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SHORT- AND LONG-RUN PLANT CAPACITY NOTIONS: DEFINITIONS AND COMPARISON

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Abstract

Starting from the existing input- and output-oriented plant capacity measures, this paper proposes new long-run input- and output-oriented plant capacity measures. While the former leave fixed inputs unchanged, the latter allow for changes in all input dimensions to gauge either a maximal plant capacity output or a minimal input combination at which non-zero production starts. The paper also establishes a formal relation between the existing short-run and the new long-run plant capacity measures. Furthermore, for a standard nonparametric frontier technology, all linear programs as well as their variations are specified to compute all efficiency measures defining these short- and long-run plant capacity concepts. Finally, we numerically illustrate this basic relationship between these short-run and long-run technical concepts of capacity utilisation.

Keywords: efficiency; plant capacity utilisation.

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1. INTRODUCTION

The notion of plant capacity was introduced by Johansen (1968, p. 362) as "... the maximum amount that can be produced per unit of time with existing plant and equipment, provided that the availability of variable factors of production is not restricted." Färe (1984) established necessary and sufficient conditions for the existence of plant capacity. For instance, he shows that the plant capacity notion cannot be obtained for certain popular parametric technology specifications (e.g., Cobb-Douglas). Färe, Grosskopf and Kokkelenberg (1989) and Färe, Grosskopf and Valdmanis (1989) introduce a nonparametric frontier framework in which plant capacity as well as a measure of the capacity utilisation can be determined from data on observed inputs and outputs using a pair of output-oriented efficiency measures.

For over 25 years, no major methodological innovation has occurred related to this plant capacity concept. While input- and output-oriented efficiency measurement models have become widely available in most frontier models (e.g., Hackman (2008) or Zhu (2014)), only an output-oriented plant capacity concept was existent. Recently, Cesaroni, Kerstens and Van de Woestyne (2016) use the same framework to define a new input-oriented measure of plant capacity utilisation based on a couple of input-oriented efficiency measures.

In addition to this engineering notion of plant capacity, one can mention at least three ways of defining an economic, cost-based capacity concept in the literature (e.g., Nelson (1989)). Each of these cost-based notions attempts to determine the short run inadequate or excessive utilisation of existing fixed inputs. A first concept concentrates on the outputs produced at short-run minimum average total cost given existing input prices (e.g., Hickman (1964)). A second definition focuses on the outputs for which short- and long-run average total costs curves are tangent (e.g., Segerson and Squires (1990)). A third capacity notion considers the outputs determined by the minimum of the long-run average total costs (e.g., Klein (1960)). Alternative economic capacity concepts are discussed in Grifell-Tatjé and Lovell (2014).

Each of these capacity notions has its advantages and disadvantages.[1] Estimates of plant capacity have regularly been reported in the literature, though it cannot be denied that the plant capacity notion is nowhere as popular as some of the cost-based notions of capacity. Since this paper focuses on plant capacity, we discuss some empirical studies based on this concept. Since the large majority of empirical plant capacity studies focuses on fisheries and health care, we briefly summarise some of these studies.

[1] A brief summary of how these different engineering and economic capacity concepts can be transposed in a nonparametric frontier framework is found in De Borger et al. (2012) and Grifell-Tatjé and Lovell (2014).

Felthoven (2002) analyses the impact of the American Fisheries Act (AFA) of 1998 on the Pollock fishery and finds that decommissioned vessels exhibited a lower level of technical efficiency and that the capacity utilization of the AFA-eligible vessels increased after the law came into effect. Other fisheries studies include Guyader and Daurès (2005) analysing the French seaweed fleet, Kirkley et al. (2003) focusing on the Malaysian purse seine fishery, Reid et al. (2003) reporting on the Western and Central Pacific Ocean tuna fishery, and Walden and Tomberlin (2010) discussing US bottom trawl gear fishing.

Valdmanis, Bernet and Moises (2010) compute state-wide hospital capacity in Florida based on the whole hospital population as part of an emergency preparedness plan. Starting from a scenario involving patient evacuations from Miami due to a major hurricane event, they assess whether hospitals in proximity to the affected market can absorb the excess patient flow. Alternative health care studies are Magnussen and Rivers Mobley (1999) comparing Norwegian and Californian hospitals, Karagiannis (2015) analysing Greek public hospitals, Kerr et al. (1999) focusing on Northern Irish acute hospitals, and Valdmanis, DeNicola and Bernet (2015) reporting on Florida's public health departments.

Apart from the use of basic plant capacity estimates, one can also mention some methodological refinements making use of the plant capacity concept. These plant capacity estimates are also parameters in a so-called short-run industry model trying to reallocate outputs and resources across units in an effort to reduce excess capacity at the industry level. For instance, Yagi and Managi (2011) explore such model in a fishery context. Another methodological refinement using the plant capacity notion is its inclusion in a decomposition of the Malmquist productivity index (see De Borger and Kerstens (2000) and the extension by Yu (2007)). Färe, Grosskopf and Kirkley (2000) suggest integrating the plant capacity notion into the revenue function and the cost indirect output distance function and they derive a decomposition of the corresponding Malmquist productivity indices.

This paper develops two new plant capacity measures using nonparametric frontier technologies that take a long run instead of a short run perspective: one output-oriented, and one input-oriented. Furthermore, this paper compares both these short- and long-run plant capacity notions to one another.

The paper is structured as follows. Section 2 introduces technologies and their representations using efficiency measures, the inverses of distance functions. Section 3 defines the traditional short-run input- and output-oriented plant capacity measure. Then, the new long-run plant capacity measures are proposed. Also some relations between short- and long-run plant capacity measures are established. For a standard nonparametric frontier technology, Section 4 specifies all linear programs as well as their variations needed to compute all efficiency measures defining these short- and long-run plant capacity concepts. It also establishes a relation with the literature on frontier models without inputs and without outputs. A numerical example in Section 5 illustrates these relations between short-run and long-run plant capacity concepts. Some concluding remarks are made in the final section.

2. TECHNOLOGY: DISTANCE FUNCTIONS AND EFFICIENCY MEASURES

We start by defining technology and some basic notation. Given an N-dimensional input vector ($x \in \mathbb{R}_+^N$) and an M-dimensional output vector ($y \in \mathbb{R}_+^M$), the production possibility set or technology can be defined: $S = \{(x,y) \mid x \text{ can produce } y\}$. Associated with this technology S, the input set denotes all input vectors $x \in \mathbb{R}_+^N$ that can produce a given output vector $y \in \mathbb{R}_+^M$: $L(y) = \{x \mid (x,y) \in S\}$. Analogously, the output set associated with S denotes all output vectors $y \in \mathbb{R}_+^M$ that can be produced from a given input vector $x \in \mathbb{R}_+^N$: $P(x) = \{y \mid (x,y) \in S\}$. Furthermore, the output set $P = \{y \mid \exists x : (x, y) \in S\}$ denotes the set of all possible outputs regardless of the needed inputs.

It is common to partition the input vector into a fixed and variable part ($x = (x^f,x^v)$), with $x^v \in \mathbb{R}_+^{N_v}$ and $x^f \in \mathbb{R}_+^{N_f}$ with $N = N_v + N_f$. We define a short run technology $S^f = \{(x^f,y) \mid x^f \text{ can produce } y\}$ and the corresponding input set $L^f(y) = \{x^f \mid (x^f,y) \in S^f\}$ and output set $P^f(x^f) = \{y \mid (x^f,y) \in S^f\}$.

Note that technology S^f is obtained by projection of technology $S \in \mathbb{R}^{N+M}$ into $\mathbb{R}^{N_f+M}$ (e.g., by setting all variable inputs equal to zero). By analogy, the set P is realized by projection of technology $S \in \mathbb{R}^{N+M}$ into $\mathbb{R}^M$ (e.g., by setting all inputs equal to zero). We return to the precise relations between the set S and its projections S^f and P in Section 5.

One can define the radial input efficiency measure as:

$$DF_i(x, y) = \min\left\{\lambda \mid \lambda \geq 0, \lambda\, x \in L(y)\right\}. \tag{1}$$

It offers a complete characterisation of the input set $L(y)$. The main properties are that it is situated between zero and unity ($0 < DF_i(x,y) \leq 1$), with efficient production on the boundary (isoquant) of the input set $L(y)$ represented by unity, and that the radial input efficiency measure has a cost interpretation (see, e.g., Hackman (2008)).

By analogy, denote the radial input efficiency measure of the input set $L^f(y)$ by $DF_i^f(x^f,y)$. This is defined as follows: $DF_i^f\left(x^f, y\right) = \min\left\{\lambda : \lambda \geq 0, \lambda x \in L^f(y)\right\}$.

Next, one can define the radial output efficiency measure as:

$$DF_o\left(x, y\right) = \max\left\{\theta : \theta \geq 0, \theta y \in P(x)\right\}. \tag{2}$$

It offers a complete characterization of the output set $P(x)$. Its main properties are that it is larger than or equal to unity ($DF_o(x,y) \geq 1$), with efficient production on the boundary (isoquant) of the output set $P(x)$ represented by unity, and that the radial output efficiency measure has a revenue interpretation (e.g., Hackman (2008)).

By analogy, denote the radial output efficiency measure of the output set $P^f(x^f)$ by $DF_o^f(x^f,y)$. Then, this efficiency measure can be defined as $DF_o^f\left(x^f, y\right) = \max\left\{\theta : \theta \geq 0, \theta y \in P^f(x^f)\right\}$. Next, denote $DF_o\left(y\right) = \max\left\{\theta : \theta \geq 0, \theta y \in P\right\}$. Contrary to the radial output efficiency measure (2), this new efficiency measure $DF_o\left(y\right)$ does not depend on a particular input vector x. Hence, this measure is allowed to choose the inputs needed for maximizing θ.

Furthermore, we need the following particular definitions of technologies. First, $L(0) = \{x \mid (x,0) \in S\}$ is the input set with zero output level. Second, $DF_i^{SR}(x^f, x^v, y) = \min\left\{\lambda : \lambda \geq 0, (x^f, \lambda x^v) \in L(y)\right\}$ is a sub-vector input efficiency measure reducing only the variable inputs. Third, $DF_i^{SR}(x^f, x^v, 0) = \min\left\{\lambda : \lambda \geq 0, (x^f, \lambda x^v) \in L(0)\right\}$ is the sub-vector input efficiency measure reducing variable inputs evaluated relative to this input set with a zero output level.

3. PLANT CAPACITY UTILISATION: DEFINITIONS

The existing plant capacity measures can in fact be interpreted as focusing on the short run, where a subvector of fixed inputs cannot be changed. The new plant capacity measures take a long run perspective and assume that all inputs can be varied when determining plant capacity measures. We first treat the existing short-run plant capacity measures. Thereafter, the new long-run plant capacity measures are defined.

3.1 Short-Run Plant Capacity Utilisation

We now first recall the definition of the short-run output-oriented plant capacity utilisation measure (see Färe, Grosskopf and Kokkelenberg (1989) and Färe, Grosskopf and Valdmanis (1989)). The definition of the output-oriented measure of plant capacity utilisation

($PCU_o^{SR}(x, x^f, y)$) requires solving an output efficiency measure relative to both a standard technology and the same technology without restrictions on the availability of variable inputs and is defined as:

$$PCU_o^{SR}(x, x^f, y) = \frac{DF_o(x, y)}{DF_o^f(x^f, y)}, \qquad (3)$$

where $DF_o(x,y)$ and $DF_o^f(x^f, y)$ are output efficiency measures relative to technologies including respectively excluding the variable inputs as defined before. Notice that $0 < PCU_o^{SR}(x, x^f, y) \leq 1$, since $1 \leq DF_o(x,y) \leq DF_o^f(x^f,y)$. Thus, output-oriented plant capacity utilisation has an upper limit of unity, but no lower limit.

Following Färe, Grosskopf and Kokkelenberg (1989: 660), this leads to the following short-run output-oriented decomposition:

$$DF_o(x, y) = DF_o^f(x^f, y).PCU_o^{SR}(x, x^f, y). \qquad (4)$$

Thus, the traditional output-oriented efficiency measure $DF_o(x,y)$ can be decomposed into a biased plant capacity measure $DF_o^f(x^f, y)$ and an unbiased plant capacity measure $PCU_o^{SR}(x, x^f, y)$, following the terminology introduced by Färe, Grosskopf and Kokkelenberg (1989), Färe, Grosskopf and Valdmanis (1989) and Färe, Grosskopf and Lovell (1994).

Cesaroni, Kerstens and Van de Woestyne (2016) offer a definition of the input-oriented plant capacity measure ($PCU_i(x,x^f,y)$):

$$PCU_i^{SR}(x, x^f, y) = \frac{DF_i^{SR}(x^f, x^v, y)}{DF_i^{SR}(x^f, x^v, 0)}, \qquad (5)$$

where $DF_i^{SR}(x^f, x^v, y)$ and $DF_i^{SR}(x^f, x^v, 0)$ are both sub-vector input efficiency measures reducing only the variable inputs relative to the technology, whereby the latter efficiency measure is evaluated at a zero output level. Notice that $PCU_i^{SR}(x, x^f, y) \geq 1$, since $0 \leq DF_i^{SR}(x^f, x^v, 0) \leq DF_i^{SR}(x^f, x^v, y) \leq 1$. Thus, input-oriented plant capacity utilisation has a lower limit of unity, but no upper limit.

This leads to the following short-run input-oriented decomposition:

$$DF_i^{SR}(x^f, x^v, y) = DF_i^{SR}(x^f, x^v, 0).PCU_i^{SR}(x, x^f, y). \qquad (6)$$

Thus, the traditional sub-vector input-oriented efficiency measure $DF_i^{SR}(x^f, x^v, y)$ is decomposed into a biased plant capacity measure $DF_i^{SR}(x^f, x^v, 0)$ and an unbiased plant capacity measure $PCU_i^{SR}(x, x^f, y)$.

3.2 Long-Run Plant Capacity Utilisation

A new definition of a long-run output-oriented measure of plant capacity utilisation ($PCU_o^{LR}(x,y)$) involves an output efficiency measure relative to both a standard technology and the same technology without restrictions on the availability of inputs and is defined as:

$$PCU_o^{LR}(x,y) = \frac{DF_o(x,y)}{DF_o\ (y)}, \qquad (7)$$

where $DF_o(x,y)$ and $DF_o\ (y)$ are output efficiency measures relative to technologies including all inputs respectively ignoring all inputs.

This leads to the following long-run output-oriented decomposition:

$$DF_o(x,y) = DF_o\ (y).PCU_o^{LR}(x,y). \qquad (8)$$

Thus, the traditional output-oriented efficiency measure $DF_o(x,y)$ can be decomposed into a biased plant capacity measure $DF_o\ (y)$ and an unbiased plant capacity measure $PCU_o^{LR}(x,y)$,

A new definition of the long-run input-oriented plant capacity measure ($PCU_i^{LR}(x,y)$) is:

$$PCU_i^{LR}(x,y) = \frac{DF_i\ (x,y)}{DF_i\ (x,0)}, \qquad (9)$$

where $DF_i\ (x,y)$ and $DF_i\ (x,0)$ are both input efficiency measures aimed at reducing all input dimensions relative to the technology, whereby the latter efficiency measure is evaluated at a zero output level. This definition presupposes the following definition of an input efficiency measure reducing all inputs relative to an input set with a zero output level: $DF_i\ (x,0) = \min\{\lambda : \lambda \geq 0, \lambda x \in L(0)\}$.

This leads to the long-run input-oriented decomposition:

$$DF_i\ (x,y) = DF_i\ (x,0).PCU_i^{LR}(x,y). \qquad (10)$$

Thus, the input-oriented efficiency measure $DF_i\ (x,y)$ is decomposed into a biased plant capacity measure $DF_i\ (x,0)$ and an unbiased plant capacity measure $PCU_i^{LR}(x,y)$.

3.3 Relations between Short- and Long-Run Plant Capacity Utilisation

Figure 1 develops the geometric intuition behind the short-run and long-run plant capacity measures. The isoquant denoting the combinations of fixed and variable inputs yielding a given output level $L(y)$ is represented by the polyline *abcd* and its vertical and horizontal extensions at *a* and *d* respectively. We focus on observation *e* to illustrate first the short-run output-oriented plant capacity utilisation measure: for a given fixed input vector, it scales up the use of variable inputs to reach a translated point *e′* that allows maximizing the vector of outputs. For the development of the short-run input-oriented plant capacity measure, it therefore seems logical to look for a reduction in variable inputs for given fixed inputs towards the translated point e'' that is situated outside the isoquant $L(y)$ because it produces an output vector of zero (it is compatible with the isoquant $L(0)$ that is situated lower).

<FIGURE 1 ABOUT HERE>

In brief, while the short-run output-oriented plant capacity measure evaluates capacity by contrasting the frontier outputs for a given observation with respect to the maximal outputs available (represented by the horizontal segment starting at point *d* of the frontier in Figure 1) net of inefficiency, the short-run input-oriented plant capacity measure assesses capacity by contrasting the minimum variable inputs for an observation with given outputs with respect to the minimal variable inputs for a translated observation producing a zero output (represented by point *a* on the vertical segment *ab* of the frontier in Figure 1), also net of inefficiency. Otherwise stated, while the output-oriented plant capacity measure compares output levels relative to the maximum level of outputs available, the input-oriented plant capacity measure compares variable input levels relative to the amount of variable inputs compatible with a zero output level.

The long-run plant capacity notions are now straightforward to illustrate. The long-run output-oriented plant capacity measure scales up all inputs to reach a translated point e''' that allows maximizing the vector of outputs. The long-run input-oriented plant capacity measure now equally looks for a reduction in all inputs towards the translated point e'''' that is situated outside the isoquant $L(y)$ because it corresponds to a zero output level.

We now establish a relation between the short- and long-run output-oriented as well as input-oriented plant capacity measures. Recalling that the short-run plant capacity measures leave a subvector of fixed inputs unaltered while the long-run plant capacity measures assume that all input dimensions can be varied to gauge plant capacity, the following proposition follows suit:

Proposition 1: (i) The following relation can be established between short- and long-run output-oriented plant capacity measures (3) and (7) respectively:

$$PCU_o^{SR}(x, x^f, y) \le PCU_o^{LR}(x, y) \le 1 \qquad (11)$$

(ii) The following relation can be established between short- and long-run input-oriented plant capacity measures (5) and (9) respectively:

$$PCU_i^{SR}(x, x^f, y) \ge PCU_i^{LR}(x, y) \ge 1 \qquad (12)$$

Proof: Since the numerator in the short-run output-oriented plant capacity measure (3) equals the numerator in the long-run output-oriented plant capacity measure (7), (i) follows from

$1 \le DF_o(x^f, y) \le DF_o(y)$. Since the numerator in the short-run input-oriented plant capacity measure (5) equals the numerator in the long-run input-oriented plant capacity measure (9), (ii) follows from $DF_i^{SR}(x^f, x^v, 0) \le DF_i\ (x, 0) \le 1$.

4. NONPARAMETRIC TECHNOLOGIES

We choose to specify these plant capacity notions using nonparametric frontier technologies, because these primal capacity notions are difficult to estimate using traditional parametric specifications. For instance, Färe (1984) shows that a plant capacity notion cannot be obtained for certain popular parametric specifications of technology (e.g., the Cobb-Douglas).

Therefore, plant capacity is measured relative to a nonparametric frontier technology obtained from K observations (x_k, y_k), (k = 1, …, K) imposing strong disposal of both inputs and outputs, convexity and variable returns to scale (see Hackman (2008) or Zhu (2014)):

$$S^{VRS} = \left\{ (x, y): \quad x \ge \sum_{k=1}^{K} x_k\ z_k,\ y \le \sum_{k=1}^{K} y_k\ z_k, \sum_{k=1}^{K} z_k = 1, z_k \ge 0 \right\}. \tag{13}$$

We now turn to the computation of all plant capacity notions with respect to this variable returns to scale technology. Note that alternative assumptions on technology (e.g., constant returns to scale) are ignored.

4.1 Short-Run Plant Capacity Utilisation

For the sake of clarity, we explicitly add the two linear programs (LPs) for computing the short-run output-oriented plant capacity measure. For an evaluated observation (x_o, y_o), one can obtain the radial output measure $DF_o(x_o,y_o)$ as follows:

$$\begin{aligned} DF_o(x_o, y_o) &= \max_{\theta, z} \theta \\ \text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \ge \theta y_{om} \quad m = 1, ..., M, \\ & \sum_{k=1}^{K} x_{kn} z_k \le x_{on} \quad n = 1, ..., N, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \theta \ge 0, z_k \ge 0, \quad k = 1, ..., K. \end{aligned} \tag{14}$$

The efficiency measure $DF_o^f(x_o^f, y_o)$ is computed for observation (x_o, y_o) as:

$$
\begin{aligned}
DF_o^f(x_o^f, y_o) &= \max_{\theta, z} \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq \theta y_{om} \quad m = 1, \ldots, M, \\
& \sum_{k=1}^{K} x_{kn}^f z_k \leq x_{on}^f \quad n = 1, \ldots, N^f, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned}
\tag{15}
$$

Observe that there are no input constraints on the variable inputs. Note that Färe, Grosskopf and Lovell (1994) introduce an alternative LP with a scalar for each variable input dimension. This LP and (15) are equivalent to making each variable input a decision variable. Thus, (15) can be solved as:

$$
\begin{aligned}
DF_o^f(x_o^f, y_o) &= \max_{\theta, z, x^v} \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq \theta y_{om} \quad m = 1, \ldots, M, \\
& \sum_{k=1}^{K} x_{kn}^f z_k \leq x_{on}^f \quad n = 1, \ldots, N^f, \\
& \sum_{k=1}^{K} x_{kn}^v z_k \leq x_n^v \quad n = 1, \ldots, N^v, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned}
\tag{16}
$$

Turning now to the short run input-oriented plant capacity measure, one computes the radial sub-vector input measure $DF_i^{SR}(x_o^f, x_o^v, y_o)$ for an evaluated observation (x_o, y_o):

$$
\begin{aligned}
DF_i^{SR}(x_o^f, x_o^v, y_o) &= \min_{\lambda, z} \lambda \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq y_{om} \quad m = 1, \ldots, M, \\
& \sum_{k=1}^{K} x_{kn}^f z_k \leq x_{on}^f \quad n = 1, \ldots, N^f, \\
& \sum_{k=1}^{K} x_{kn}^v z_k \leq \lambda x_{on}^v \quad n = 1, \ldots, N^v, \quad N^f + N^v = N, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \lambda \geq 0, z_k \geq 0, \quad k = 1, \ldots, K.
\end{aligned}
\tag{17}
$$

The sub-vector efficiency measure $DF_i^{SR}(x_o^f, x_o^v, 0)$ is obtained for observation (x_o, y_o) by solving:

$$DF_i^{SR}(x_o^f, x_o^v, 0) = \min_{\lambda, z} \lambda$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq 0 \quad m = 1,...,M, \\
& \sum_{k=1}^{K} x_{kn}^f z_k \leq x_{on}^f \quad n = 1,...,N^f, \\
& \sum_{k=1}^{K} x_{kn}^v z_k \leq \lambda x_{on}^v \quad n = 1,...,N^v, \quad N^f + N^v = N, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \lambda \geq 0, z_k \geq 0, \quad k = 1,...,K.
\end{aligned} \tag{18}$$

Note that the observed output levels on the right-hand side of the output constraints are set equal to zero.[2] In fact, since the output constraints are redundant, this problem can be rewritten:

$$DF_i^{SR}(x_o^f, x_o^v, 0) = \min_{\lambda, z} \lambda$$

$$\begin{aligned}
\text{s.t.} \quad & \sum_{k=1}^{K} x_{kn}^f z_k \leq x_{on}^f \quad n = 1,...,N^f, \\
& \sum_{k=1}^{K} x_{kn}^v z_k \leq \lambda x_{on}^v \quad n = 1,...,N^v, \quad N^f + N^v = N, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \lambda \geq 0, z_k \geq 0, \quad k = 1,...,K.
\end{aligned} \tag{19}$$

Observe that the LPs (15) and (19) are similar in that certain constraints are suppressed: the variable input constraints in LP (15) and the output constraints in LP (19). Given the nature of the inequality constraints, this is again similar to making the variable inputs decision variables in LP (16) and to setting the outputs equal to zero in LP (18): both approaches allow for an arbitrary scaling of inputs downwards and of outputs upwards.

4.2 Long-Run Plant Capacity Utilisation

To obtain the long-run plant capacity measures, just three more efficiency measures need to be computed. For the output-oriented case, $DF_o(x_o,y_o)$ has already been computed in (14). One just needs to compute the efficiency measure $DF_o(y_o)$ for a given observation (x_o, y_o):

[2] The determination of input utilization rates for the variable inputs is straightforward in the output-oriented case (e.g., Färe, Grosskopf and Lovell (1994: § 10.3)), the determination of optimal variable inputs is equally trivial in this input-oriented case.

$$
\begin{aligned}
& DF_o(y_o) = \max_{\theta,x,z} \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq \theta y_{om} \quad m = 1,\ldots,M, \\
& \sum_{k=1}^{K} x_{kn} z_k \leq x_n \quad n = 1,\ldots,N, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, \quad k = 1,\ldots,K, \\
& x_n \geq 0, \quad n = 1,\ldots,N.
\end{aligned} \tag{20}
$$

Obviously, the input constraints in (20) are redundant, since these constraints are allowed to take any arbitrary value. Hence, by omitting these input constraints, LP (20) simplifies to

$$
\begin{aligned}
& DF_o\ (y_o) = \max_{\theta,\, z} \theta \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq \theta y_{om} \quad m = 1,\ldots,M, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \theta \geq 0, z_k \geq 0, \quad k = 1,\ldots,K.
\end{aligned} \tag{21}
$$

Finally, for the input-oriented case, the efficiency measure $DF_i\ (x_o, y_o)$ is calculated for a given observation (x_o, y_o) as follows:

$$
\begin{aligned}
& DF_i\ (x_o, y_o) = \min_{\lambda,\, z} \lambda \\
\text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq y_{om} \quad m = 1,\ldots,M, \\
& \sum_{k=1}^{K} x_{kn} z_k \leq \lambda x_{on} \quad n = 1,\ldots,N, \\
& \sum_{k=1}^{K} z_k = 1, \\
& \lambda \geq 0, z_k \geq 0, \quad k = 1,\ldots,K.
\end{aligned} \tag{22}
$$

Last but not least, the efficiency measure $DF_i\ (x_0, 0)$ is obtained for observation (x_o, y_o) by solving:

$$DF_i\ (x_o, 0) = \min_{\lambda, z} \lambda$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{k=1}^{K} y_{km} z_k \geq 0 \quad m = 1, \dots, M, \\ & \sum_{k=1}^{K} x_{kn} z_k \leq \lambda x_{on} \quad n = 1, \dots, N, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \lambda \geq 0, z_k \geq 0, \quad k = 1, \dots, K. \end{aligned} \tag{23}$$

Note again that the observed output levels on the right-hand side of the output constraints are constrained to equal zero. Again, since the output constraints are redundant, this problem simplifies as follows:

$$DF_i\ (x_o, 0) = \min_{\lambda, z} \lambda$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{k=1}^{K} x_{kn} z_k \leq \lambda x_{on} \quad n = 1, \dots, N, \\ & \sum_{k=1}^{K} z_k = 1, \\ & \lambda \geq 0, z_k \geq 0, \quad k = 1, \dots, K. \end{aligned} \tag{24}$$

Observe that the LPs (21) and (24) are similar in that some constraints are eliminated: all input constraints in LP (21) and again all output constraints in LP (24). Given the nature of the inequality constraints, we again make all inputs decision variables in LP (20) and we set all outputs equal to zero in LP (23). This makes an arbitrary scaling of the inputs downwards and of the outputs upwards possible.

4.3 Relation with the Literature

Remark that LP (21) is formally identical to the output-oriented efficiency measure computed relative to a variable returns to scale technology without inputs proposed by Lovell and Pastor (1999) and further refined by Liu et al. (2001). An early application is Lovell and Pastor (1997) who have applied such a model to a target setting procedure established by a large Spanish savings bank. We are inclined to think that in a clear production setting where inputs can be specified (but are not for whatever reason), such a model can be interpreted as an estimate of the long run output-oriented plant capacity. Obviously, such model without inputs is also often used when assessing the efficiency of accounting ratios (e.g., see Cai and Wu (2001) or Halkos and Salamouris (2004)) or when evaluating social indicators like the Human Development Index (e.g., see Lefèbvre, Coelli and Pestieau (2010)). When we leave a clear production setting and inputs can simply not be specified, then of course the above interpretation does not hold.

Further remark that the LPs (19) and (24) are related to the input-oriented efficiency measure computed relative to a variable returns to scale technology without outputs proposed by Lovell and Pastor (1999). We are unaware of any other economic context in which these specific models have ever been used.

5. NUMERICAL ILLUSTRATION

We illustrate the ease of implementing some of the new plant capacity definitions introduced in this contribution by using a small set of artificial data. Table 1 contains 16 fictitious observations with two inputs generating a single output: one input is variable, the other one is fixed. A three-dimensional representation of the technology resulting from these 16 fictitious observations is provided by Figures 2 and 3.

<TABLE 1 ABOUT HERE>

Figure 2 demonstrates the relation between the set S and its projections S^f and P (mentioned in Section 2) in case of a variable returns to scale technology obtained from the 16 available observations (grey coloured dots). Technology S consists of two inputs (the variable input x^v and the fixed input x^f) and one output (y) and is visible by means of its convex boundary. Setting all variable inputs equal to zero yields the short run technology S^f visualised by the red piecewise linear convex region in the fixed input output plane. The projections of the original 16 observations are visible by means of red coloured boxes. Finally, setting all inputs equal to zero results in the output set P visible as the green interval on the y-axis. The original 16 observations are now projected onto the corresponding points indicated by green diagonal crosses.

<FIGURES 2 TO 3 ABOUT HERE>

Having explained the relations between the technology S and its projections, we now turn to an illustration of all plant capacity measures. The short-run and long-run output-oriented plant capacity measures are illustrated using Figure 2. By contrast, both input-oriented plant capacity measures are elucidated using Figure 3.

First, Figure 2 illustrates the components of the output-oriented capacity measures defined by (3) and (7). Consider observation a with inputs x_v = 7.5, x_f = 5.5, and output y = 3.5. Then, $DF_o(x\ ,y) = \dfrac{|a_1 b|}{|a_1 a|} = 1.4505$ and $DF_o^f(x^f, y) = \dfrac{|a_3 c_2|}{|a_3 a_2|} = \dfrac{|c_1 c|}{|a_1 a|} = 1.6429$. Using (3), we conclude that $PCU_o^{SR}(x, x^f, y) = \dfrac{1.4505}{1.6429} = 0.8829$. Since $DF_o(y) = \dfrac{|d_1 d|}{|a_1 a|} = 1.7143$, equation (7) yields $PCU_o^{LR}(x, y) = \dfrac{1.4505}{1.7143} = 0.8462$.

Second, Figure 3 illustrates the components of the input-oriented capacity measures defined by (5) and (9). To serve this illustration, two sections are added to Figure 3: the section by the plane α parallel to the variable input axis represents the short-run plant capacity measure; the section by the plane β going through the origin intends to illustrate the long-run plant capacity measure. These two sections have been projected in two dimensions in Figure 4: the horizontal axis represents the variable input, the vertical axis denotes the output. The section representing the short-run plant capacity measure is denoted by the black polyline; the section depicting the long-run plant capacity measure is denoted by the red dashed polyline.

<FIGURE 4 ABOUT HERE>

Again, consider observation a with inputs x_v = 7.5, x_f = 5.5, and output y = 3.5. This observation is visible both in Figures 3 and 4. Then, $DF_i^{SR}(x^f, x^v, y) = \frac{|a_2 a_1|}{|a_2 a|} = 0.4000$ while $DF_i^{SR}(x^f, x^v, 0) = \frac{|b_2 b_1|}{|b_2 b|} = 0.2333$. Hence, $PCU_i^{SR}(x, x^f, y) = \frac{0.4000}{0.2333} = 1.7143$ using equation (5). Since $DF_i(x, y) = \frac{|a_4 a_3|}{|a_4 a|} = 0.6241$ and $DF_i(x, 0) = \frac{|b_4 b_3|}{|b_4 b|} = 0.5103$, equation (9) returns $PCU_i^{LR}(x, y) = \frac{0.6241}{0.5103} = 1.2230$.

Similar computations as those illustrated above can be executed on all observations provided in Table 1. The resulting plant capacity measures and its components are reported in Tables 2 (output-oriented) and 3 (input-oriented).

<TABLES 2 AND 3 ABOUT HERE>

6. CONCLUSIONS

This contribution introduces new output- and input-oriented plant capacity measures taking a long-run perspective complementing the existing short-run output- and input-oriented plant capacity measures. While the short-run output- and input-oriented plant capacity measures leave a subvector of fixed inputs unaltered, the new long-run plant capacity measures allow for changes in all input dimensions to determine either a maximal plant capacity output in the output-oriented case or a minimal input combination at which non-zero production starts in the input-oriented case.

Also a relation between these short- and long-run plant capacity measures has been established. For a standard nonparametric frontier technology with variable returns to scale, all linear programs (including some variations) are discussed computing the efficiency measures defining these plant capacity concepts. We also develop a relation with frontier models without inputs and without outputs. A numerical example has served to clarify the geometric intuition behind these new plant capacity measures and 5 illustrates these relations between short-run and long-run plant capacity concepts.

Though the existing short-run plant capacity measures have enjoyed some popularity among applied economists, it is fair to say that these concepts have mainly been employed in a specialised efficiency literature. We hope these new long-run plant capacity definitions can contribute to enlarge the empirical toolbox available for practitioners in production economics at large.

REFERENCES

Cai, Y., W. Wu (2001) Synthetic Financial Evaluation by a Method of Combining DEA with AHP, *International Transactions in Operational Research*, 8(5), 603–609.

Cesaroni, G., K. Kerstens, I. Van de Woestyne (2016) A New Input-Oriented Plant Capacity Notion: Definition and Empirical Comparison, *Pacific Economic Review*, forthcoming. DOI: 10.1111/1468-0106.12201

De Borger, B., K. Kerstens (2000) The Malmquist Productivity Index and Plant Capacity Utilization, *Scandinavian Journal of Economics*, 102(2), 303–310.

De Borger, B., K. Kerstens, D. Prior, I. Van de Woestyne (2012) Static Efficiency Decompositions and Capacity Utilisation: Integrating Technical and Economic Capacity Notions, *Applied Economics*, 44(31), 4125-4141.

Färe, R. (1984) The Existence of Plant Capacity, *International Economic Review*, 25(1), 209-213.

Färe, R., S. Grosskopf, J. Kirkley (2000) Multi-Output Capacity Measures and Their Relevance for Productivity, *Bulletin of Economic Research*, 52(2), 101-112.

Färe, R., S. Grosskopf, E. Kokkelenberg (1989) Measuring Plant Capacity, Utilization and Technical Change: A Nonparametric Approach, *International Economic Review*, 30(3), 655-666.

Färe, R., S. Grosskopf, V. Valdmanis (1989) Capacity, Competition and Efficiency in Hospitals: A Nonparametric Approach, *Journal of Productivity Analysis*, 1(2), 123-138.

Färe, R., S. Grosskopf, C.A.K. Lovell (1994) *Production Frontiers*, Cambridge, Cambridge University Press.

Felthoven, R.G. (2002) Effects of the American Fisheries Act on Capacity, Utilization and Technical Efficiency, *Marine Resource Economics*, 17(3), 181-205.

Grifell-Tatjé, E., C.A.K. Lovell (2014) Productivity, Price Recovery, Capacity Constraints and their Financial Consequences, *Journal of Productivity Analysis*, 41(1), 3-17.

Guyader, O., F. Daurès (2005) Capacity and Scale Inefficiency: Application of Data Envelopment Analysis in the Case of the French Seaweed Fleet, *Marine Resource Economics*, 20(4), 347-365.

Hackman, S.T. (2008) *Production Economics: Integrating the Microeconomic and Engineering Perspectives*, Berlin, Springer.

Halkos, G.E., D.S. Salamouris (2004) Efficiency Measurement of the Greek Commercial Banks with the Use of Financial Ratios: A Data Envelopment Analysis Approach, *Management Accounting Research*, 15(2), 201-224.

Hickman, B.G. (1964) On a New Method for Capacity Estimation, *Journal of the American Statistical Association*, 59(306), 529-549.

Johansen, L. (1968) Production Functions and the Concept of Capacity, Namur, Recherches Récentes sur la Fonction de Production (Collection "Economie Mathématique et Econometrie", n° 2) [reprinted in F.R. Førsund (ed.) (1987) *Collected Works of Leif Johansen, Volume 1*, Amsterdam, North Holland, 359-382].

Karagiannis, R. (2015) A System-of-Equations Two-Stage DEA Approach for Explaining Capacity Utilization and Technical Efficiency, *Annals of Operations Research*, 227(1), 25-43.

Kerr, C.A., J.C. Glass, G.M. McCallion, D.G. McKillop (1999) Best-Practice Measures of Resource Utilization for Hospitals: A Useful Complement in Performance Assessment, *Public Administration*, 77(3) 639–650.

Kirkley, J.E., D.E. Squires, M.F. Alam, H.O. Ishak (2003) Excess Capacity and Asymmetric Information in Developing Country Fisheries: The Malaysian Purse Seine Fishery, *American Journal of Agricultural Economics*, 85(3), 647-662.

Klein, L.R. (1960) Some Theoretical Issues in the Measurement of Capacity, *Econometrica*, 28(2), 272-286.

Lefèbvre, M., T. Coelli, P. Pestieau (2010) On the Convergence of Social Protection Performance in the European Union, *CESifo Economic Studies*, 56(2), 300-322.

Liu, W.B., D.Q. Zhang, W. Meng, X.X. Li, F. Xu (2011) A Study of DEA Models Without Explicit Inputs, *Omega*, 39(5), 472-480.

Lovell, C.A.K., J.T. Pastor (1997) Target Setting: An Application to a Bank Branch Network, *European Journal of Operational Research*, 98(2), 290-299.

Lovell, C.A.K., J.T. Pastor (1999) Radial DEA Models Without Inputs or Without Outputs, *European Journal of Operational Research*, 118(1), 46-51.

Magnussen, J., L. Rivers Mobley (1999) The Impact of Market Environment on Excess Capacity and the Cost of an Empty Hospital Bed, *International Journal of the Economics of Business*, 6(3), 383-398.

Nelson, R. (1989) On the Measurement of Capacity Utilization, *Journal of Industrial Economics*, 37(3), 273-286.

Reid, C., D. Squires, Y. Jeon, L. Rodwell, R. Clarke (2003) An Analysis of Fishing Capacity in the Western and Central Pacific Ocean Tuna Fishery and Management Implications, *Marine Policy*, 27(6), 449-469.

Segerson, K., D. Squires (1990) On the Measurement of Economic Capacity Utilization for Multi-Product Industries, *Journal of Econometrics*, 44(3), 347-361.

Valdmanis, V., P. Bernet, J. Moises (2010) Hospital Capacity, Capability, and Emergency Preparedness, *European Journal of Operational Research*, 207(3), 1628-1634.

Valdmanis, V., A. DeNicola, P. Bernet (2015) Public Health Capacity in the Provision of Health Care Services, *Health Care Management Science*, 18(4), 475-482.

Walden, J.B., D. Tomberlin (2010) Estimating Fishing Vessel Capacity: A Comparison of Nonparametric Frontier Approaches, *Marine Resource Economics*, 25(1), 23-36.

Yagi, M., S. Managi (2011) Catch Limits, Capacity Utilization and Cost Reduction in Japanese Fishery Management, *Agricultural Economics*, 42(5), 577-592.

Yu, M.-M. (2007) The Capacity Productivity Change and The Variable Input Productivity Change: A New Decomposition of the Malmquist Productivity Index, *Applied Mathematics and Computation*, 185(1), 375-381.

Zhu, J. (2014) *Quantitative Models for Performance Evaluation and Benchmarking: Data Envelopment Analysis with Spreadsheets*, 3rd Edition, Berlin, Springer.

Figure 1: Isoquant with Input and Output-oriented Plant Capacity Measures

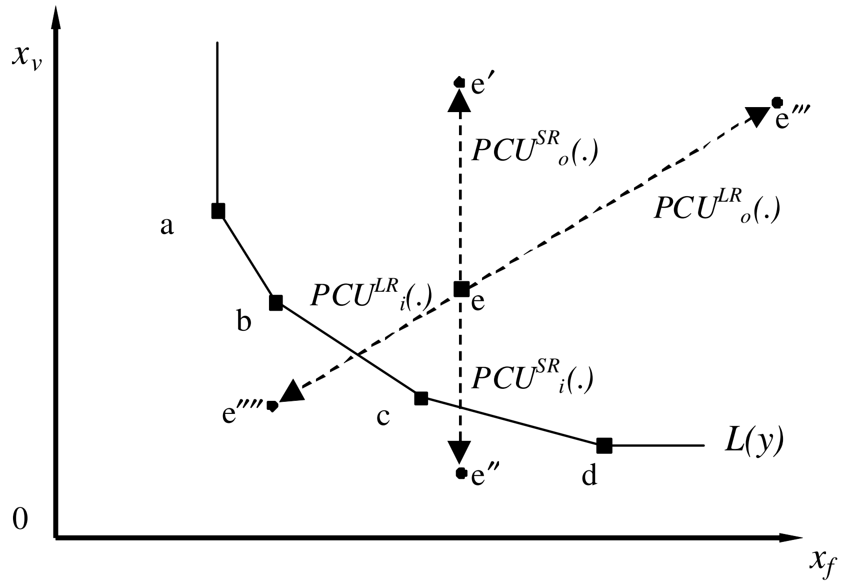

Figure 2: Technology *S* and its Projections S^f and *P*: Output-Oriented Plant Capacity

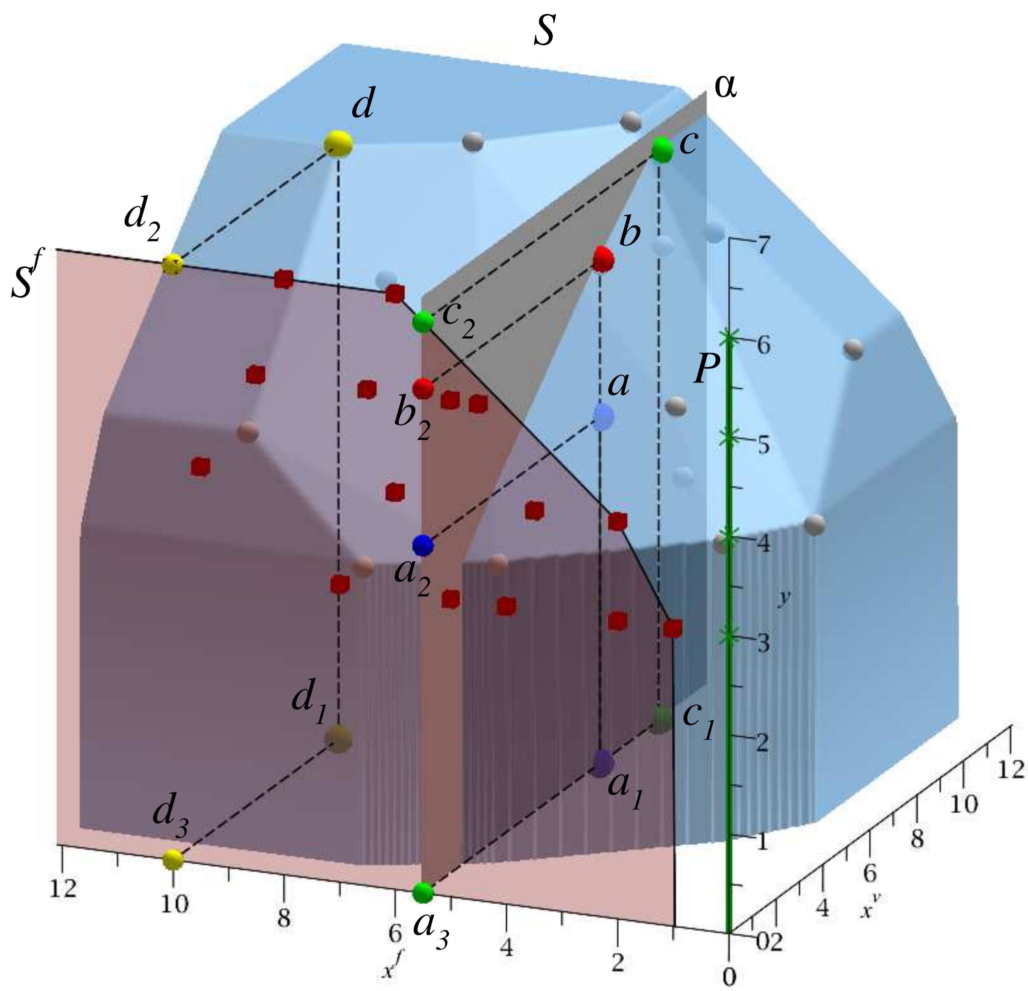

Figure 3: Technology *S*: Input-Oriented Plant Capacity

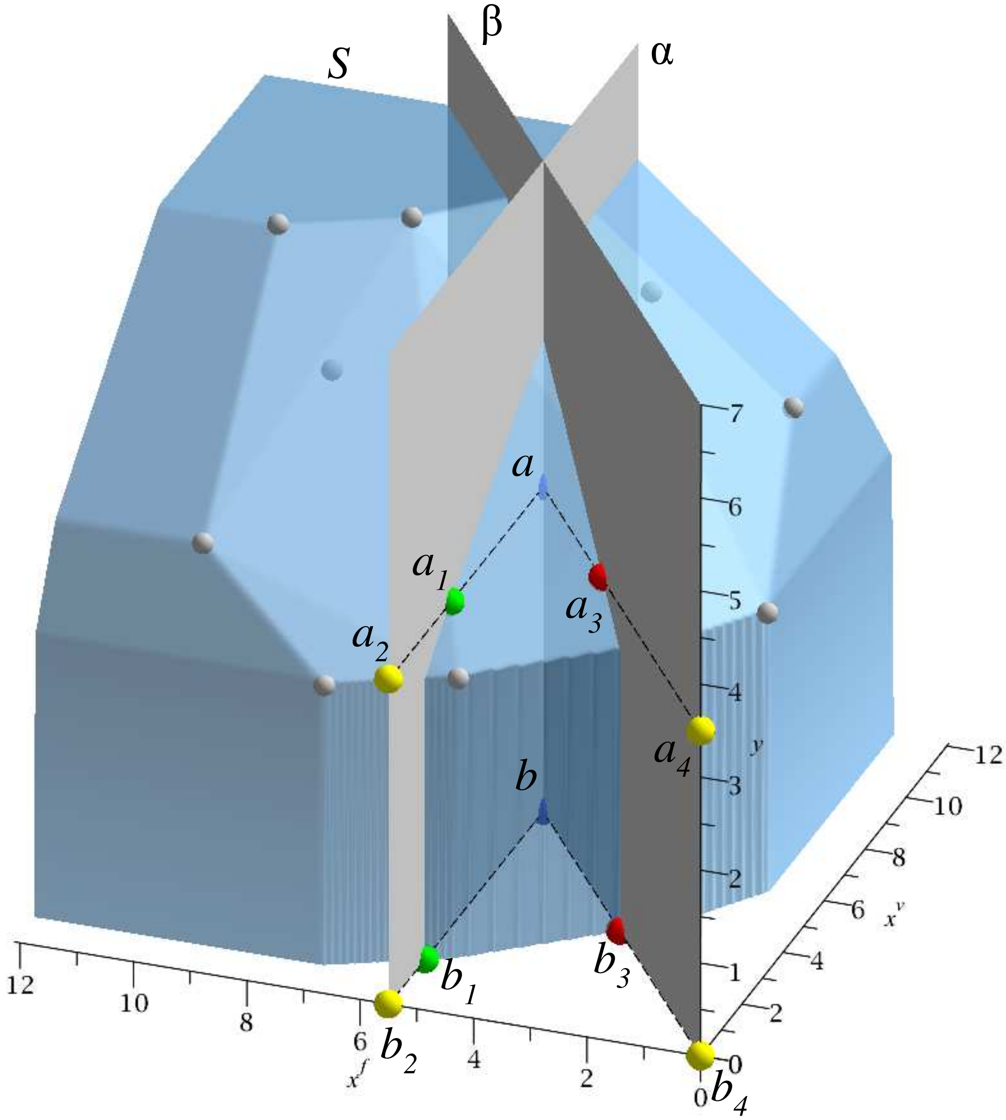

Figure 4: Short Run Technology S^f Constructed from Numerical Example

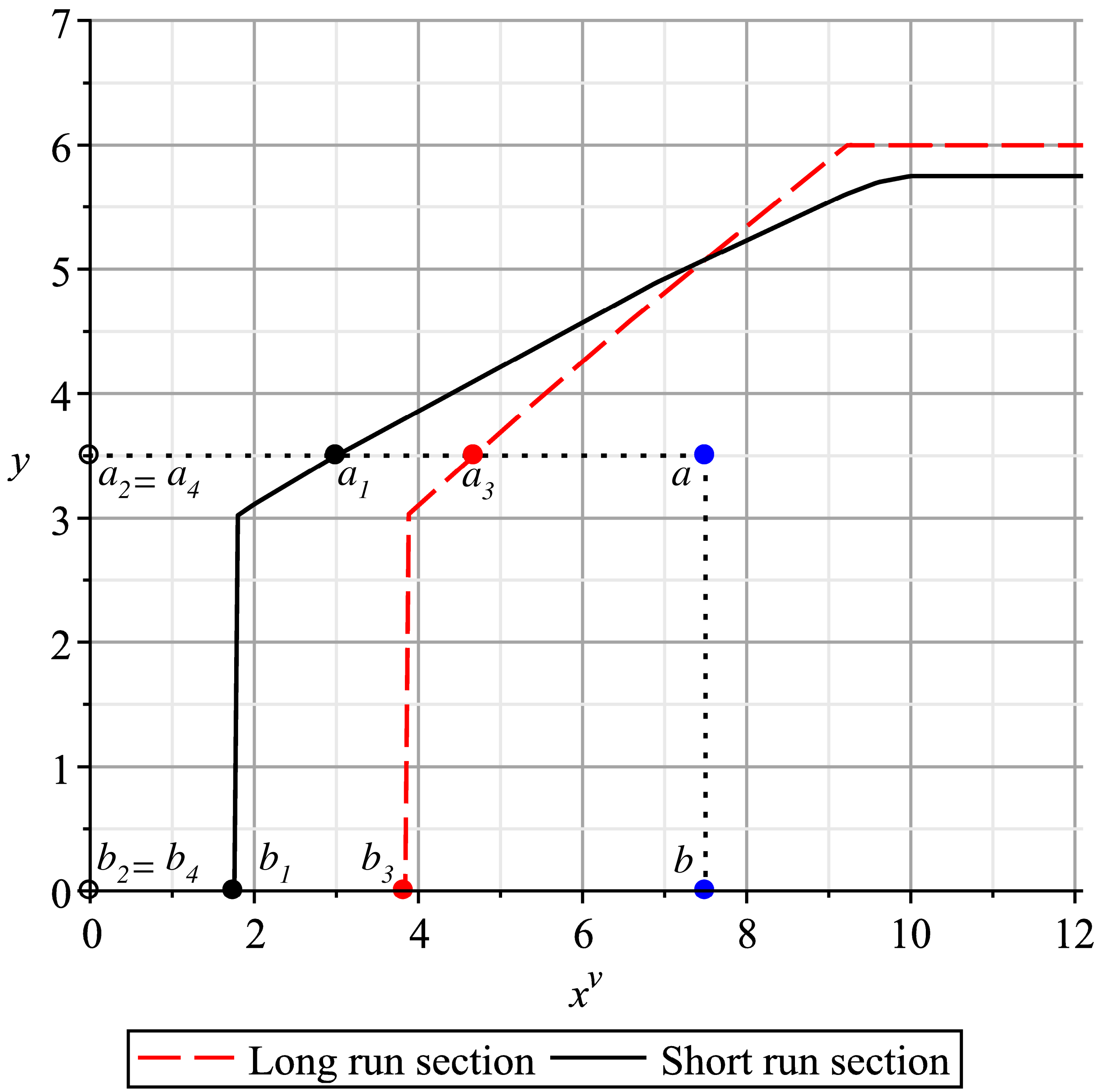

Table 1: Numerical Example Containing 16 observations

Nr	x^v	x^f	y
1	1.0	7.0	3.0
2	2.0	5.0	3.0
3	4.5	2.0	3.0
4	6.0	1.0	3.0
5	7.5	4.0	3.0
6	2.0	9.5	4.0
7	10.0	2.0	4.0
8	5.5	6.0	4.0
9	6.0	3.5	4.0
10	6.5	6.5	5.0
11	5.5	8.5	5.0
12	9.0	5.0	5.0
13	10.0	4.5	5.0
14	7.0	10.0	6.0
15	8.0	8.0	6.0
16	10.0	6.0	6.0

Table 2: Output-oriented Short- and Long-run Efficiency Results and Plant Capacity Utilisation

Nr	$DF_o(x,y)$	$DF_o^f(x^f,y)$	$DF_o(y)$	$PCU_o^{LR}(.)$	$PCU_o^{SR}(.)$
1	1.0000	2.0000	2.0000	0.5000	0.5000
2	1.0000	1.8333	2.0000	0.5000	0.5455
3	1.0000	1.3333	2.0000	0.5000	0.7500
4	1.0000	1.0000	2.0000	0.5000	1.0000
5	1.5250	1.6667	2.0000	0.7625	0.9150
6	1.0000	1.5000	1.5000	0.6667	0.6667
7	1.0000	1.0000	1.5000	0.6667	1.0000
8	1.1339	1.5000	1.5000	0.7560	0.7560
9	1.0000	1.1875	1.5000	0.6667	0.8421
10	1.0071	1.2000	1.2000	0.8393	0.8393
11	1.0278	1.2000	1.2000	0.8565	0.8565
12	1.0700	1.1000	1.2000	0.8917	0.9727
13	1.0500	1.0500	1.2000	0.8750	1.0000
14	1.0000	1.0000	1.0000	1.0000	1.0000
15	1.0000	1.0000	1.0000	1.0000	1.0000
16	1.0000	1.0000	1.0000	1.0000	1.0000

Table 3: Input-oriented Short- and Long-run Efficiency Results and Plant Capacity Utilisation

Nr	$DF_i(x, y)$	$DF_i(x,0)$	$DF_i^{SR}(x^f, x^v, y)$	$DF_i^{SR}(x^f, x^v, 0)$	$PCU_i^{LR}(.)$	$PCU_i^{SR}(.)$
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.5692	0.5692	0.3778	0.3778	1.0000	1.0000
6	1.0000	0.6667	1.0000	0.5000	1.5000	2.0000
7	1.0000	0.5769	1.0000	0.4500	1.7333	2.2222
8	0.8544	0.5873	0.7273	0.2727	1.4547	2.6667
9	1.0000	0.6916	1.0000	0.5417	1.4459	1.8462
10	0.9915	0.5175	0.9846	0.1923	1.9159	5.1200
11	0.9655	0.4901	0.9351	0.1818	1.9702	5.1429
12	0.9176	0.4684	0.8611	0.2222	1.9593	3.8750
13	0.9286	0.4485	0.8400	0.2417	2.0705	3.4759
14	1.0000	0.4022	1.0000	0.1429	2.4865	7.0000
15	1.0000	0.4205	1.0000	0.1250	2.3784	8.0000
16	1.0000	0.4111	1.0000	0.1500	2.4324	6.6667