frac{[1-a]^\sigma}{[\zeta\varphi\beta]^{1-\sigma}}\right]^{-1} > 1. \tag{51}$$

Proof: See Appendix 7.5 ■

The following lemma is intuitive and useful to the interpretation of Proposition 2 and the numerical experiments that will follow.

Lemma 2 *Stationary level of the economy:*

The stationary levels of RE production F_* and final output Y_* are increasing functions of

- *the long-termism of private agents β;*
- *the productivity of investment goods φ in the formation of the capital stock;*
- *the productivity of energy A^* and capital ζ in final production;*
- *the productivity factor of capital $1/b_*$ and the share of capital γ in RE production*
- *the renewable energy flow $\mathcal{F}$;*
- *the elasticity of substitution σ between energy and capital in final production.*

Proof: See Appendix 7.6 ■

Let us make the following comments on Proposition 2 and Lemma 2:

1. The inverse stationary relationship (50) between ε_* and s_*^g can be understood as follows. Recall that ε_* is the ratio between the quantity of energy ($X_* = F_*$) delivered by the energy production process and the quantity of energy consumed by this process. Here, this quantity corresponds to the energy content of the G_* units of capital invested in energy production. G_* units of capital are made from G_*/φ units of final output and each unit of final output has an energy content of X_*/Y_*. Accordingly, the energy content of G_* is $[G_*/\varphi].[X_*/Y_*]$. The ratio between X_* and this energy content is $\varphi Y_*/G_*$, i.e. the inverse of s_*^g. When the EROEI is lower, a larger share of output must thus be allocated to investment in the energy sector.

2. Relationship (50) also outlines that at given φ, G_*/Y_* is the driver of the EROEI value: the EROEI is lower when the capital requirement of the energy sector per unit of final output is higher. This helps to understand why ε_* depends on some parameter models and not on others as shown by (49). Any parameter change that leads to an increase in G_*/Y_* leads to a decrease in the EROEI value: higher values of β, ζ, φ, γ and a have such an implication. For example, a better productivity of

investment goods (a higher φ) raises the production of the two sectors (cfr. Lemma 2) and G_*/Y_*: thereby, it implies a lower ε_* [10].

The same type of reasoning explains why productivity factors A_*, b_* and the RE flow $\mathcal{F}$ do not affect the long-run EROEI value: although they all stimulate the production of final goods and of energy, they do not change G_*/Y_* and so leave the EROEI unchanged.

The impact of σ on ε_* is more ambiguous. If a higher σ increases the production of the two sectors (cfr. Lemma 2), its impact on G_*/Y_* may be positive or negative: it will be positive if $\zeta\gamma\beta < [1-a]^{-1}$ and negative if $\zeta\gamma\beta > [1-a]^{-1}$. ε_* decreases in the first case and increases in the second one.

3. Technical progress induces a rebound effect in the use of energy: even though it reduces the dependency of the economy on energy *ceteris paribus*, it stimulates output sufficiently strongly to increase energy consumption. This general equilibrium effect neutralizes the positive partial equilibrium impact of technical progress on the EROEI *ceteris paribus*. As a result, technical progress (i.e. a higher A_* or a lower b_*) allows the economy to reach a higher long-run level of final output for a same EROEI ratio.

4. Equation (49) shows that the EROEI value measured at a macroeconomic level does not depend exclusively on technological variables (here γ, ζ, φ) but also on β. Futhermore, in the same line as the above remark 3, it also puts forward that a partial equilibrium reasoning about the determinants of the EROEI ratio may be quite misleading at a general equibrium level. Indeed, for a given production level F_*, a higher γ (or a higher ζ or a higher φ) reduces the quantity of final output that must be invested in the RE sector. A higher value of γ, ζ or φ thus increases the EROEI value at given F_*. But F_* changes endogenously and a rebound effect in the production of F_* actually makes the global EROEI decreasing in γ, ζ and φ.

5. Because u_* is propotional to q_* (cfr. (41)), (49) implies a negative relationship between the long run EROEI value and the real price of energy q_* (at given A_*). This confirms, in a general equilibrium framework, the inverse relationship established between these two variables by King and Hall (2011) and by Heun and de Wit (2012) in a partial equilibrium analysis. The next section will explore whether such a negative relationship is also observed during the energy transition.

6. Obviously, an EROEI value above 1 is a necessary condition for survival of a(n economic) system. In our decentralized economy, it must be strictly above 1 as established by (51). This result echoes the literature in energy science which argues that an economy cannot survive with a global EROEI lower that 3 and that an EROEI close to 5 is even necessary "to maintain what we call civilisation" (Hall et al., 2009, p.45).

[10] A higher φ has two opposite effects on s^g_*: at given G_*/Y_*, it decreases s^g but it also leads to a higher G_*/Y_*, which increases s^g_*. The second effect dominates.

4 Dynamics

As Propositions 1 and 2 (point 2) have outline, an energy transition characterised by a significant fall in the EROEI of the energy production process leads to an important reallocation of capital towards the energy sector. This reallocation could negatively impact the final sector production and imply a downward adjustment of output and private consumption. We study hereafter this possibility and the elements that affect the magnitude of the contraction in consumption and output if such a contraction occurs. We also analyze what influences the ability of the economy to ultimately recover an income level (at least) as high as the peak level reached during the age of high EROEI resources.

Our analysis of the model dynamics mainly relies on numerical simulations of a calibrated version of the model. We can however establish two general properties regarding the possibility of an overshooting of consumption and output during the energy transition.

Proposition 3 *1. A contraction of private consumption occurs when the real unit cost of energy is above its stationary state value, i.e. when*

$$u_t = a^{\sigma}\left[\frac{q_t}{A_t}\right]^{1-\sigma} > u_*, \tag{52}$$

or, equivalently, when the price of an efficient unit of energy (i.e. q_t/A_t) is above its stationary state value $q_t/A_t > q^*/A^*$.

2. If a contraction of consumption and output happens, it necessarily starts before the completion of the energy transition, i.e. when energy production still relies on the non-renewable resource.

Proof:

1. Given (22), $C_t < C_{t-1}$ is observed if $v_t < 1/[\beta\varphi] = v_*$. This inequality and (32) imply that

$$a^{\sigma}\left[\frac{q_t}{A_t}\right]^{1-\sigma} = 1-(1-a)^{\sigma}\left[\frac{v_t}{\zeta}\right]^{1-\sigma} > 1-(1-a)^{\sigma}\left[\frac{v^*}{\zeta}\right]^{1-\sigma},$$

which is equivalent to (52).

2. From the above point 1, we thus know that a contraction in consumption starts in a period t where $v_t < v_* < v_{t-1}$ (since there is no contraction in $t-1$) and thus when $u_t > u_* > u_{t-1}$ or, equivalently, $q_t/A_t > q_*/A_* > q_{t-1}/A_{t-1}$. Since $A_t > A_{t-1}$, this set of inequalities implies that

$$\frac{q_t}{v_t} > \frac{A_t}{A_{t-1}}\frac{q_{t-1}}{v_{t-1}} > \frac{q_{t-1}}{v_{t-1}}. \tag{53}$$

When (53) holds, a comparison between the expressions of (26) in t and $t-1$ implies that

$$\frac{G_t}{F_t} > \frac{G_{t-1}}{F_{t-1}},$$

which, using (27), also implies that

$$F_t > F_{t-1} \qquad \text{and thus} \qquad G_t > G_{t-1}. \tag{54}$$

The contraction in C_t thus starts in a period where RE grows and where more capital is allocated to RE production. Moreover, the inequality $q_t/A_t > q_{t-1}/A_{t-1}$ and (30) imply that

$$\frac{Y_t}{A_t X_t} > \frac{Y_{t-1}}{A_{t-1}X_{t-1}} \qquad \text{or} \qquad \frac{Y_t}{Y_{t-1}} > \frac{A_t}{A_{t-1}}\frac{X_t}{X_{t-1}}.$$

Since $A_t/A_{t-1} > 1$, total energy production necessarily decreases more than output in a situation where $Y_t/Y_{t-1} < 1$:

$$1 > \frac{Y_t}{Y_{t-1}} > \frac{X_t}{X_{t-1}}. \tag{55}$$

The combination of (54) and (55) proves ad absurdio that if a contraction in consumption and output happens, it necessarily starts at a time where NRE is still used. Assume indeed that the decrease in consumption and output occurs when NRE resource is not be extracted anymore, i.e. at time where $X_t = F_t$ and $X_{t-1} = F_{t-1}$. (54) would then imply that $X_t > X_{t-1}$, i.e. $X_t/X_{t-1} > 1$, which contradicts (55).

■

4.1 The baseline scenario

As the model is highly stylized, its calibration is unavoidably rough and we will pay more attention to the general shape of the simulated trajectories of the different variables than to their numerical values. As we have supposed a unitary depreciation rate of capital, we consider a period length of 16 years, which means that a unitary depreciation rate per period corresponds to an annual depreciation rate of about 6%. The calibration is made while assuming that the model describes the World economy. It relies on different current values of observed variables at the World level (GDP, energy consumptions, savings rate,...) and several "reasonable" assumptions about the initial NRE stock $\mathcal{S}$, the RE flow $\mathcal{F}$, the elasticity of substitution between energy and capital σ and the potential of technical progress in the energy and non-energy sectors. It is detailed in Appendix 7.7.

As far as the key choice of σ is concerned, we do not know any paper that has estimated σ at a World level using a CES production function. However, there are some attempts relying on nested CES functions that indicate that the possibility of substitution between energy and a mix of capital and labour is rather weak. Kemfert (1998) obtains $\sigma = .458$ for the West-German Industry. Using industry-level data from 12 OECD countries, van der Werf (2008) obtains industry estimates ranging from .17 to .65 and country estimates ranging from .17 to .61. He also tests the hypothesis of a unitary elasticity (the Cobb-Douglas case) and rejects it for all the countries and industries of his data set. Using a four-input translog function applied to the manufacturing industry of seven OECD countries, Forito and van den Bergh (2016) conclude that capital and energy are complementary factors.

The baseline scenario describes the transition dynamics of an economy with a rather low initial capital stock ($K^*/K_1 \approx 6$) and a potential of energy efficiency gains of 50%

($A^*/A_1 = 1.5$) but no technical progress in the energy sectors. Figures 1.a to 1.f hereafter illustrate this baseline scenario. Figure 1.a illustrates the progressive change in the energy mix of the economy. NRE production (E_t) peaks at the beginning of the trajectory ($t = 3$) and next declines monotonically until NRE extraction becomes unprofitable (end of period 12). RE production (F_t) evolves itself non monotonically. It first increases and overtakes E_t during period 5. But it peaks two periods later and next decreases towards its asymptotic value. The figures put forward several striking features of the transition in the baseline scenario.

1. A rapid development of RE production is possible when NRE production is still relatively abundant (Fig. 1.a). The end of the NRE era puts a drag on this development.

2. The energy transition is characterized by a downward trend of the EROEI ratio (Fig. 1.b) and a progressive rise in the real energy price (Fig. 1.d).

   Initially, both NRE and RE production offers high EROEIs (Figure 1.b)[11] but the decrease of the NRE stock and the rise of RE production progressively make the two types of energy production more capital intensive[12] and, thereby, indirectly more energy consuming (in spite of the energy efficiency gains that progressively reduce the energy content of each unit of capital). The EROEI of each energy production process declines accordingly[13]. The low EROEI values reached during the last periods where NRE is still used mark the end of a "golden" age characterised by abundant and cheap energy. This is in line with the literature on EROEI which expects a fall in this variable because RE technologies will not offer as high EROEI ratios as those offered by conventional fossil energies (Murphy and Hall, 2010).

   The stationnary state and transitory dynamics analyses confirm, in a macroecomic framework, the inverse relationship between the energy price and the EROEI ratio established by King and Hall (2011) and Heun and de Wit (2012) in partial equilibrium settings.

3. Even though the output level is, in the long run, well above its initial value, it evolves non monotonically: before the completion of the transition (i.e. at a time when non-renewables are still used), final output, private consumption and investment (Fig. 1.c) overshoot the level that the economy is able to sustain in the long run. Furthermore, the downward adjustment in output and consumption starts after the peak in NRE production but well before the end of the NRE extraction (Fig. 1.a and 1.c).

   This non monotonic evolution of Y_t, C_t and I_t (Fig. 1.c) follows from the fact that final production and energy production make a rival use of the capital stock. Output growth requires more capital and more energy (in spite of the energy efficiency

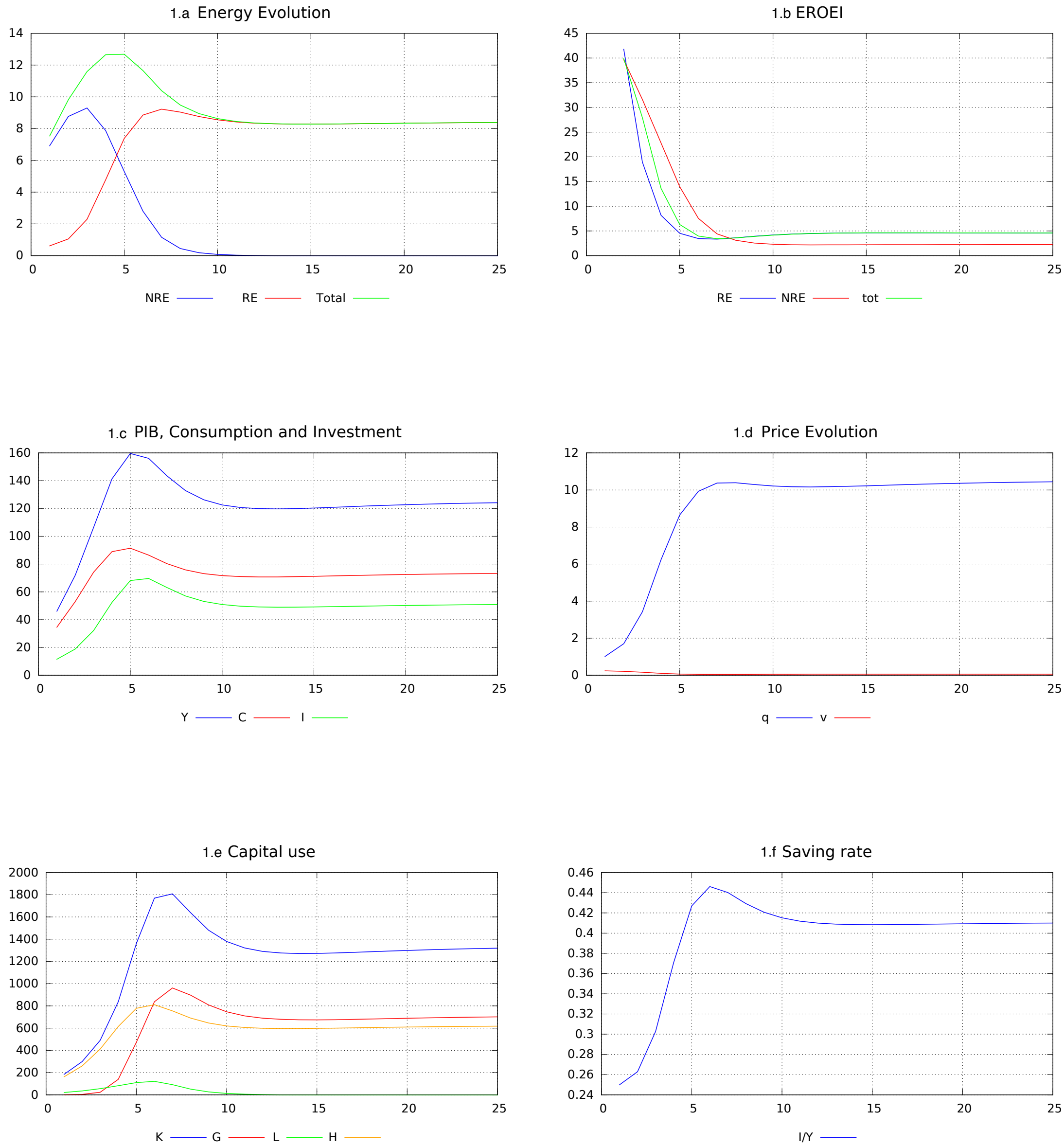
[11] As hydroelectricity represents the main part of the RE mix in the first period, the initial (calibrated) value of RE EROEI is even initially higher than the one of NRE EROEI.

[12] First, the capital requirements of NRE extraction increases when the NRE stock decreases. Second, RE production is characterised by decreasing returns to scale. In a more disaggregated model of the RE sector, this decreasing trend in the RE EROEI would also stem from the fact that the development of RE production would rely on lower EREOI energy sources than hydropower.

[13] The fall in the EROEI of RE however stops when RE production peaks. After this peak, the EROEI of RE improves slightly not only because F_t decreases but also because A_t increases (the energy efficiency gains in final production reduce the energy content of the capital used in the RE sector).

gains) and producing more energy requires itself more capital. But energy growth and output growth remain easily compatible as long as the conditions of energy production do not deteriorate too much.

Figure 1: The baseline scenario



However, during the energy transition, the capital requirements of the energy sector do increase non only because more energy must be produced but also because energy production becomes more capital intensive in both energy subsectors. In order to make energy and output growth possible in this context, a significant increase in the savings rate (Fig. 1.f) is necessary but not always enough. From period 6, the intensification of the capital requirements of the economy puts a halt on economic growth and final output starts to contract. This negative growth of output induces in turn a decrease in the capital stock (Fig. 1.e), which strengthens the economic contraction. In the last periods of NRE extraction, the economy thus experiences a prolonged period of negative growth (in spite of the energy efficiency gains). It stops when the energy transition is completed, output being then 25% lower than at its peak level (but nevertheless much higher than its initial level).

4. The energy transition is accompanied by a very important reallocation of capital between the sectors of the economy (Fig. 1.e) and a strong increase in the savings rate (Fig. 1.f).

4.2 Sensitivity analysis

4.2.1 The impact of the NRE stock

Since the NRE source is no longer extracted in the long run, its initial stock has no impact on the steady state. But it obviously affects the transitory dynamics. It has an unambiguously positive impact on the intertemporal utility level of an infinitely lived representative agent. But if we interpret the infinitely lived agent as a dynasty of finitely lived altruists agents (as e.g. in Michel *et alii* (2006)), we can question how the initial abundance of the resource stock affects the material well-being of the generations born during the possible consumption peak and downturn.

We can formulate three conjectures that all our numerical experiments confirmed.

1. *The condition of a downward adjusment of consumption and output during the energy transition is more easily met when the initial NRE stock is high.*

   Using point 2 of Proposition 3, we can be sure that with a zero initial stock $\mathcal{S} = 0$, an undercapitalized economy (for which $K_1 < K^*$) will not overshoot its steady consumption and output levels during the transitory dynamics. This is in accordance with Germain (2012) who shows that the transitory dynamics of output in an economy that only uses a constant renewable resource flow is monotonic. By continuity, this remains certainly true if the NRE stock is positive but small enough. Otherwise said, an overshooting of output and consumption can only occur in an economy with a sufficiently large initial NRE stock and, in such a case, necessarily at a time where the NRE stock is still exploited.

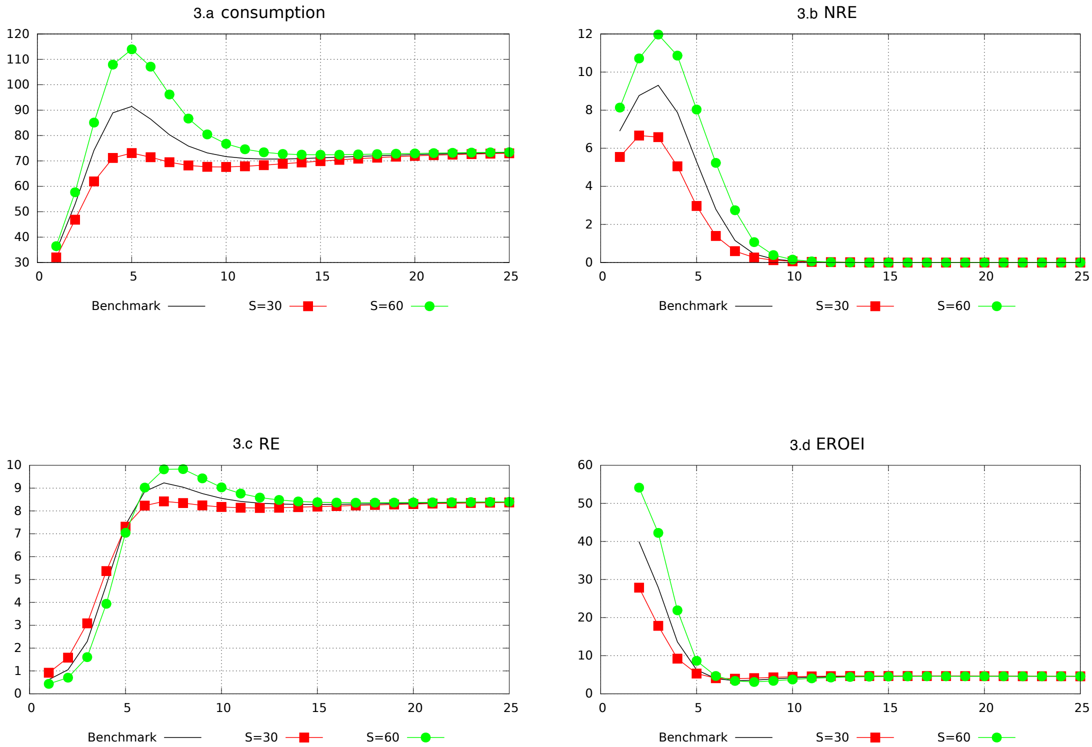
2. *If a contraction of consumption and output occurs during the energy transition of an economy with an initial NRE stock $\mathcal{S}$, its magnitude is increasing in $\mathcal{S}$.*

   With a bigger $\mathcal{S}$, the NRE production cost and the energy price q_t are initially lower at a given level of final output. Consequently, the condition of an overshoothing (52) will be satisfied at a higher output level -and possibly at a later time- in a better endowed economy. Moreover, as the initial NRE stock does not affect the long run equilibrium, the output contraction will also be deeper in the best endowed economy.

3. *A bigger initial stock of NRE resource delays the development of the RE sector.*

As a bigger $\mathcal{S}$ initially implies a lower q_t, it also reduces the profitability of the RE sector and initially slows down its development.

Figure 2: The impact of the NRE stock



Figures 3.a-d compare the economy dynamics for three values of $\mathcal{S}$: $\mathcal{S} = 60$, $\mathcal{S} = 43.5$ (baseline scenario) and $\mathcal{S} = 30$. All the other model parameters are unchanged. In the best endowed economy, the overshooting of consumption and output is amplified as expected (see Fig. 3.a for C_t). For the generations born at (or close to) the peak, one may speak of a "malediction of abundance": with a higher initial NRE stock, they inherit a higher level of consumption but experience a more severe consumption loss later during the transition.

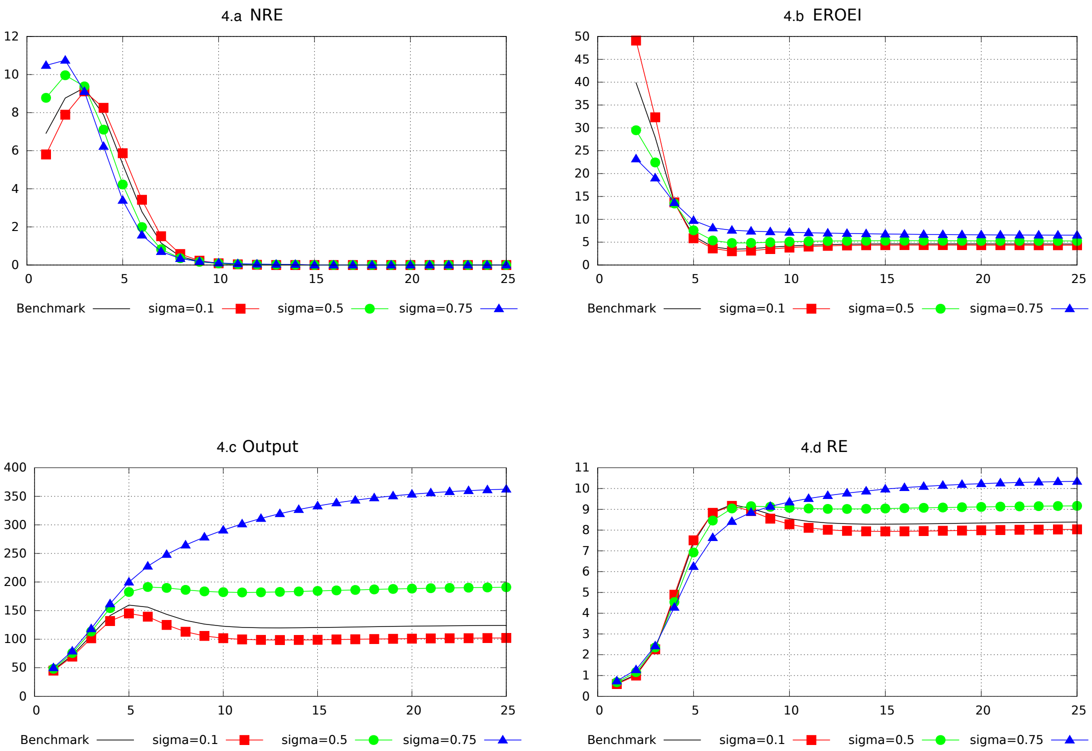
In any period where NRE extraction is positive, it is more important in the best endowed economy, in which it even lasts one period longer (Fig. 3.b). The abundance of the NRE stock impacts RE production in two steps. As conjectured, it delays the development of the RE sector (Fig. 3.c). However, at a later stage, it allows a faster and stronger growth of RE production: from period 6, RE production becomes larger in the best endowed economy where output growth is stronger and NRE production remains higher, which progressively ease the allocation of capital to the RE sector. As already noticed in the baseline scenario, this stressed that the point up to which the RE sector can develop during the transition depends positively on the availability of the NRE resource.

The EROEI (Fig. 3.d) is initially higher in the best endowed economy since NRE production is less capital demanding, which allows a stronger consumption growth with a lower savings rate. However, it drops more quickly because NRE is extracted more intensively. It even falls below the value observed in the baseline scenario when RE production becomes higher in the last periods before the completion of the transition.

4.2.2 The role of the substitution between capital and energy

With a higher elasticity of substitution σ, a given combination of capital and energy allows the economy to produce a higher final output level. Furthermore, the stationary analysis has shown that a higher σ allows the economy to produce higher long run levels output and energy. In the case of our calibration based on the World economy, $\zeta\gamma\beta > [1-a]^{-1}$ and a higher σ also implies a higher value of the long run EROEI. All this suggests that *a higher σ makes the energy transition easier.* A sensivity analysis comparing the transitory dynamics of the economy for 4 different values of σ, $0.1, 0.25(\text{baseline}), 0.5, 0.75$ confirms this conjecture as shown in Fig. 4.a-d.

Figure 3: The role of the substitution between capital and energy

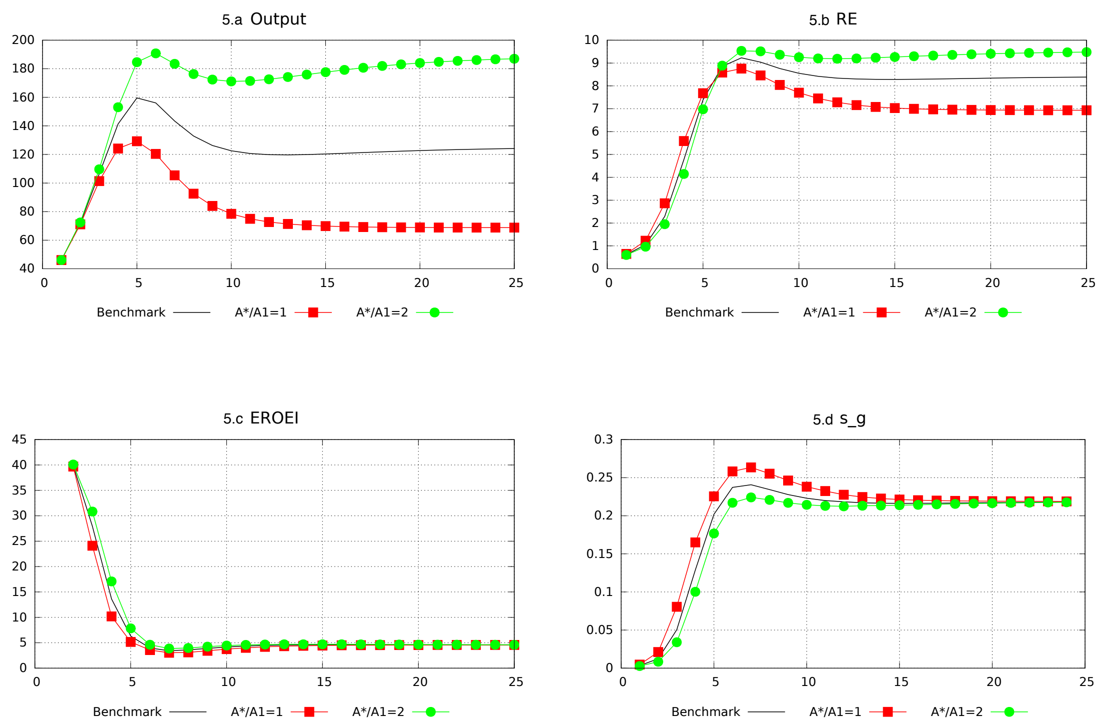


In an initially undercapitalized economy, a higher σ leads to a much less conservative management of the NRE stock at the beginning of the simulation (see $t = 1, 2$ in Fig. 4.a). This initially makes NRE production more capital intensive and implies a lower EROEI ratio (Fig. 4.b). From period 3 to 8, the opposite happens: the economy is now better capitalized and a higher σ enables it to reach a higher production level with less energy and a higher EROEI value. After period 8, NRE production is close to zero and X_t is essentially supplied by the RE sector (Fig. 4.d). Since a higher σ makes final production (Fig. 4.c) and investment easier, it also eases RE production. These evolutions contribute to reducing the magnitude of the downward adjustment in output, which even disappears when $\sigma = .75$. If σ is sufficiently high, the energy transition of the economy becomes smooth, without any overshooting of output and consumption. Unfortunately, such high values of σ do not seem consistent with the empirical estimates (see our discussion on σ).

4.2.3 The role of technical progress

Proposition 2 and Lemma 2 have shown that for a same long run EROEI, an economy with a larger potential of technical progress (either in the use of energy in the final sector or in the production of energy) can reach a higher long run output level. One thus understands that even if an output contraction occurs during the transition, the potential of technical progress plays a key role in the ability of the economy to recover long run output levels at least equal to the peak levels reached before the end of the NRE production. More generally, we can conjecture that *a larger potential of technical progress (either in the use of energy in the final sector or in the RE production) eases the energy transition.*

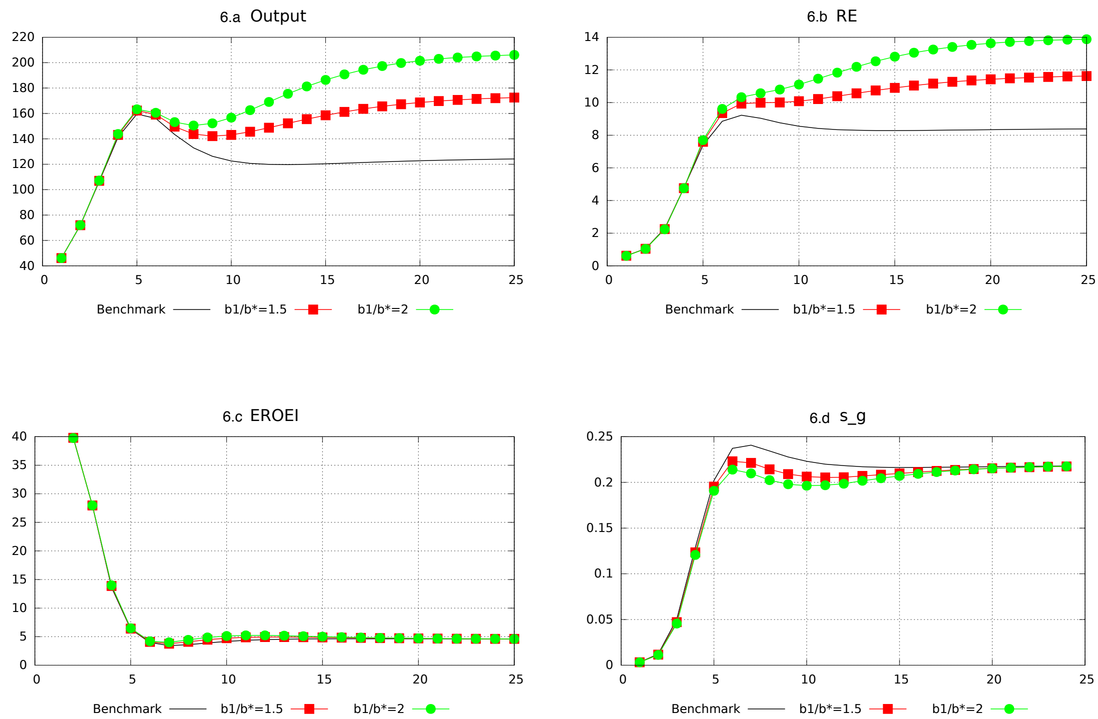
Figure 4: The role of energy efficiency in the final sector



Figures 4.a-d compare the trajectories of final output, RE production, global EROEI and s_t^g for the three values of A_*: $A_* = 2A_1$, $A_* = 1.5A_1$ (which is the reference scenario) and $A_* = A_1$ (no technical progress). Given the law of motion of A_t (see (16)), a higher A^* implies stronger energy efficiency gains during the whole trajectory of A_t. As a result, when A_* is higher, 1) output growth is initially stronger and output peaks at a higher level during the transition; 2) the consecutive output contraction is relatively weaker; 3) the economic recovery is stronger once the transition towards renewables is completed.

Initially (i.e. as long as NRE remains relatively abundant), the development of the RE sector (Fig. 5.b) is somewhat delayed when energy efficiency gains are stronger. But later (here from period 6), these stronger efficiency gains make RE production easier. Indeed, even though the long-run EROEI does not depend on A_*, the EROEI remains transitorily higher when efficiency gains are stronger[14]. The economy with stronger efficiency gains is able to produce more final output and more RE while investing a lower share of final output in RE production (see s_t^g in Fig. 5.d). Energy efficiency gains thus induce a positive feedback loop between the production of capital goods and the production of energy and so ease the energy transition.

Figure 5: The role of technical progress in the RE sector



[14]The energy content of capital goods is lower when energy efficiency is higher.

The impact of technological progress in the RE sector is not very different. Figures 6.a-d compare the trajectories of final output, RE production, global EROEI and s_t^g for three values of the ratio b_1/b^*: $b_1/b^* = 1$ (baseline scenario), $b_1/b^* = 1.5$ and $b_1/b^* = 2$. Qualitatively, they look rather similar to Fig. 5.a-d. A lower b^* makes the economy converge towards higher levels of output and RE production (Fig. 6.a-b). However, as the productivity gains in the RE sector follow from a learning by doing process and as RE production is relatively small in the first periods, the difference between the variants and the baseline scenario is initially very weak. It only becomes very clear[15] after the output peak (and thus later than in the sensitivity analysis on A_*). Stronger productivity gains in the RE sector unsurprisingly ease the transition by allowing the economy to produce more RE while investing a lower share of final output in the RE sector (Fig. 6.d). The EROEI evolution is very similar in the variants and in the baseline scenario: during the transition, stronger productivity gains improve the EROEI *ceteris paribus* but this effect is almost counterbalanced by the impact of a higher level of RE production on the capital intensity of the energy sector.

5 Conclusion

In a decentralised two-sector model (energy and final good sectors), we have analysed the macroeconomic implications of an energy transition characterized by a progressive rise of renewables in the energy mix of the economy. Such a transition progressively marks the end of high EROEI energy resources. On the one hand, NRE extraction becomes increasingly capital intensive when the stock of the NRE resource decreases. On the other hand, the capture of the RE flow is subject to decreasing returns to scale. Consequently, the extraction of the residual NRE stock and the progressive rise in RE production both decrease the global EROEI of energy production, which constrains the economy to allocate more capital to energy production and to save more. For an economy which initially enjoyed non-renewable resources with a high EROEI, the transition towards a 100%-renewable energy supply is thus characterized by important changes both in the allocation of capital between the energy and non-energy sectors and in the allocation of output between consumption and investment. The progressive decline of the EROEI thus exacerbates an unavoidable feedback loop between the production of energy and the production of capital goods: when energy production becomes more capital intensive, it crowds capital out of the final good sector *ceteris paribus* and thereby puts a drag on the production of future capital goods, which further strengthens the tensions between the use of capital in the energy and non-energy sectors. This can put a halt to the growth process fueled by the access to high EROEI resources and can even make the economy enter into a phase of contraction of economic activity and private consumption. Such a contraction would necessarily begin (well) before the end of NRE extraction.

Numerical simulations of a calibrated version of the model have confirmed that the transition can be characterized by a contraction of economic activity and private consumption. The magnitude of this contraction has been shown to be larger when (i) the initial stock of NRE is higher, (ii) the potentials of technical progress in the energy and non energy sectors are lower and (iii) the substitutability between capital and energy in

[15]The comparison between Fig 6.a and 6.b shows that the divergence between the output curves is contemporary to the one in RE production.

final production is weaker[16]. A contraction during the transition is however not always observed. For instance, for sufficiently high values of the elasticity of substitution, the energy transition can unwind smoothly, without any overshooting of GDP and consumption. Unfortunately, such high values seem inconsistent with empirical estimates.

The contraction phase may be followed by a phase of recovery. The possibility to recover long run output and consumption levels at least as high as the peak levels reached when non-renewables were still abundant crucially depends on (i) the potential of energy efficiency gains in the final sector and/or (ii) the potential of technical progress in the RE sector when NRE extraction is stopped.

Simulations have also shown that a rapid development of RE production was dependant on the availability of NRE. This is another illustration of the impact of the feedback loop between the production of energy and the production of capital goods: as RE and final productions make a rival use of capital, it is easier to expand the productive capacity of the RE sector when final production can still rely on a relatively high EROEI non-renewable resource.

Let us end while outlining that we have assumed a perfect substitutability between non-renewable and renewable energies. This is certainly a very optimistic assumption and one can suspect that an imperfect substitutability could worsen the economic implications of the energy transition.

6 Conflict of Interest Statement

On behalf of all authors, the corresponding author states that there is no conflict of interest.

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8 Appendix

8.1 NRE Sector: Derivation of (4)

Using (1) and (2), problem (3) can be reduced to :

$$\max_{\{S_t\}_{t=2,\dots,T+1}} \sum_{t=1}^{T} R_t \left[q_t \left[S_t - S_{t+1} \right] - v_t \rho_t \left[\frac{S_t}{S_{t+1}} - 1 \right] \right]$$

under the constraints

$$S_t - S_{t+1} \geq 0, \ t = 1, ..., T. \tag{56}$$

We can write the Lagrangian of this problem as:

$$\mathcal{L} = \sum_{t=1}^{T} R_t \left[q_t \left[S_t - S_{t+1} \right] - v_t \rho_t \left[\frac{S_t}{S_{t+1}} - 1 \right] \right] + \sum_{t=1}^{T} \lambda_t \left[S_t - S_{t+1} \right]$$

where the λ_t's are multipliers. Given that $S_1 = \mathcal{S} > 0$, (56) implies that the positivity constraints $S_t \geq 0, \ t = 2, ..., T+1$ are satisfied. The first order optimal conditions are:

$$\frac{\partial \mathcal{L}}{\partial S_t} = \begin{cases} R_{t-1} \left[-q_{t-1} + v_{t-1} \rho_{t-1} \frac{S_{t-1}}{S_t^2} \right] + ... \\ \qquad ...R_t \left[q_t - v_t \rho_t \frac{1}{S_{t+1}} \right] - \lambda_{t-1} + \lambda_t = 0, \ \forall t = 2, ..., T \end{cases} \tag{57}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = S_t - S_{t+1} \geq 0, \lambda_t \geq 0, \lambda_t \frac{\partial \mathcal{L}}{\partial \lambda_t} = 0, \ \forall t = 1, ..., T \tag{58}$$

$$\frac{\partial \mathcal{L}}{\partial S_{T+1}} = R_T \left[-q_T + v_T \rho_T \frac{S_T}{S_{T+1}^2} \right] - \lambda_T = 0 \tag{59}$$

We look for a solution where $S_{t+1} > S_t$ $(t = 1, ..., T_e)$ and $S_{t+1} = S_t$ $(t = T_e + 1, ..., T)$ where T_e is the final period of extraction and $1 \leq T_e \leq T$. In other terms :

$$\mathcal{S} = S_1 > S_2 > ... > S_{T_e - 1} > S_{T_e} > S_{T_e + 1} = ... = S_T = S_{T+1} \geq 0. \tag{60}$$

We characterize the solution starting from the last period.

In $t = T$, (59) and $S_T = S_{T+1}$ (see (60)) imply that $R_T \left[-q_T + v_T \rho_T \frac{1}{S_{T+1}}\right] - \lambda_T = 0$. Then, using (57) in $t = T$, we obtain $R_{T-1}\left[-q_{T-1} + v_{T-1}\rho_{T-1}\frac{S_{T-1}}{S_T^2}\right] - \lambda_{T-1} = 0$. Since $S_{T-1} = S_T$ (see (60)), it also means that $R_{T-1}\left[-q_{T-1} + v_{T-1}\rho_{T-1}\frac{1}{S_T}\right] - \lambda_{T-1} = 0$. Next using (57) in $t = T-1$, we obtain $R_{T-2}\left[-q_{T-2} + v_{T-2}\rho_{T-2}\frac{S_{T-2}}{S_{T-1}^2}\right] - \lambda_{T-2} = 0$. Since $S_{T-2} = S_{T-1}$ (see (60)), we can equivalently write $R_{T-2}\left[-q_{T-2} + v_{T-2}\rho_{T-2}\frac{1}{S_{T-1}}\right] - \lambda_{T-2} = 0$. Repeating the same reasoning until $T_e + 2$, we obtain:

$$R_{T_e+1}\left[-q_{T_e+1} + v_{T_e+1}\rho_{T_e+1}\frac{1}{S_{T_e+2}}\right] - \lambda_{T_e+1} = 0. \tag{61}$$

For $t = 1, ..., T_e$, $S_{t+1} > S_t$ and (58) implies that $\lambda_t = 0$. Therefore, (57) leads to

$$R_{t-1}\left[-q_{t-1} + v_{t-1}\rho_{t-1}\frac{S_{t-1}}{S_t^2}\right] + R_t\left[q_t - v_t\rho_t\frac{1}{S_{t+1}}\right] = 0,\ t = 2, ..., T_e. \tag{62}$$

In $t = T_e+1$, (57) implies that $R_{T_e}\left[-q_{T_e} + v_{T_e}\rho_{T_e}\frac{S_{T_e}}{S_{T_e+1}^2}\right] + R_{T_e+1}\left[q_{T_e+1} - v_{T_e+1}\rho_{T_e+1}\frac{1}{S_{T_e+2}}\right] - \lambda_{T_e} + \lambda_{T_e+1} = 0$. Using (61) and $\lambda_{T_e} = 0$, this expression can be rewritten as

$$-q_{T_e} + v_{T_e}\rho_{T_e}\frac{S_{T_e}}{S_{T_e+1}^2} = 0. \tag{63}$$

Using the definition of R_t, we then obtain (4).

8.2 RE Sector: Derivation of (8) and (10)

After substitution of f_t by (6) into (7), the problem of a RE firm rewrites as:

$$\max_{g_t}\ q_t B_t g_t^{\gamma} - v_t g_t$$

The first-order optimality condition is $\gamma q_t B_t g_t^{\gamma-1} - v_t = 0$, or $\gamma q_t \frac{f_t}{g_t} = v_t$, which leads to

$$g_t = \gamma \frac{q_t}{v_t} f_t.$$

Summing these individual relationships overs all firms implies (8).

Similarly, summing the technological relationships (6) over all firms gives the following aggregate technology: $F_t = N f_t = N B_t g_t^{\gamma} = N B_t \left[G_t/N\right]^{\gamma}$ or

$$F_t = N^{1-\gamma} B_t G_t^{\gamma}.$$

Using assumption (9), we obtain $F_t = N^{1-\gamma}\frac{1}{d_t}\left[1 - \frac{F_t}{\mathcal{F}}\right] G_t^{\gamma}$, which gives (10).

8.3 Final Good Sector

The expressions of the marginal productivity of X_t and H_t writes respectively as

$$\frac{\partial Y_t}{\partial X_t} = aA_t\left[\frac{Y_t}{A_tX_t}\right]^{\frac{1}{\sigma}} = a\left[A_t\right]^{\frac{\sigma-1}{\sigma}}\left[\frac{Y_t}{X_t}\right]^{\frac{1}{\sigma}} \tag{64}$$

$$\frac{\partial Y_t}{\partial H_t} = [1-a]\,\zeta\left[\frac{Y_t}{\zeta H_t}\right]^{\frac{1}{\sigma}} = [1-a]\,\zeta^{\frac{\sigma-1}{\sigma}}\left[\frac{Y_t}{H_t}\right]^{\frac{1}{\sigma}}, \tag{65}$$

which lead straightforwardly to the first-order conditions (12) and (13).

The Euler theorem for homogeneous function of degree 1 next implies:

$$\frac{\partial Y_t}{\partial X_t}X_t + \frac{\partial Y_t}{\partial H_t}H_t = Y_t \tag{66}$$

$$q_t\frac{X_t}{Y_t} + v_t\frac{H_t}{Y_t} = 1, \tag{67}$$

where the last expression follows from the equality between the marginal product of each factor and its price. Using (12) and (13), X_t/Y_t and X_t/Y_t can be substituted by a function of each factor price and the productivity factors, which leads to

$$q_t\left[\frac{a}{q_t}\right]^{\sigma}A_t^{\sigma-1} + v_t\left[\frac{1-a}{v_t}\right]^{\sigma}\zeta^{\sigma-1} = 1.$$

(15) follows straightforwardly.

8.4 Proof of Lemma 1

1. A stationary state equilibrium must be such that $q_* > 0$ or, equivalently, $u_* > 0$. Using (41), this implies (48) when $\sigma < 1$ (what we assume). As $\beta < 1$, (48) also requires that $\zeta\varphi$ is sufficiently larger than the right-hand-side of the inequality.

2. With the stationary value of q_* and v_* (uniquely given by (40) and (41)), equations (42)-(44) determine F_*/Y_*, H_*/Y_* and G_*/F_*. For a given ratio G_*/F_*, (45) is an equation in F_* which admits a unique solution. Since $\gamma < 1$, the right-hand-side of (45) is monotonically increasing in F_* from 0 (when $F_* = 0$) to infinity (when $F_* \to \mathcal{F}$). For a value of the left-hand-side given by (44), there is thus a unique value of F_* that satisfies (45). Variables G_*, H_*, Y_*, Y_* and C_* can next be obtained.

8.5 Proof of Proposition 2

1. At a stationary state, $L^* = 0$ and (39) implies that

$$\epsilon_* = \varphi\frac{Y_*}{G_*} = \varphi\frac{Y_*}{F_*}\frac{F_*}{G_*}.$$

Using (43) and (44), the last equality becomes

$$\begin{aligned}\epsilon_* &= \varphi\left[A_*^{\sigma-1}\left[\frac{a}{q_*}\right]^{\sigma}\right]^{-1}\frac{v_*}{\gamma q_*}\\ &= \frac{\varphi}{\gamma\beta\varphi}a^{-\sigma}\left[\frac{A_*}{q_*}\right]^{1-\sigma},\end{aligned}$$

where the last equality follows from (40). Using (41), this expression leads to (49).

2. Since s_*^ℓ, (50) follows straightforwardly from (39).

3. (41) and (48) imply that $0 < u_* < 1$. Using this inequality and $\beta\gamma < 1$, (49) leads to condition (51).

8.6 Proof of Lemma 2

(40) shows that v_* is decreasing in β or φ but does not depend on any other parameter. The different statements in Lemma 2 are then proved as follows:

- Given (41), a higher value of β or φ implies an increase in u_*. With $\sigma < 1$, it also implies a higher q_*. These changes in v_*, u_* and q_* imply higher values of H_*/Y_* and G_*/F_* and a lower F_*/Y_*. Via (45), a higher G_*/F_* implies a higher F_*. An increase in F_* combined with a decrease in F_*/Y_* implies a more than proportional increase in Y_*.

- A higher A_* leaves u_* unchanged but raises q_* (see (41)). This implies an increase in q_*/v_* and thereby in G_*/F_* (via (44)) and so in F_* (via (45)) and G_*. As ε_* and thus $s_*^g (= G_*/[\varphi Y_*])$ do not depend on A_* (see (49)-(50)), an increase in A_* raises Y_* in the same proportion as G_*.

- A higher ζ increases u_* and q_* if $\sigma > 1$ (see (41). This implies

  1. an increase in G_*/F_* (via (44)) and so in F_* (via (45)) and in G_*;
  2. a decrease in F_*/Y_* (via (43)).

  An increase in F_* combined with a decrease in F_*/Y_* implies a more than proportional increase in Y_*.

- The positive impact of $1/b_*$ and $\mathcal{F}$ on both F_* and Y_* is obvious: equations (40) to (44) do not depend on these two parameters, which do not affect v^*, q^* and the ratios H_*/Y_*, F_*/Y_* and G_*/F_*. At unchanged G_*/F_*, (45) implies a positive effect of both $1/b_*$ and $\mathcal{F}$ on F_* and thereby on Y_* (via (42)).

  A higher γ leaves v_* and q_* unchanged but raises G_*/F_* (see (44)). With a bigger G_*/F_*, (45) implies a bigger F_* (and thus also a bigger G_*). As γ does not affect F_*/Y_* (see (43)), Y_* gets bigger too.

- In the case of a change in σ, the differential of (41) leads to

$$\frac{\mathrm{d}\, u_*}{\mathrm{d}\, \sigma} = -[1-a]^\sigma \left[\frac{v_*}{\zeta}\right]^{1-\sigma} \ln \frac{1-a}{v_*/\zeta} = [u_* - 1] \ln \frac{1-a}{v_*/\zeta}. \tag{68}$$

  This derivative is positive if the logarithmic term is negative i.e. if (using (40))

$$\frac{1-a}{v_*/\zeta} = [1-a]\zeta\varphi\beta < 1 \qquad \Leftrightarrow \qquad \zeta\varphi\beta < \frac{1}{1-a}.$$

If this inequality is not satisfied, u^* is decreasing in σ. However, q^* is necessarily increasing in σ. Indeed, in the case of a change in σ, the differential of the first equality of (41) writes as

$$\mathrm{d}u_* = u^* \ln \frac{a}{q_*/A_*}\mathrm{d}\sigma + [1-\sigma]\frac{u_*}{q_*}\mathrm{d}q_*.$$

After dividing this equality by $\mathrm{d}\sigma$ and using (68), one obtains

$$[1-\sigma]\frac{u_*}{q_*}\frac{\mathrm{d}q_*}{\mathrm{d}\sigma} = [1-u_*]\ln\frac{v_*/\zeta}{1-a} + u^* \ln\frac{q_*/A_*}{a}. \tag{69}$$

When $\sigma < 1$, the expression multiplying $\mathrm{d}q_*/\mathrm{d}\sigma$ is positive. In order to show that the right-side of (69) is necessarily positive, let us first note that

$$\begin{aligned}\frac{q_*/A_*}{a} &= a^{-1}\left[\frac{u}{a^{\sigma}}\right]^{\frac{1}{1-\sigma}} = \left[\frac{u}{a}\right]^{\frac{1}{1-\sigma}} \\ \frac{v_*/\zeta}{1-a} &= [1-a]^{-1}\left[\frac{1-u}{[1-a]^{\sigma}}\right]^{\frac{1}{1-\sigma}}\left[\frac{1-u}{1-a}\right]^{\frac{1}{1-\sigma}},\end{aligned}$$

where the first line of equalities uses the definition of u^* in (41) and the second line uses the link between the price of capital v_* and the real unit cost of capital in final production (which is necessarily equal to $1-u^*$).

Using the last two expressions, the right-side of (69) can be rewritten as follows

$$u_* \ln\frac{q_*/A_*}{a} + [1-u_*]\ln\frac{v_*/\zeta}{1-a} = \frac{H(u_*)}{1-\sigma} \quad \text{with} \quad H(u_*) = u_* \ln\frac{u_*}{a} + [1-u^*]\ln\frac{1-u_*}{1-a}.$$

Let us show that $H(u)$ reaches a minimum value of 0 in $u_* = a$ and is strictly positive for any other value of $u \in]0,1[$.

$$H'(u) = \ln\frac{u}{a} + u\frac{1/a}{u/a} - \ln\frac{1-u}{1-a} + [1-u]\frac{-1/1-a}{1-u/1-a} = \ln\frac{u}{a} - \ln\frac{1-u}{1-a}.$$

The first order condition for a minimum of $H(u)$ ($H'(u) = 0$) gives $u = a$. Futhermore, $H(u)$ is convex since is

$$H''(u) = \frac{1/a}{u/a} - \frac{-1/1-a}{1-u/1-a} = \frac{1}{u} + \frac{1}{1-u} > 0.$$

For $u = a$, $H(u)$ thus reaches a minimum of $H(a) = 0$. It is strictly positive otherwise. Hence, q^*/v^* and thereby G_*/F_* (via (44)) are increasing in σ. (45) then implies that F_* is itself increasing in σ.

Other parameters being given, (50) and the second equality in (49) imply that a change in σ affects s^g (and thereby G_*/Y_*) in the same way as u^*. If $\zeta\varphi\beta > 1/[1-a]$, u^* and G^*/Y^* are decreasing in σ. An increase in σ (which increases G_* as we have seen) then implies an increase in Y_* more than proportional to the one in G_*. If $\zeta\varphi\beta < 1/[1-a]$, u^* and G^*/Y^* are increasing in σ. Y_* increases as well but to a lesser extent than G^*.

8.7 Calibration

Given that a period ($\Delta t = 1$) lasts 16 years, periods $t = 0$, $t = 1$,... correspond respectively to the intervals 1981-1996, 1997-2012,... In the following (as well as in the simulations), the value of a variable for a given period is the average annual value of this variable over the period.

According to IEA (2014), world secondary energy consumption in 2012 is $8,979$ million tons of oil equivalent (Toe hereafter) and total primary energy supply is $13,371$ MToe. Moreover the average annual secondary energy consumption during period 1, X_1, is approximately 7.8 GToe. On the basis of annual world GDP values, we compute $Y_1 = 46\ 10^9$ M\$[17]. We also obtain values for X_0 and Y_0 in the same way.

Considering the value of 4.3 from Piketty (2013) as an upper bound for the world capital ratio, we estimate the initial ratio between global production (GDP) and capital (K_1/Y_1) to be 4.0. With this ratio and given Y_1, we directly obtain a value for the aggregate capital stock: $K_1 = 184.5$.

According to BP (2014), the share of renewable energy (with hydroelectricity) in the world's primary energy mix is around 9.7 % in 2014 and the annual average during period 1 is around 6.9 %. Assuming that the share of renewable resources in secondary energy is close to their share in primary energy, we use this last data to fix the ratio F_1/X_1. Then we obtain $E_1 = 7.25$ and $F_1 = 0.54$ (10^9 Toe). Assuming the initial non-renewable stock to be equivalent to a hundred times the current annual secondary energy consumption (which is very uncertain), we have $\mathcal{S} = (100/16) \times E_1 \simeq 43.5\ 10^9$ Toe.

From BP (2014), it appears that for EU27, USA, Japan, China and Russia, the manufacturing sector (an energy-intensive sector) is characterised by a real unit energy cost (RUEC hereafter) much larger than 10 %. Thus 10 % can be considered as a lower bound for the aggregate RUEC. We fix the RUEC, which is equal to the share of energy, to $u_1 = 0.15$. Then we are able to compute the initial price of energy $q_1 = u_1 * Y_1/X_1 = 0.89$.

The investment (in percent of GDP) varies between 22.2 % and 25 % during the interval 1997-2012 according to WEO (2015). The average over period 1 (resp. 0) is $s_1 = 23.82$ % (resp. $s_0 = 24.49$ %). We so obtain a value of φ given by:

$$s_0 = \frac{K_1}{\varphi Y_0} \quad \Longleftrightarrow \quad \varphi = \frac{1}{s_0}\frac{Y_1}{Y_0}\frac{K_1}{Y_1} \simeq 25.96. \tag{70}$$

In order to evaluate the EROEIs of NRE and RE, we use equations (37) and (38). We consider first RE production. We segment RE production in two parts: production by hydroelectricity and by renewable sources without hydroelectricity (wind, solar, geothermal, etc.). We use the superscripts "hy" and "re" to refer respectively to hydroelectricity and

[17] In dollars of 2005.

other renewable sources. Therefore, we can write the aggregate EROEI for RE as follow:

$$\varepsilon_t^{RE} = \frac{1}{s_{t-1}^g}\frac{F_t}{X_{t-1}} = \left(\frac{G_t^{hy}+G_t^{re}}{\varphi Y_{t-1}}\right)^{-1}\frac{F_t^{hy}+F_t^{re}}{X_{t-1}} \tag{71}$$

$$= \frac{\varphi Y_{t-1}}{X_{t-1}}\left(\frac{F_t^{hy}}{G_t^{hy}}\frac{G_t^{hy}}{G_t^{hy}+G_t^{re}} + \frac{F_t^{re}}{G_t^{re}}\frac{G_t^{re}}{G_t^{hy}+G_t^{re}}\right) \tag{72}$$

$$= \varepsilon_t^{hy}\frac{G_t^{hy}}{G_t^{hy}+G_t^{re}} + \varepsilon_t^{re}\frac{G_t^{re}}{G_t^{hy}+G_t^{re}}. \tag{73}$$

Assuming that the ratio $G_t^{re}/(G_t^{hy}+G_t^{re})$ is close to $F_t^{re}/(F_t^{hy}+F_t^{re})$, we get the value for the aggregate RE EROEI as follows:

$$\varepsilon_t^{RE} \simeq \frac{\varepsilon_t^{hy}F_t^{hy}+\varepsilon_t^{re}F_t^{re}}{F_t^{hy}+F_t^{re}}. \tag{74}$$

From BP (2014), we observe the share of hydroelectricity in total RE production, which is close to 86,9 % (average on period 1). We do not observe the share of wind power, solar and geothermal and hence we assume that their shares are equal. Based on ranges from Gupta and Hall (2011) and Boyd (2014) we estimate the following EROEIs: wind: 30, solar: 10, geothermal: 25. Taking the average of these values, the EROEI for RE without hydroelectricity is 21.67. Assuming a EROEI for hydroelectricity equal to 80 (a plausible guess), we obtain the following estimate for the EROEI of RE:

$$\varepsilon_1^{RE} \simeq 80 * 0.868 + 21.67 * 0.131 = 72.34. \tag{75}$$

The approach is the same for NRE. We assume the following EROEIs: oil: 30, coal: 70, gas: 30, nuclear: 6. Given the shares of each source during period 1 given by BP (2014) (oil: 38.5 %, coal: 30.6 %, gas: 24.7 % and nuclear: 6 %), we obtain the following estimate for the EROEI of NRE:

$$\varepsilon_1^{NRE} \simeq 30 * 0.385 + 70 * 0.306 + 30 * 0.247 + 6 * 0.06 = 40.78. \tag{76}$$

Hence, we can fix the value of L_1[18]. Then we obtain $r_1 = L1 * (S - E1)/E1 \simeq 111.5$. In the same way, we estimate G_1 by $(\frac{Y_0}{X_0}) * F1 * (\frac{\varphi}{\varepsilon_1^{NE}}) \simeq 0.94$. Finally we find $H_1 = K_1 - L_1 - G_1 \simeq 161.3$. Using this value, we estimate $v_1 : v_1 = (1-u_1)\dfrac{Y_1}{H_1} \simeq 0.24$.

Based on estimates from the literature (see subsection 4.1), we set the elasticity of substitution between capital and energy in the final sector to $\sigma = .25$.

Tehnical progress in the final and RE sector is described by the following functional forms:

$$A_t = A^* + \frac{A_1 - A^*}{\exp_{a_1}\left(\sum_{i=1}^{t-1} X_i\right)} \tag{77}$$

$$b_t = b^* + \frac{b_1 - b^*}{\exp_{a_2}\left(\sum_{i=1}^{t-1} F_i\right)} \tag{78}$$

$$\tag{79}$$

[18]Indeed $\varepsilon_1^{NRE} = \varphi\frac{Y_0}{X_0}\frac{E_1}{L_1}$ so that $L_1 = \varphi\frac{Y_0}{X_0}\frac{E_1}{\varepsilon_1^{NRE}} \simeq 22.3$.

where A_1 and b_1 are initial conditions the calibration of which is detailed below and a_1 and a_2 are strictly positive parameters.

On the basis of Valero et alii (2010), we estimate to 1/45 the ratio between current *exergy* obtained from RE (geothermal, tidal power, solar, water power, ocean waves, biomass, etc.) and the potential exergy that could be obtained from these resources. To fix $\mathcal{F}$, we assume that the ratio $F_1/\mathcal{F}$ is equal to this value of 1/45. Therefore, we set the value of $\mathcal{F}$ to $45F_1 = 24.33$. Given $\mathcal{F}, v_1, q_1, G_1, F_1$, we determine γ and b_1: $\gamma = \frac{G_1}{F_1}\frac{v_1}{q_1} \simeq 0.47$ and $b_1 = G_1^{\gamma}\left(\frac{1}{F_1} - \frac{1}{\mathcal{F}}\right) \simeq 1.75$.

Given Y_1, H_1, X_1, q_1, v_1, σ and considering equations (30), (31), (32) in $t = 1$, we obtain a system of three equations with three unknowns : a, A_1 and ζ. We solve the system by a simple all-in-one algorithm and we obtain : $a = 0.34$, $A_1 = 7.81$ and $\zeta = 0.26$.

The optimality condition (22), with the given values of Y_1, Y_0, s_1, s_0, φ and v_1 impose a constraint on the choice of α and β. Fixing $\alpha = 3$, we obtain $\beta = 0.653$, which corresponds to a time preference rate of 2.6 % on an annual basis.