http://lem.cnrs.fr

Document de travail du LEM
Discussion paper LEM
2016-11

Stratified higher education, social mobility at the top and efficiency: The case of the French 'Grandes écoles'

Joël HELLIER

joel.hellier@univ-nantes.fr – LEM UMR 9221 / LEMNA University of Nantes

URL de téléchargement direct / URL of direct download:
http://lem.cnrs.fr/IMG/pdf/dp2016-11.pdf

Stratified higher education, social mobility at the top and efficiency:

The case of the French '*Grandes écoles*'

Joël Hellier*

Abstract

We show that the system of 'Grandes écoles' (GEs) is a key determinant of social stratification, low intergenerational mobility at the top and low educational efficiency in France. A stylised intergenerational model of the French higher education system is constructed. This system is composed of two types of establishment, the GEs and the universities, which differ (i) in the strictness and shape of their admission, and (ii) in their per-student expenditures. We compare this system with a unified two-level higher education structure in which there is one type of establishment only that is comprised of two successive levels with two admission procedures. The GE system favours family backgrounds and the unified system personal aptitudes, which leads to lower intergenerational mobility in the former. Rising expenditure on the highest education level favours skill upgrading of the population in the unified system, whereas it insulate a narrow elite in the GE system. Finally, with similar education expenditures, the unified system results in higher human capital accumulation than the GE system in both the upper skill group and the whole population. These results suggest that the GE system hurts both social mobility and human capital accumulation.

The US and the UK display tertiary education systems which are close to the GE system in terms of selective admission and results.

Our approach provides theoretical bases for the analysis of selective versus comprehensive education systems (Turner, 1960) and a demonstration that highly stratified and selective systems reinforce family backgrounds and reduce mobility (Kirckhoff, 1995).

Keywords. *Education efficiency; Family background; Grandes écoles; Higher education; Intergenerational mobility*.

JEL Classification. I21, I28, J24, J62.

* *LEM-CNRS* (UMR 9221), University of Lille, and LEMNA, University of Nantes.
Pers. Address. 28 rue de Sévigné 75004 Paris. joel.hellier@wanadoo.fr

1. Introduction

This paper shows that, in contrast with their meritocratic claim, the '*Grandes écoles*' are key determinants of low intergenerational mobility and low educational efficiency in France.

France is, with the US and the UK, one of the advanced countries in which intergenerational earnings and skills mobility is the lowest (Corak, 2013) and this mobility is particularly low at the top of the earnings distribution (Raitano et al., 2015). In addition, recent studies indicate that mobility has fallen in France in the last two decades (Lefranc, 2011; BenHalima et al., 2014).

The literature on intergenerational mobility puts forward several determinants.[1] By definition, a weighty impact of family backgrounds is the key element of low mobility. Costly education prevents skill upgrading for children born into modest families, and this limitation is magnified when income inequality is high. A reduction in redistribution and in the Welfare state tends to erase the pro-mobility impacts of public policies. Finally, deficient primary education does not permit to offset the impact of family differences in culture and education.

When considering the above-mentioned determinants, the fact that intergenerational mobility has decreased in France and is comparable to that of the US and the UK is rather surprising. First, schooling is freely provided in France in primary, secondary and tertiary education. If there are fees in the *Grandes écoles*, these remain limited compared to the universities in the US and the UK, and students from modest families are exempted. Second, income inequality has continuously decreased in the seventies and eighties and is now rather low in France.[2] This should have resulted in higher mobility for the generation born since the seventies, which is not what has been observed. Third, France can be seen as an exception in the backward move of the Welfare state observed in many advanced economies. Finally, the only usual explanation that fits with the French case is the role of primary education. France is one of the OECD countries in which the per-student expenditure in primary education (in percent of GDP per capita) is the lowest. However, other countries display low public expenditure in primary school without displaying such a low mobility.

One key characteristic of the French education system is the existence of '*Grandes écoles*' (GEs), which are highly selective tertiary establishments that aim at producing the French elite. The *Grandes écoles* display several key characteristics. First, they are distinct from the universities and the separation between the two occurs right after secondary school. Their

[1] See section 2 for a broader presentation.

[2] This does not appear in Corak's 'Great Gatsby curve' (Corak, 2013) because he considers inequality at a moment when France was still an inequality-oriented country.

admission is highly selective and is not based on a minimum human capital level but on a pre-determined number of intakes that has not increased much over the last decades, whereas the number of students admitted in universities has exploded. Finally, their per-student expenditure is significantly higher than that of the universities.

This paper develops an approach which shows that the division of higher education between *Grandes écoles* and universities is a key factor of low intergenerational mobility at the top and low educational efficiency in France. A stylised intergenerational model of the French higher education system is constructed. This system is composed of two types of establishment, the GEs and the universities, which differ (i) in the strictness and shape of their admission procedures that both occur at the end of basic education, and (ii) in their per-student expenditures. We compare this system with a unified two-level higher education structure in which there is one type of establishment only, the universities, that are comprised of two successive levels (U1 and U2) with two admission procedures, one at the end of basic education to enter U1, and the other at the end of U1 to enter U2.

In the case of equal intakes of students in GEs and U2, the GE system favours family backgrounds and the unified system personal abilities, which results in lower intergenerational mobility in the former. This stems from the earlier selection to enter the highest level in the GE system. Higher expenditure on basic education favours upward mobility to the entry in both U1 and U2 in the unified system, whereas in the GE system it boosts upward mobility to the university but not to the GEs. Increasing per-student expenditure on the highest level (GE or U2) reduces mobility in the GE system whereas it increases the intake of students in U2 in the unified system. Finally, when assuming similar education expenditures, the unified system results in higher human capital accumulation in both the upper skill group and the whole population. These results suggest that the GE system (i) engenders a narrow self-reproducing elite and lessens thereby intergenerational mobility, and (ii) could have a negative effect on human capital accumulation. We finally note that the US and the UK, despite the lack of *Grandes écoles*, display tertiary education systems which are close to the GE system in terms of selective admission and budgets.

Section 2 briefly exposes the literature on the subject and the characteristics of the French higher education system. Section 3 builds the two models of higher education and Section 4 describes the related education decision. Section 5 compares these models in terms of weight of personal aptitude vs. family background in the educational attainment and in terms of efficiency. Section 6 compares the model in terms of intergenerational mobility and social stratification. We finally discuss our major findings and we conclude in Section 6.

2. Literature and the French higher education system

The approach developed hereafter utilises the economic modelling of human capital accumulation and intergenerational mobility to analyse the impacts of the French system of higher education characterised by the key role of the *Grandes écoles*.

2.1. Human capital accumulation and intergenerational mobility

Since the seminal works of Becker (1964), Ben Porah (1967) and Becker & Tomes (1976, 1979), the economic analysis of human capital accumulation and intergenerational mobility has known substantial developments (see the review by Chusseau & Hellier, 2013). From a theoretical point of view, the analysis has moved from intergenerational dynamics that generate human capital convergence to the exploration of the factors determining lasting polarisation in human capital. The empirical literature has been centred on the analysis of the determinants of human capital attainment, with a special emphasis on the impact of family backgrounds.

2.1.1. Theoretical approaches

If Ben Porah (1967) analysed the distribution of education throughout the life cycle, Becker & Tomes (1979) were the firsts to model the impact of education decisions within an intergenerational perspective. From an approach in perfect competition, they showed that the different dynasties (successive generations linked by a parent-child relationship) converge toward the same steady human capital, which could be reached after a limited number of generations. In the case of imperfections on the credit market, Loury (1981) and Becker & Tomes (1986) showed that this convergence is preserved but takes a longer time.

The subsequent theoretical literature has focused on the factors and mechanisms that could explain the persistence of human capital differences and the emergence of different groups tending towards different steady human capital levels (polarization). These factors are several. First, Galor & Zeira (1993) and Barham et al. (1995) showed that imperfections on the credit market (with a fixed cost of education in the former) hamper children from low income and low cultural background families to pursue further education. Second, an *S*-shaped education function can generate human capital polarization with a high skilled and a low skilled group (Galor & Tsiddon, 1997)[3]. Third, neighbourhood effects, peer effects and local externalities can considerably slow down the convergence of human capital or even create under-education

[3] In Galor & Tsiddon (1997), this polarization is however transitional.

traps (Benabou, 1993, 1994, 1996; Durlauf, 1994, 1996). Such traps can also emerge from differences in altruism across families (Das, 2007). A last explanation for social segmentation and low intergenerational mobility can be found in the very structure of education systems.

The economic analysis of the relations between the structure of education systems, social stratification and intergenerational mobility is rather recent. Key issues are the influences of the division of education between several levels and cycles, and of the funding allowed for each of them, on inequality, social stratification and welfare. Driskill & Horowitz (2002) and Su (2004) analysed the impact of the allocation of public funding between basic and further education upon growth, welfare and income distribution. Bertocchi & Spagat (2004) generated social stratification at the different stages of economic development from a model with basic and secondary education, the latter being divided into vocational and general studies. Su (2006) showed that the upper class in developing countries imposes larger expenditure on higher education for a minority at the expense of basic education, whereas public allocation to education is more balanced in developed countries. From an intergenerational model with three education cycles (basic education, vocational studies and university, with a selective admission to the latter) Chusseau & Hellier (2011) generate different social stratifications with under-education traps depending on the public funding allocated to each cycle and on the strictness of admission. Su et al. (2012) distinguish between standard and elite colleges to analyse the U-shape relationship between wages and skills observed in the US in the last two decades. From a calibrated overlapping generation model, Herrington (2015) shows that differences in public spending and in public contribution to early childhood education are key determinants of the divergence in inequality and intergenerational mobility between the US and Norway. The crucial role of early childhood education is confirmed by Restuccia & Urrutia (2004) and Blankenau & Youderian (2015).

As regards higher education, its twofold objective of training and signalling was initially highlighted by Arrow (1973), Spence (1973) and Stiglitz (1975). A number of works have been devoted to the analysis of higher education costs and the way to avoid their crowding-out effect on modest families (e.g., Caucutt & Kumar, 2003, Akyol & Athreya, 2005, Gilboa & Justman, 2009). Another strand of literature has focused on the tightness of admission and selection procedures. Gary-Bobo & Trannoy (2008) explain the concomitance of admission rules and tuition fees by double-sided asymmetric information in the university enrolment process. Distinguishing admission requirements from graduation requirements, Gilboa & Justman (2005) showed that a more lenient admission without change in graduation requirements promotes earnings equality but reduces intergenerational mobility. This result is obtained within a model where personal ability combines family backgrounds and a random component. By disentangling family backgrounds and i.i.d. innate personal abilities, Brezis & Hellier (2013) show in contrast that, within a two-tier higher education with standard and elite universities, highly selective admission to elite establishments results in permanent social stratification with low intergenerational mobility and large self-reproduction of the upper class. Finally, to our knowledge, no theoretical model of the French system of '*Grandes écoles*' has been proposed so far.

In contrast with the economic theory, sociology has for a long time analysed the impact of education systems upon social structures, stratification and mobility. These analyses were initiated in the early XXth century by Durkheim and Weber. We mention here a limited number of works which are relevant for the approach developed in this paper.

An essential distinction is made by Turner (1960) between comprehensive education systems that bring a large proportion of children to the education level necessary to integrate the highest positions, and selective systems that recruit a limited number of the best students to enter the upper class. In the same vein, Hopper (1968) distinguishes different levels of education stratification based on the selection and differentiation processes.

Kerckhoff (1995) suggests that the impact of family backgrounds could be magnified when the education system is highly stratified and selective. This argument has been confirmed by a number of empirical works (Hanushek & Woessmann, 2006; Marks et al., 2006; Pfeffer, 2008; Dunne, 2010; Dronkers et al., 2011). Based on the PISA surveys, most of these analyses are however centred on the education system up to secondary school.

The above-mentioned sociological literature focuses on differentiation and selectivity of admission procedures in education as key elements of the elite self-reproduction, and therefore of mobility at the top of the social ladder. The model developed in this paper tackles similar issues.

2.1.2. Empirical works

There is a large empirical literature on intergenerational mobility.[4] We shall limit our brief presentation to the main methods and the key findings of these works.

As regards the measurement of intergenerational mobility, two major methods have typically been utilised by economists and sociologists.

Following Solon (1992) and Zimmerman (1992), the first is based on the calculation of the elasticity of earnings or education levels of individuals relative to their parents'. It typically

[4] Reviews of this literature can be found in Solon (1999), Bjorklund & Jantti (2000, 2009), Fields (2008), Causa & Johansson (2009), Black & Devereux (2011), and Chusseau & Hellier (2013).

consists in estimating the equation $y_t = \sum_k \alpha_k x_k + \beta y_{t-1} + \gamma$ where y_t depicts the (log of) earnings or education level of an individual, y_{t-1} that of her parents and x is a vector of control variables. This calculation determines the intergenerational elasticity β (IGE) and the intergenerational correlation, which are the most utilised indicators in the economic literature.[5]

On top of IGEs, sociologists often utilise mobility tables. These are based on the division of the population between different groups (earnings deciles, education levels, social classes, etc.) and mobility is measured by the probability to switch groups from one generation to the next. The mobility matrix $\{a_{ij}\}$ depicts the proportion of individuals in group *i* with parents in group *j*. One indicator usually calculated from mobility matrices is the odds ratio.

Even if the preceding two types of measurement are different in their construction and interpretation, they lead to the same general diagnosis as regards intergenerational mobility:

1) In all countries, family backgrounds have a significant impact on earnings and education

2) The impact of family backgrounds, and thereby intergenerational mobility, considerably differs across countries (Corak, 2013; Blanden, 2013). In advanced countries, Italy, the UK, the US and France display the highest IGEs (between 0.4 and 0.5 for income), which indicates low intergenerational mobility. In contrast, Scandinavian countries and Canada have the lowest elasticities (between 0.15 and 0.25) and thus a rather high mobility.

3) Intergenerational mobility is typically lower at the top of income distribution, and this difference is substantial in France (Raitano et al., 2015), the US and the UK (Björklund et al., 2012, for both countries; Chetty et al., 2014a, for the US; Blanden & Macmillan, 2014 for the UK).

4) Several works suggest that intergenerational mobility has decreased in the last decade in France (Lefranc, 2011, and Ben-Halima et al., 2014), but also in the US and the UK. [6]

Finally, it can be noted that the latter three countries are characterised by elitist tertiary education systems, wherein a limited number of prestigious establishments select a feebly increasing number of students while standard universities have considerably augmented their intakes. This is particularly the case in France with the system of *Grandes écoles.*

---

[5] Some works also calculate the rank-rank slope (slope of the relation that binds the rank of children to that of their parents. E.g., Chetty et al., (2014a).

[6] Aaronson & Mazumder, 2008, for the US; Blanden et al., 2004, 2007, and Nicoletti & Ermisch, 2007, for the UK. In contrast, Breen & Golthorpe (1999, 2001) found no change in mobility in the UK between cohorts born in 1958 and 1970. Chetty et al. (2014b) found no decrease in intergenerational earnings mobility in the US, for cohorts born between 1971 and 1993. In addition, the impact of family income has increased in both the US (Belley & Lochner, 2007) and France (Ben-Halima et al., 2014)

2.2. The French higher education and the '*Grandes écoles*'

Even if the French higher education has experienced a succession of reforms that have considerably modified its structure and size since World War II, the existence of *Grandes écoles* which integrate a limited number of students with highly selective admissions is a persistent characteristic of this system.

The French higher education can be broadly divided into three types of study which all necessitate the prior obtaining of the *baccalauréat* ('bac').[7]

Short vocational studies (BTS, DUT, DEUST) deliver purely professional and technical degrees that are obtained in two years. There is a selection to entry and the two-year degree can now be extended to a third year sanctioned by a vocational bachelor level.

The University is opened to anyone having obtained the bac and is comprised of three cycles. The *licence* (bachelor's level) is obtained after 3 years, followed by the *master* (2 years) which can give access to the *doctorat* (PhD, 3 years). There are exams to pass from one year to the next, with additional admission procedures to enter the master level and to register at the *doctorat* level. About one third of the students entering the university are eliminated after one or two years at the licence level.

The *Grandes écoles* are highly selective tertiary establishments that aim at producing the French elite. Even if the first Grande école was created in the late XVIth century, their development goes back to the French revolution and nineteenth century with the creation of the major *Grandes écoles d'ingénieurs*. Their purpose was to provide the state and the country with highly skilled specialists necessary for economic and military purposes.

The creation of the *Grandes écoles* was initially justified by the promotion of meritocracy and personal aptitudes. A lightening example is that of l'ENA, which was created after World War II to escape from a situation in which the appointment to the highest public positions was discretionarily decided by politicians through personal and family links. The basic idea was that anonymous exams erase social, personal and family determinants and favours capacities and work. In addition, by limiting the amount of admissions to the amount of available positions, this should prevent the restoration of personal links in the filling of posts among the admitted candidates.

There are now two major types of GEs, business schools and engineering schools, both leading to top executive positions. In addition, 'Science Po' and l'ENA (*Ecole Nationale*

---

[7] Medical studies, architecture, accounting studies and a few other fields have specific shapes. The baccalauréat is the final degree that sanctions the completion of secondary school. Presently, about 70% of a generation obtain one of the 3 types of bac (general, technical and professional), whereas they were about 15% fifty years ago.

d'Administration) aim at training high level civil servants, and the ENS (*Ecole Normale Supérieure*) top researchers.

The GEs display two essential characteristics. First, their admission is highly selective and operates through *concours*. This means that each GE decides for its number N of intakes and the N candidates with the highest marks at the entry exam can join the GE. Hence, admission is not based on a minimum level, but on a pre-determined number of intakes. The normal way to integrate a GE is to enter first a '*classe préparatoire*' that prepares the candidates to the exams during two years (typically more because few are admitted the first time they apply). The *classes préparatoires* are themselves very selective. This selective procedure has permitted to maintain a narrow number of intakes in the GEs whereas the number of students in tertiary education has been multiplied by more than 3 since the early seventies in France. From a true sample of the French employed population, BenHalima et al. (2014) find that those with a GE degree moved from 2.78 percent in 1977 to 2.82 percent in 2003, whereas those with a tertiary education degree (higher than the bac) increased from 12 to 31 percent. This recruitment is even tighter when considering the most prestigious GEs. Albouy & Wanecq (2003) define the '*Très Grandes écoles*' (Top GEs), which are the most prestigious leading to the highest top executive and public positions. They show that, for men, the share of a generation entering a top GE decreased from 0.8 for the generations born between 1929 and 1938 to 0.6% for those born between 1959 and 1968. For the same generations, the share of those entering a *grande école* (but not a top one) increased from 2.3% to 3.2%, and the share of those completing tertiary education was multiplied by more than 3.5.

The second specificity of the GEs is the level of their per-student expenditures, which is significantly higher than that of universities. There are to our knowledge no yearly standardised data permitting to compare per-student expenditure in the GEs and the universities. We however have data on per-student expenditures for several GEs that can be compared to per-student expenditure in universities published every year by the French ministry of education. The Observatoire Boivigny[8] reports that, in 2002, the per student budget is of 50,380 Euro for the *Ecole des Mines*, 50,000 Euro for the ENA, 24,000 Euro for HEC (one of the most prestigious business schools), 19,000 Euro for the *Ecole Centrale Paris*, 12,600 Euro for *Sciences-Po Paris*, against less than 7000 Euro on average in universities. In 2013, the first 20 engineering GEs reported by *l'Usine Nouvelle*[9] gather

[8] http://www.boivigny.com/Le-budget-des-etablissements_a25.html
[9] http://www.usinenouvelle.com/comparatif-des-ecoles-d-ingenieurs-2013

19,300 students with an average per-student expenditure of 48,500 Euro (respectively 65,136 students and 33,800 Euro for the first fifty), against 1.5 million students and a per student expenditure of 11,000 Euro in Universities (MENESR DEPP/ Compte de l'éducation). Even if those data are not fully standardised, the differences are substantial and indicate per-student expenditures that are at least three times higher in the GEs compared to the universities.

In the stylised model built in the next section, we shall focus on the two major types of tertiary establishments, namely, the universities and the GEs. We shall consequently assume that the French GE system of higher education is composed of two branches that differ in their admission procedures and budgets.

3. The model

The approach aims at comparing the GE system with a two-level unified higher education system. We therefore model each system and analyse their respective impacts on intergenerational mobility and educational efficiency.

In the GE system, there are two types of establishments, *Grandes écoles* and universities, with different admission procedures which both take place at the end of basic education. In addition, the per-student expenditure is higher in the GEs.

The benchmark to which the GE system is compared is a two-level unified higher education system. Contrary to the GE system, this structure is comprised of one type of establishment only, universities, but these have two levels, U1 and U2. Hence, there is a top level in the unified system as well, but the admission to the top is based on the human capital at the end of the first level U1 which is common to everyone admitted in tertiary education.

We assume overlapping generations with each individual having one child, and a constant number of dynasties (successive generations linked by a parent-child relationship) normalised to 1. The dynasties are initially (generation 0) continuously distributed over a bounded human capital interval. The individual of the t-th generation of dynasty i is denoted ‘individual (i,t)’.

Individuals accumulate human capital through education, and education is comprised of two phases, i.e., basic and higher education.

Being young (child), all individuals receive the same basic education and their needs are provided by their parents. At the end of basic education, individual (i,t) has accumulated a human capital level denoted h_{it}^{B} and she becomes an adult. She then lives one period of time and chooses whether to pursue further education or to join directly the labour market.

When completing her overall education (basic education, or one of the higher education opportunities described below), individual (*i*,*t*) possesses the final human capital level h_{it}. Then, she spends the whole of her remaining time working.

Prior to education decisions, individuals are heterogeneous and they differ in two respects:

1) Their family backgrounds which encompass the influence of intra-family human capital externalities and transfers and act through several channels: intra-family direct transmission of human capital, intra-family transmission of capacity to learn, information about the best education strategy, affiliation with influential networks etc. All these intra-family externalities and transfers are directly linked to the parent's human capital h_{it-1}.

2) Their personal innate aptitude[10], a_{it} for individual (*i*,*t*), which are independent from family backgrounds (Maoz & Moav, 1999, and Lochner, 2004 for models with the same assumption) and randomly distributed across individuals within each generation inside the segment $\left[\underline{a},\overline{a}\right]\subset\mathbb{R}_*^+$.

In summary, the couple of attributes (family background, personal aptitude), i.e. $\left(h_{it-1},a_{it}\right)$, fully defines individual (*i*,*t*), and her educational decision will be based on these attributes, on the cost of education and on the shape of the education system.

3.1. Basic education

The State provides all individuals with basic education. The individual's human capital at the end of basic education, h_{it}^B, depends on three elements: 1) her family background h_{it-1}; 2) her personal innate aptitude a_{it}; 3) the expenditure on basic education, which is depicted by coefficient δ_B, assumed to be proportional to the per-pupil public expenditure.

The human capital at the end of basic education h_{it}^B is given by the function:

$$h_{it}^B=\delta_B {a_{it}}^{\beta}(h_{it-1})^{\eta}, \qquad \text{with } 0<\beta<1,\ \ 0<\eta<1 \tag{1}$$

3.2. Two higher education systems

We consider two systems of higher education. In the first called 'GE system', there are two types of establishments, GEs (*G*) and universities (*U*), with two different admission procedures that both take place at the end of basic education. In addition, the per-student expenditure is higher in GEs than in universities.

---

[10] We select the tem 'aptitude' rather than 'ability' because, in a number of works, ability encompasses both the family backgrounds and a randomly distributed element (Becker & Tomes, 1979, 1986; Gradstein et al., 2005).

The second higher education structure is called ‘unified system’ and characterised by one type of establishment only, universities. Universities do not differ in their quality but they combine two successive levels of studies. There is one admission procedure at the end of basic education to enter the university level 1 (U1), and an additional admission procedure at the end of level 1 to enter level 2 (U2). We finally assume to simplify that the time spent in higher education is institutionally determined, and this time is φ_1 for the university in the GE system and for U1 in the united system, and φ_2 for a GE and for achieving U2, with $\varphi_2 > \varphi_1$.

There are thus two levels in each system. The lower level is university in the GE system and U1 in the unified system, and the higher level is G in the GE system and U2 in the unified one. In both systems, the education functions that define the human capital achievement at the end of each level depend on three determinants:

1) The human capital attained by the individual at the end of basic education.
2) The individual’s personal aptitude.
3) The public expenditure on the type of higher education, $\delta_j, \quad j = G, U, U1, U2$.

3.2.1. The GE system: Grandes écoles versus Universities

The GE system is a simplified and stylised model of the French higher education structure.

There are two types of establishments, *Grandes écoles* and Universities.

Given the low tuition fees in the French tertiary education, we suppose that the only cost of further studies is the opportunity cost linked to the time spent in higher education.

To enter the university, a child must have a minimal human capital $\underset{\sim}{h}$ at the end of basic education.

To enter a GE, on top of having the minimal human capital $\underset{\sim}{h}$, one must belong to the $\alpha < 1$ children with the highest human capital at the end of basic education. We suppose that α is sufficiently small so that a limited number of children among those having attained $\underset{\sim}{h}$ at the end of basic education can enter a GE.

Let us rank the children by increasing order of human capital at the end of basic education. Then, there is at each generation t a unique human capital value $h_{\alpha,t}$ such that there are α children with a human capital higher than or equal to $h_{\alpha,t}$, and hence $1-\alpha$ children with a human capital below $h_{\alpha,t}$, at the end of basic education.

The education functions for each level of the GE system are:

$$h_{it}^{U} = \left(1 + \delta_U a_{it}^{1-\beta}\right) h_{it}^{B}, \qquad \text{if individual } (i,t) \text{ enters the University} \qquad (2)$$

$$h_{it}^{G} = \left(1 + \delta_G a_{it}^{1-\beta}\right) h_{it}^{B}, \qquad \text{if individual } (i,t) \text{ enters a GE} \qquad (3)$$

where δ_j, $j = U, G$, depicts the quality of the j-study, which is directly and positively related to the per-student expenditure on each type of education.

In line with the observed facts exposed in Section 2, we assume that the per-student expenditure is higher in the GE than in the university: $\delta_G > \delta_U$.

Functions (2) and (3) indicate that, on top of the human capital acquired in basic education h_{it}^B, higher education j brings the additional human capital $\delta_j a_{it}^{1-\beta} h_{it}^B = \delta_j \delta_B a_{it} (h_{it-1})^\eta$. This additional skill depends (i) on the already acquired human capital h_{it}^B, on the quality of the j-education δ_j, and (iii) on the individual's aptitude a_{it}. As a consequence, aptitude has a higher relative impact in human capital creation in higher education than in basic education only.

3.2.2. The two-level unified higher education

In the unified higher education, the university is divided into the successive two levels $U1$ and $U2$. As previously, the admission to the first level is conditioned by a minimal human capital attainment $\underset{\sim}{h}$ at the end of basic education. In addition, to be admitted in level 2, a student must have achieved a minimal human capital $\tilde{h}$ at the conclusion of the first level of university, $U1$. The related education functions are:

$$h_{it}^{U1} = \left(1 + \delta_{U1} a_{it}^{1-\beta}\right) h_{it}^B, \qquad \text{if individual } (i,t) \text{ attends } U1 \qquad (4)$$

$$h_{it}^{U2} = \left(1 + (\delta_{U1} + \delta_{U2}) a_{it}^{1-\beta}\right) h_{it}^B, \qquad \text{if individual } (i,t) \text{ attends } U2 \qquad (5)$$

The interpretation of functions (4) and (5) is similar to that of functions (2) and (3).

4. Education choice

We firstly determine the individual's optimal choice without the admission constraints $\underset{\sim}{h}$, α and $\tilde{h}$. We subsequently introduce the admission rules to determine the individual's final decision.

4.1. Optimal choice without admission constraint

Once they have achieved basic education, individuals possess one unit of time they can allocate to working and higher education.

Individuals maximise their lifetime income, which depends on their human capital, on earnings per unit of human capital, and on their working time over their life. We denote w_t the (after-tax) earnings per unit of human capital at the beginning of generation *t*'s adult life. To simplify, we assume an exogenous and constant rate of growth υ of unit earnings.

Consider individual (*i*,*t*) with human capital h_{it}^B at the end of basic education. If she joins directly the labour market, her lifetime earnings are $I_{it}^B = \int_0^1 w_t e^{(\upsilon-r)\theta} h_{it}^B d\theta$, where *r* is the discount factor. In the GE system, her lifetime earnings is $I_{it}^U = \int_{\varphi_1}^1 w_t e^{(\upsilon-r)\theta}(1+\delta_U a_{it}^{1-\beta}) h_{it}^B d\theta$ if she enters a university and $I_{it}^G = \int_{\varphi_2}^1 w_t e^{(\upsilon-r)\theta}(1+\delta_G a_{it}^{1-\beta}) h_{it}^B d\theta$ if she enters a GE. In the unified system, her lifetime earnings is $I_{it}^{U1} = \int_{\varphi_1}^1 w_t e^{(\upsilon-r)\theta}(1+\delta_{U1} a_{it}^{1-\beta}) h_{it}^B d\theta$ if she enters *U*1 only and $I_{it}^{U2} = \int_{\varphi_2}^1 w_t e^{(\upsilon-r)\theta}\left(1+(\delta_{U1}+\delta_{U2}) a_{it}^{1-\beta}\right) h_{it}^B d\theta$ if she pursues both *U*1 and *U*2.

Lemma 1: *In the GE system:*

1) *There is a threshold value of aptitude* a_U *such that individual (i,t) prefers basic education only to the university iif* $a_{it} < a_U$ *, and prefers the university iif* $a_{it} > a_U$ *.*
2) *There is a threshold value of aptitude* a_G *such that individual (i,t) prefers the GE to the university iif* $a_{it} > a_G$ *, and prefers the university to the GE iif* $a_{it} < a_G$ *.*
3) *There is a threshold value of aptitude* $a_{G/B}$ *such that individual (i,t) prefers basic education only to the GE iif* $a_{it} < a_{G/B}$*, and prefers basic education only to GE iif* $a_{it} > a_{G/B}$*.*

with:

$$a_U = \left(\frac{1}{\delta_U} \frac{E^{\varphi_1}-1}{E-E^{\varphi_1}} \right)^{1/(1-\beta)}, \tag{6}$$

$$a_G = \left(\frac{E^{\varphi_2}-E^{\varphi_1}}{\delta_G\left(E-E^{\varphi_2}\right)-\delta_U\left(E-E^{\varphi_1}\right)} \right)^{1/(1-\beta)}, \tag{7}$$

$$a_{G/B} = \left(\frac{1}{\delta_G} \frac{E^{\varphi_2}-1}{E-E^{\varphi_2}} \right)^{1/(1-\beta)}, \tag{8}$$

and: $E \equiv \exp\left[\upsilon - r\right]$.

Proof. Appendix A.

It can be easily verified that $\frac{\partial a_U}{\partial \delta_U} < 0$, $\frac{\partial a_G}{\partial \delta_G} < 0$, $\frac{\partial a_G}{\partial \delta_U} > 0$ and $\frac{\partial a_{G/B}}{\partial \delta_G} < 0$.

Lemma 2: *In the unified system:*

1) *There is a threshold value of aptitude* a_{U1} *such that individual (i,t) prefers basic education only to U1 iif* $a_{it} < a_{U1}$, *and prefers U1 iif* $a_{it} > a_{U1}$.
2) *There is a threshold value of aptitude* a_{U2} *such that individual (i,t) prefers U1 to U2 iif* $a_{it} < a_{U2}$, *and prefers U2 to U1 iif* $a_{it} > a_{U2}$.
3) *There is a threshold value of aptitude* $a_{U2/B}$ *such that individual (i,t) prefers basic education only to U2 iif* $a_{it} < a_{U2/B}$, *and prefers U2 iif* $a_{it} > a_{U2/B}$.

with:

$$a_{U1} = \left(\frac{1}{\delta_{U1}} \frac{E^{\varphi_1} - 1}{E - E^{\varphi_1}} \right)^{1/(1-\beta)} \tag{9}$$

$$a_{U2} = \left(\frac{E^{\varphi_2} - E^{\varphi_1}}{\delta_{U2}\left(E - E^{\varphi_2}\right) - \delta_{U1}\left(E^{\varphi_2} - E^{\varphi_1}\right)} \right)^{1/(1-\beta)} \tag{10}$$

$$a_{U2/B} = \left(\frac{1}{\delta_{U1} + \delta_{U2}} \frac{E^{\varphi_2} - 1}{E - E^{\varphi_2}} \right)^{1/(1-\beta)} \tag{11}$$

Proof. Appendix A.

It can be easily verified that $\frac{\partial a_{U1}}{\partial \delta_{U1}} < 0$, $\frac{\partial a_{U2}}{\partial \delta_{U1}} > 0$, $\frac{\partial a_{U2}}{\partial \delta_{U2}} < 0$, $\frac{\partial a_{U2/B}}{\partial \delta_{U1}} < 0$, and $\frac{\partial a_{U2/B}}{\partial \delta_{U2}} < 0$.

In the unified system, there are two possible cases (Appendix A), namely, $a_{U2} > a_{U2/B} > a_{U1}$ and $a_{U2} < a_{U2/B} < a_{U1}$. The case $a_{U2} < a_{U1}$ corresponds to a high human capital gain of U2 compared to the extra time spent in education when attending U2 (high δ_{U2} compared to $\varphi_2 - \varphi_1$). In this case, it is only the tighter admission to U2 that prevents all the students to enter U2 at the end of U1.

4.2. Admission constraints and final decision

For given personal attributes (personal aptitude a_{it} and family background h_{it-1}), the above-determined optimal choices depend on the expenditures on each cycle (δ_B, δ_U and δ_G in the GE system; δ_B, δ_{U1} and δ_{U2} in the unified system) and thereby on their respective quality. By considering the admission rules, we introduce an additional constraint.

Study-*j* admission rule is effective if there are children who wish to pursue the study *j* and are not admitted. We shall henceforth suppose that all the admission rules are effective. Otherwise, the admission rules would have no impact.

The admission rule to study *j* is fully determining if (i) it is effective and (ii) all the individuals who are admitted wish to pursue *j*. In this case, we can ignore the individuals' education choice and make as if the entry in study *j* is fully determined by the admission rule.

If $a_U < \underline{a}$, then $a_{it} > a_U$, $\forall it$, and everyone prefer the university to basic education only. Then the admission rule $h_{it}^B > \underset{\sim}{h}$ is fully determining.

If $a_U > \bar{a}$, then $a_{it} < a_U$, $\forall it$, and everyone prefer basic education only to the university. The admission rule $h_{it}^B > \underset{\sim}{h}$ is not effective since no one wants enter the university. This case is inappropriate by assumption.

In what follows, we shall consider the most usual case in which $\underline{a} < a_U < \bar{a}$ and $\underline{a} < a_{U1} < \bar{a}$. Individual (*i,t*) wishes to enter the university if $a_{it} \geq a_U$ and she is admitted if $h_{it}^B \geq \underset{\sim}{h}$. So, there are both individuals who wish to enter the university ($a_{it} > a_U$) and are not admitted $(h_{it}^B < \underset{\sim}{h})$ and individuals who could be admitted ($h_{it}^B \geq \underset{\sim}{h}$) but do not enter the university $(a_{it} < a_U)$. The latter combine a high family background with a low aptitude.

a) GE system

To wish to attend the university and be admitted, one must combine an aptitude $a_{it} \geq a_U$ and a human capital at the end on basic education $h_{it}^B = \delta_B a_{it}{}^{\beta} (h_{it-1})^{\eta} \geq \underset{\sim}{h} \Leftrightarrow h_{it-1} \geq \left(\underset{\sim}{h} / \delta_B\right)^{1/\eta} a_{it}{}^{-\beta/\eta}$. The relation $h_{it-1} = A_U(a_{it}) = \left(\underset{\sim}{h} / \delta_B\right)^{1/\eta} a_{it}{}^{-\beta/\eta}$ defines the *admission function* to the university in the GE system. In the map (a_{it}, h_{it-1}), all individuals located above the admission curve $A_U(a_{it})$ fulfil the admission condition to enter the university, and all those below cannot enter the university.

The number of children admitted to the GE is α. We shall further assume that all the individuals who are admitted to a GE at the end of basic education do enter a GE, i.e., that the conditions $a > a_G$ and $a > a_{G/B}$ are fulfilled for all the individuals who belong to the best α in terms of human capital at the end of basic education. Given the very limited amount of students selected at the entry of the GEs, this assumption is justified. Hence, all the individuals with a basic education $h_{it}^B > h_{\alpha,t} \Leftrightarrow h_{it-1} > \left(h_{\alpha,t} / \delta_B\right)^{1/\eta} a_{it}{}^{-\beta/\eta}$ enter a GE. The relation $h_{it-1} = A_G(a_{it}) = \left(h_{\alpha,t} / \delta_B\right)^{1/\eta} a_{it}{}^{-\beta/\eta}$ defines the *admission function* to the GE. In the map

(a_{it}, h_{it-1}), all individuals located above the admission curve $A_G(a_{it})$ fulfil the admission condition and enter the GE, and all those below cannot enter the GE.

Figure 1 depicts the distribution of individuals (defined by their attributes (h_{it-1}, a_{it})) in generation *t* between the three types of study (basic education only, university and GE).[11] The parents' human capital (generation *t*-1) is distributed between $h_{t-1}^{\min}$ and $h_{t-1}^{\max}$. The admission curve A_U separates the individuals who fulfil the admission to the university (above A_U) from those who do not (below A_U). Similarly, the admission curve A_G separates the individuals who fulfil the admission threshold to the GE (above A_G) from those who do not (below A_G). As all the admitted enter the GE, the lowest aptitude of the students who can enter the GE is higher than a_G.

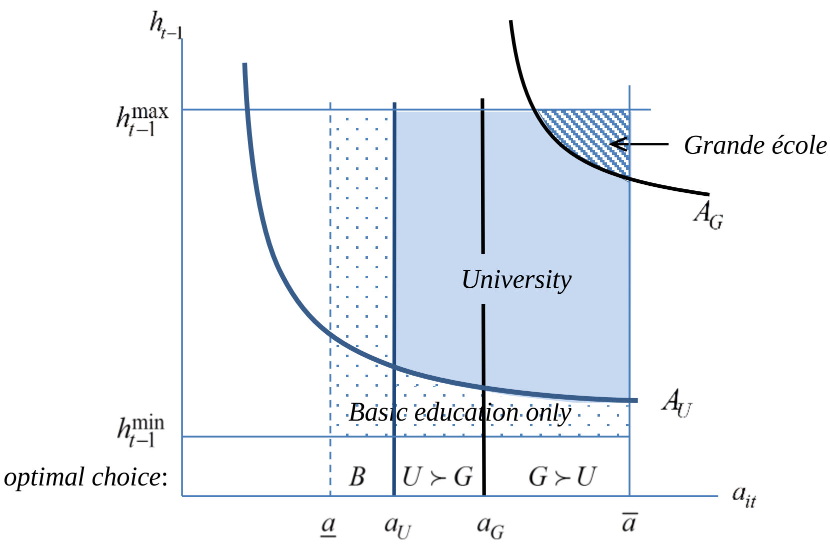

Figure 1. Distribution of students in the GE system

In Figure 1, the dotted surface depicts the set of individuals who do not enter tertiary education ($a_{it} < a_U$ and/or $h_{it}^B < \underset{\sim}{h}$), the dimmed surface those who enter the university ($a_{it} \geq a_U$ and $\underset{\sim}{h} < h_{it}^B < h_\alpha$), and the lined surface depicts those who enter a GE ($h_{it}^B > h_\alpha$).

b) Unified system

In the unified system, the conditions for an individual to attend U1 are similar to those for attending the university in the GE system. She must firstly wish to enter U1 and secondly attain a human capital higher than $\underset{\sim}{h}$. However, individuals may dislike U1 and nevertheless wish to enter U1 because this is a prerequisite to enter U2. This is the case when, for the

---

[11] We limit our presentation to the case $a_G > a_U$, the analysis of case $a_G < a_U$ being similar.

individual, we have ($\succ$ means '*is preferred to*'): $U2 \succ B \succ U1$. We shall thus distinguish the two cases, namely, $a_{U1} < a_{U2/B} < a_{U2}$ and $a_{U2} < a_{U2/B} < a_{U1}$.

As in the GE system, the admission rules define two admission functions with their related curves. The admission function to $U1$, $A_1(a)$, is identical to $A_U(a)$ in the GE system: $A_1(a) = \left(\underset{\sim}{h} / \delta_B\right)^{1/\eta} a^{-\beta/\eta}$. The admission function to U2 is different because the admission threshold $\tilde{h}$ applies to the human capital attainment at the end of U1. To enter U2, individual (i,t) must fulfil the admission condition $h_{it}^{U1} = \left(1 + \delta_{U1} a_{it}^{1-\beta}\right) \delta_B a_{it}^{\beta} (h_{it-1})^{\eta} \geq \tilde{h}$, i.e.,

$$h_{it-1} \geq \left(\frac{\tilde{h} / \delta_B}{1 + \delta_{U1} a_{it}^{1-\beta}} \right)^{1/\eta} a_{it}^{-\beta/\eta}.$$

This defines the admission function $h_{it-1} = A_2(a_{it})$ with

$$A_2(a_{it}) = \left(\frac{\tilde{h} / \delta_B}{1 + \delta_{U1} a_{it}^{1-\beta}} \right)^{1/\eta} a_{it}^{-\beta/\eta}.$$

Both curves $A_1(a_{it})$ and $A_2(a_{it})$ being drawn in the map (a_{it}, h_{it-1}), the individuals above curve A_1 (resp. A_2) are admitted to U1 (resp. U2), and all those below A_1 (resp. A_2) are not admitted to U1 (resp. U2).

Figure 2 depicts the distribution of individuals between the different studies when $a_{U1} < a_{U2/B} < a_{U2}$. The dotted surface depicts the set of individuals who do not enter the university ($a_{it} < a_{U1}$ and/or $h_{it}^{B} < \underset{\sim}{h}$), the dimmed surface those who attend U1 only ($a_{U1} \leq a_{it} < a_{U2}$, $h_{it}^{B} > \underset{\sim}{h}$ and $h_{it}^{U1} < \tilde{h}$), and the lined surface depicts those who enter U2 ($h_{it}^{B} \geq h_{\alpha}$ and $a_{it} \geq a_{U2}$).

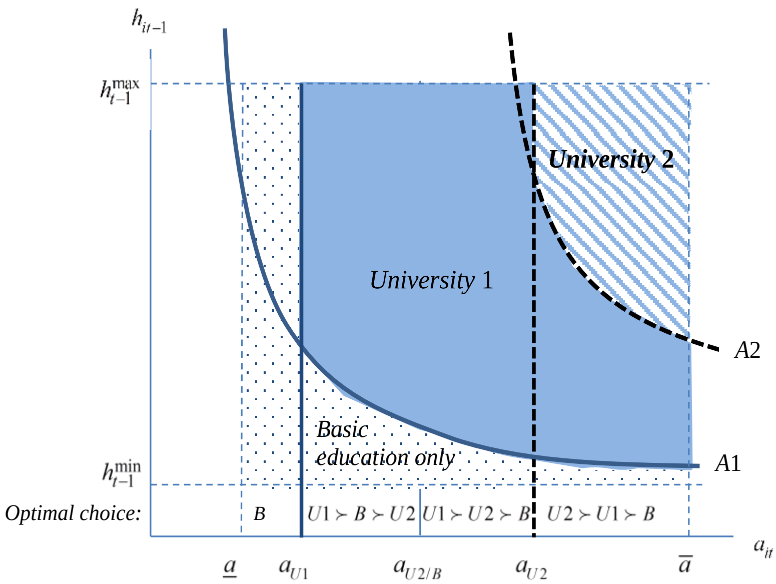

Figure 2. Distribution of students in the unified system with $a_{U1} < a_{U2/B} < a_{U2}$

Figure 3 depicts the distribution of individuals between the different studies and education groups when $a_{U2} < a_{U2/B} < a_{U1}$.

The curves A1 and A2 are identical to those in Figure 2 but the optimal choice corresponding to abilities differs.[12] As previously, the dotted surface depicts the set of individuals who do not enter the university, the dimmed surface those who attend U1 only, and the lined surface depicts those who enter U2. It must be noted that this situation leads to a large proportion of individuals with basic education only, except when a_{U1} is not far from $\underline{a}$. This is because all individuals with aptitude below a_{U1} prefer basic education to U1, which substantially restricts the number of those pursuing U1 only.

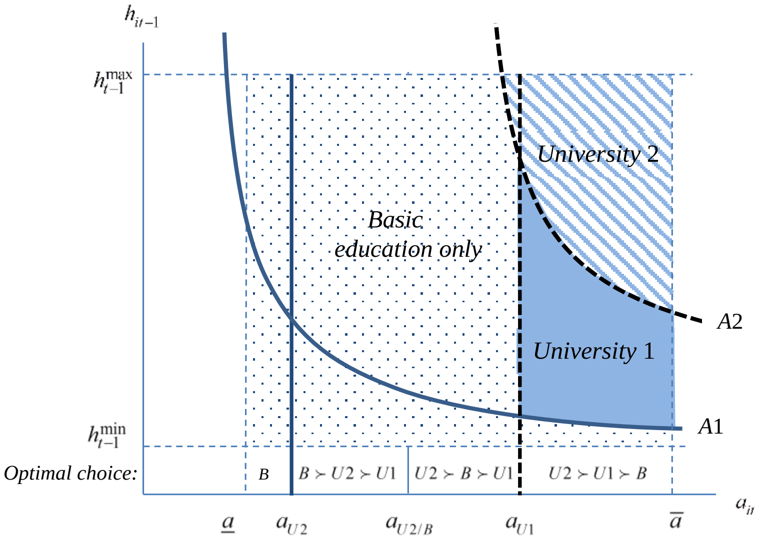

Figure 3. Distribution of students in the unified system with $a_{U2} < a_{U2/B} < a_{U1}$.

Figures 1, 2 and 3 depict the individuals' final educational attainments resulting from their choice subject to three constraints: the education functions, the education (opportunity) cost and the admission rules. In both education systems, these educational choices distribute the individuals in three skill groups (SGs):

Definition 1. We call:

1) *Low skill group* (low SG) the individuals who have a basic education only.
2) *Middle skill group* (middle SG) the individuals who have a university (in the GE system) or a U1 (in the unified system) degree.
3) *Upper skill group* (upper SG) the individuals who have a GE (in the GE system) or a U2 (in the unified system) degree.

[12] $a_{it} < a_{U2} < a_{U2/B} < a_{U1} \Rightarrow B \succ U1 \succ U2$; $a_{U2} < a_{it} < a_{U2/B} < a_{U1} \Rightarrow B \succ U2 \succ U1$; $a_{U2} < a_{U2/B} < a_{it} < a_{U1} \Rightarrow U2 \succ B \succ U1$; $a_{U2} < a_{U2/B} < a_{U1} < a_{it} \Rightarrow U2 \succ U1 \succ B$.

It can be highlighted that two types of effects determine the individuals’ educational achievement and social group membership. The incentive effect governs the individual’s choice regardless of the admission rules, and the admission effect determines the selection to entry in the different types of studies regardless of the individuals’ personal choices.

5. Aptitude, family background and educational attainment

The purpose of this section is to compare the GE system and the unified system as regards two major outcomes, i.e., (i) the respective weights of personal aptitude and family backgrounds in educational attainment and (ii) the human capital of the whole population and of the upper skill group. The results exposed here are valid at any generation. They thereby concern both the short and the longer terms.

5.1. Personal aptitude vs. family backgrounds

We analyse the respective impact of personal aptitude and family background on the capacity to attain the highest level, i.e., G in the GE system and U2 in the unified system. This restriction to the case of the highest level is logical provided that the only structural difference between the two systems is the admission rule to this level.

Definition 2. Consider two stratified education systems, S1 and S2, both comprising a basic education and a two-tier tertiary education with a lower and a higher level, with the same number of admitted to the higher level, and which only differ in their admission rules to enter the higher level.[13] Then, by assuming a given cross-distribution (personal aptitude, family background) between individuals:

1) System S1 is <u>*strictly aptitude-biased*</u> compared to S2 if *all* the students admitted to the higher level in S1 and not in S2 have a greater personal aptitude a_{it} than *all* the students admitted to the higher level in S2 and not in S1.
2) System S1 is <u>*strictly family-biased*</u> compared to S2 if *all* the students admitted to the higher level in S1 and not in S2 have a greater family background h_{it-1} than *all* the students admitted to the higher level in S2 and not in S1.

Our definitions of the aptitude and family biases are very strict. A less restrictive approach could define the aptitude (family) bias as a situation in which the ordered vector of aptitudes

[13] This signifies that (i) the education functions are the same, (ii) the expenditures on each study are identical, and (iii) the admission rule to the lower level of tertiary education is the same, in both systems. The sole admission rules to the higher level of tertiary education differ between the two systems.

(family background) of students admitted to the higher level in S1 is greater than the ordered vector of aptitudes (family backgrounds) of students admitted to the higher level in S2. An even less restrictive definition could be based on the average aptitude (family background) in each set of admitted students. In our definition, S1 is aptitude-biased (family-biased) compared to S2 if its adoption entails that all the new admitted possess a greater aptitude (family background) than the most able of the students they replace.

Proposition 1. *The Unified system is strictly aptitude-biased compared to the GE system, and the GE system is strictly family-biased compared to the unified system.*

Proof. Appendix B.

The combination of both findings described in Proposition 1 shows that, for similar admission tightness (same $\underset{\sim}{h}$ and same number of intakes in G and U2) and similar expenditures (same δ_B, $\delta_U = \delta_{U1}$ and $\delta_G = \delta_{U1} + \delta_{U2}$), the GE system favours family background at the expense of personal aptitude whereas the unified system favours personal aptitude at the expense of family backgrounds, for the entry to the highest level.

5.2. Human capital level and efficiency

So as to focus on the sole divergence in structure between the two systems, i.e., on the impact of the difference in the shape of admission to the highest level (G and U2), we shall assume:

1. Similar admission tightness (same $\underset{\sim}{h}$ and same number of intakes in G and U2) in both systems.

2. Similar expenditures in both systems (same $\delta_B, \delta_U = \delta_{U1}$ and $\delta_G = \delta_{U1} + \delta_{U2}$).

3. An identical cross-distribution of the attributes (personal aptitude, family background) across individuals.

Proposition 2. *Assume similar admission tightness and similar expenditures in both systems, and an identical cross-distribution of attributes* (a_{it}, h_{it-1}) *between individuals. Then:*

1) *The individuals in the upper SG have a higher human capital in the unified system than in the GE system.*
2) *The total (and average) human capital level is higher in the unified system than in the GE system.*

Proof. Appendix C.

Proposition 2 shows that, with similar admission tightness and expenditures, the GE system does not only lower the total human capital attainment, but it also lessens the human capital attainment of the upper skill group. As these results are obtained by assuming identical expenditures, this leads to the diagnosis that, compared to the unified system, the GE system is inefficient in terms of both the general human capital attainment of the economy and the human capital attainment of the most skilled. It should be noted that, once again, this inefficiency results from the early selection which characterises the GE system. As the admission to the GE is decided at the end of basic education, some student with a high aptitude and a rather low family background are rejected from the GE albeit they would reach higher human capital at the end of the GE study than some admitted who have a lower aptitude and higher family background.

6. Stratification and Mobility

We successively analyse (i) the impact of each higher education system on the mobility between skill groups, and (ii) the impacts of structural shifts (in admission rules and expenditures) on between-group mobility and the size of each skill group. In the latter analysis, we make a difference between the short term and the longer term. The short term approach considers the generation in which the shift is implemented and the longer term approach analyses the characteristics of the long term stratification generated by education systems. This distinction is necessary because structural shifts do not impact family backgrounds in the short term but they do in the longer term. In addition, we shall focus on the sole structural shifts which lead to notably different outcomes in the two systems.

6.1. Education systems and between-group mobility

We call upward mobility the number of moves from one skill group to another skill group with higher human capital attainment, and downward mobility the opposite number of moves. As the number of individuals per generation is given and normalised to 1, the number of moves also depicts the percentage of each move in one generation's population.

As previously, we suppose similar admission tightness (same $\underset{\sim}{h}$ and same number of intakes in G and U2) and similar expenditures (same δ_B, $\delta_U = \delta_{U1}$ and $\delta_G = \delta_{U1} + \delta_{U2}$) in both systems, and an identical cross-distribution of the attributes between individuals. We finally suppose that upward mobility exists at least in the unified system.[14] The following proposition can then be established:

Proposition 3. *The unified system generates more upward mobility to the upper skill group and more downward mobility from the upper skill group than the GE system*.

Proof. Appendix C.

Proposition 3 indicates that more children from the low and middle SG enter the upper SG and thereby more children from the upper skill group move downwards (since the intakes of students in the highest level is given) in the unified system compared to the GE system. In addition, all the individuals who benefit from upward mobility in the GE system also move upwards in the unified system. Hence, social mobility at the top is clearly strengthened in the unified system. This is a logical consequence of Proposition 1: as the GE system fosters family backgrounds for the access to the highest level, it *ipso facto* lessens mobility. Finally, as mobility between the low and the middle skill groups is the same in both system (because of similar admission tightness and expenditures), the unified system results in higher between-group mobility.

6.2. Structural shifts in the short term

We analyse the influence of structural shifts in the short term, i.e., on the first generation which is impacted. Consequently, family backgrounds are given and they are not modified by the shifts. Note that, with structural shifts, the assumption of equal intakes of students in the highest level of tertiary education must be waived because the shifts modify the intakes.

There are five possible structural shifts, two related to the admission rules and three to the expenditures on each type of study. We shall however focus on the only shifts the impact of which significantly differs between the two systems in the short term, i.e., changes in the expenditure on basic education δ_B and on the lower level of tertiary education (δ_U or δ_{U1}). The analysis of changes in the other characteristics is available from the author upon request. We can then establish the following two propositions (proofs in Appendix D):

[14] If there is no upward mobility in the unified system, the lack of mobility also applies to the GE system and the systems only differ in the respective weights of aptitude and family background in their intakes of students.

Proposition 4. *An increase in the per-student expenditure on basic education δ_B entails:*

1) In both systems*: an increase in the upward mobility of children from the low SG to the middle SG, a decrease in the downward mobility of children from the middle SG, and a decrease in the size of the low SG.*
2) In the GE system*: No change in the mobility to and from the upper SG, an increase in the size of the middle SG and no change in the size of the upper SG.*
3) In the unified system*: a decrease in the downward mobility of children from the upper SG, an increase in the upward mobility of children from the middle SG, an increase in the size of the upper SG, and an ambiguous impact on the size of the middle SG.*

Proposition 5. *An increase in the per-student expenditure on the first level of tertiary education (δ_U or δ_{U1}) entails:*

1) In both systems*: an increase in the upward mobility of children from the low SG, a decrease in the downward mobility of children from the middle skill group, and a reduction in the size of the low SG.*
2) In the GE system*: an increase in the size of the middle SG and no change in the size of the upper SG.*
3) In the unified system*: no change in the mobility of the upper skill group, an increase in the size of the upper SG and an ambiguous effect on the size of the middle SG.*

6.3. Structural shifts in the long term

Changes in the education structures modify the human capital achievements. Hence, family backgrounds are modified from the generation following that in which the structural shifts are implemented. Consequently, we cannot assume given family backgrounds in the long term.

So as to analyse the impacts of structural shifts in the long run, we firstly show that, for given characteristics, an education system tends towards a long term stratification with well-defined human capital intervals for each skill groups. We subsequently analyse the impacts of changes in the structural characteristics (admission rules and expenditures) on the between-group mobility and the size of each group.

6.3.1. Stratification in the long term

In what follows, we suppose that in both education systems the admission rules are effective but not exclusive, i.e., some individuals who wish to enter the related study are impeded by the admission rule, but not all of them.

Proposition 6. *The education dynamics tend towards a long-term stratification in which:*

1) In the GE system, the lower skill group is inside the human capital segment $S_B = \left[\underline{h}_B, \overline{h}_B \right[$ *, the middle SG in the human capital segment* $S_U = \left[\underline{h}_U, \overline{h}_U \right[$ *and the upper skill group in the segment* $S_G = \left[\underline{h}_G, \overline{h}_G \right]$.

2) In the unified system, the lower skill group is in the human capital segment S_B*, the middle skill in the segment* $S_{U1} = \left[\underline{h}_{U1}, \overline{h}_{U1} \right[$ *and the upper skill in the segment* $S_{U2} = \left[\underline{h}_{U2}, \overline{h}_{U2} \right]$.

Proof. Appendix E. The values $\underline{h}_j$ and $\overline{h}_j$, j = *B, U, U1, G, U2*, are given in the appendix.

The location of all members of a skill group in the education-related segment does not prevent between-group mobility. In fact, the model can generate a large range of configurations as regards mobility in the long term (Appendix E): no mobility at all, the three SGs being fully insulated; mobility between one couple only of adjoining skill groups (2 configurations: low and middle SG or middle and upper SG); mobility between the adjoining SGs only (low and middle SG and middle and upper SG); mobility between all the skill groups. Analysing each of these configurations would be long, fastidious and of little interest (indications are given in Appendix E, and a more comprehensive analysis is available from the author upon request). We shall therefore focus on the most likely and most interesting situation for our subject, i.e., that in which there is mobility between the adjoining skill groups only, i.e., between the low and middle SG on the one hand and between the middle and upper SG on the other hand.

6.3.2. Mobility and sizes of the skill groups in the long term

In the GE system, the selections to enter the university and the GE operate at the end of basic education. Figure 4 depicts the children's human capital at the end of basic education depending on the skill group where they are born.

We denote $\underline{h}_j^B$, $j = B,U,G$, the human capital at the end of basic education of the child of the least skilled parent with a *j*-degree (and belonging to the related SG) provided that this child has the lowest aptitude $\underline{a}$, and $\overline{h}_j^B$, the human capital at the end of basic education of the child of the most skilled parent with a *j*-degree provided that this child has the highest aptitude $\overline{a}$. Then, all the children born in the low SG (*j* = *B*) have a human capital at the end of basic education inside segment $S_B^B = \left[\underline{h}_B^B, \overline{h}_B^B \right[$, all those born in the middle SG (*j* = *U*) have a human capital at the end of basic education inside segment $S_U^B = \left[\underline{h}_U^B, \bar{h}_U^B\right[$, and all the children born in the upper SG (j = G) possess a human capital at the end of basic education in segment $S_G^B = \left[\underline{h}_G^B, \bar{h}_G^B\right]$. The limit values of the three segments are described in Appendix E.

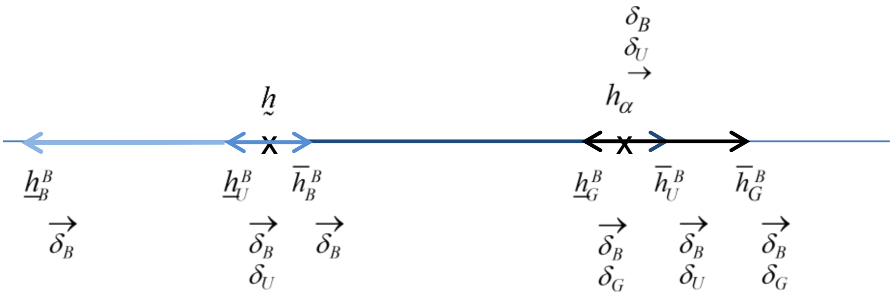

Figure 4. Mobility in the GE system in the long term

As we have assumed that the three skill groups exist and that there is mobility between adjacent skill groups, then S_B^B and S_U^B overlap ($\bar{h}_B^B > \underline{h}_U^B$) as well as S_U^B and S_G^B ($\bar{h}_U^B > \underline{h}_G^B$), and the admission threshold $\underset{\sim}{h}$ must be inside the intersection of segments S_B^B and S_U^B: $\underline{h}_U^B < \underset{\sim}{h} < \bar{h}_B^B$. By construction, threshold h_α is between $\underline{h}_G^B$ and $\bar{h}_U^B$.[15] In Figure 4, the variations of these limits in relation to the three education expenditures are indicated (for instance, $\underline{h}_B^B$ increases with δ_B, and $\underline{h}_U^B$ increases with δ_B and δ_U).

Note that individuals with personal aptitudes below a_U prefer not to enter the university even if their human capital after basic education is above $\underset{\sim}{h}$. In contrast, as we have assumed that all the admitted enter the GE, all the individuals above h_α enter the GE.

In the unified system, the admission to U1 takes place at the end of basic education as in the GE system, but the admission to the highest level U2 now occurs at the end of U1. The impact of changes in the expenditures on the mobility between the low SG and the middle skill group is similar to that determined in the GE system. Figure 5 depicts the individuals' human capital at the end of U1 depending on their families' skill group.

The segment $S_{LM}^{U1} = \left[\underline{h}_{LM}^{U1}, \bar{h}_{LM}^{U1}\right[$ gather the educational attainments of children from the low and middle skill groups at the end of U1, and the interval $S_{U2}^{U1} = \left[\underline{h}_{U2}^{U1}, \bar{h}_{U2}^{U1}\right[$ the educational attainments of children from the upper skill group at the end of U1. The values of

[15] Remember that h_α changes from one generation to the next with the random distribution of aptitudes.

the segments extremities are described in Appendix E. In Figure 5, the variations of the segments extremities with education expenditures on each study are indicated. Note that the students with an education attainment above $\tilde{h}$ and an aptitude lower than a_{U2} refuse to enter $U2$ (because of the incentive effect). These students normally come from the upper SG because the children from the low and middle SGs with a human capital higher than $\tilde{h}$ at the end of $U1$ typically have a high aptitude.

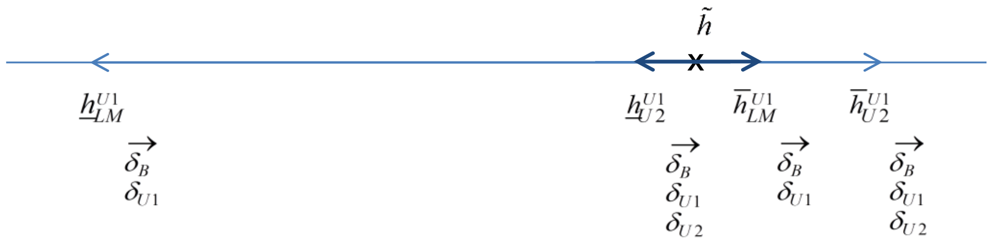

Figure 5. Mobility to the upper SG in the unified system in the long term

From the impacts of the changes in expenditures (δ_j, $j = B, U, U1, G, U2$) on the human capital segments indicated in Figures 4 and 5 and on the aptitude thresholds (a_U, a_{U1}, a_{U2}), we can establish the following three propositions (proofs in Appendix E):

Proposition 7. *An increase in expenditures on basic education (δ_B):*

1) *<u>In both systems</u>: increases the upward mobility of children from the low SG, lowers the downward mobility of children from the middle SG, and lessens the size of the low SG.*
2) *<u>In the GE system</u>: has no impact on the mobility to and from the upper SG and hence on the size of the upper SG and increases the size of the middle SG.*
3) *<u>In the unified system</u>: increases the upward mobility of children from the middle SG, decreases the downward mobility of the upper SG, augments the size of the upper SG, and has an ambiguous impact on the size of the middle SG.*

Proposition 8. *An increase in expenditures on the lower level of tertiary education (δ_U, δ_{U1}):*

1) *<u>In both systems</u>: raises the upward mobility of children from the low SG, decreases the downward mobility of children from middle SG, increases the (absolute) upward mobility of children from middle SG, and lessens the size of the low SG.*
2) *<u>In the GE system</u>: increases the downward mobility of children from upper SG, increases the size of the middle SG and leaves unchanged the size of the upper SG.*
3) *<u>In the unified system</u>: decreases the downward mobility of children from upper SG, raises the size of the upper SG, and has an ambiguous impact on the size of the middle SG.*

Proposition 9. *An increase in the expenditure on the higher level of tertiary education (δ_G and δ_{U2}):*

1) *In both systems: decreases the downward mobility of children from the upper SG.*
2) *In the GE system: decreases the upward mobility of children from the middle SG and leaves unchanged the size of the upper SG.*
3) *In the unified system: increases the upward mobility of children from the middle SG and increases the size of the upper skill group.*

The effects of changes in expenditures substantially differ between the two systems. These divergences are logically explained by the rules of admission to the highest level, i.e., the moment when the selection operates and the condition of admission (given number of intakes in the GE system vs. human capital threshold in the unified system):

1. A rise in expenditure on basic education increases the educational attainment of all individuals without changing the human capital hierarchy across them. Hence, it increases the number of children who overtake both thresholds $\underset{\sim}{h}$ and $\tilde{h}$ in the unified system, increasing thereby the intakes in both higher education levels, U1 and U2. In contrast, it only increases the number of university students in the GE system since the GEs' intakes remain unchanged.

2. A rise in expenditures on the first level of tertiary education boosts the admission to the highest level in the unified system since human capital increases for all students at the end of U1. In contrast, it has no impact on the intake of students in the GE system. In the long term, since parents in the middle skill group benefit from higher human capital, the rise in δ_U increases their upward mobility, which increases the downward mobility of children from the upper SG since the number of intakes in given in the GEs. Note that, if the absolute upward mobility of children from the middle SG rises, this is not the case for the relative mobility (the ratio of children moving upwards to the total number of children from the middle SG) which tends to decrease[16].

3. Finally, a rise in expenditure on the highest level increases its intake of students in the unified system, both through the incentive effect for the children from the middle SG and through the decrease in downward mobility of children from the upper SG (their family background increases and thereby their human capital at the end of U1). In contrast, in the GE system, the decrease in the downward mobility of children from the upper SG (due to higher parents' human capital and hence higher family backgrounds) is obtained at the expense of the upward mobility of children from the middle SG because the GEs' intakes are unchanged.

[16] Proof in Brezis & Hellier (2013).

7. Discussion and conclusion

A stylised model of the French higher education system (called GE system) has been built and compared to a benchmark structure (called unified system) in which there is one type of establishment only, universities, with two successive levels (U1 and U2). The two systems essentially differ in their admission to the highest level. The divergence in outcomes between the two systems is notable:

1. For the admission to the highest level (G or $U2$) the unified system favours personal aptitudes and the GE system family backgrounds, which restrains intergenerational mobility in the latter (Propositions 1 and 3).

2. An increase in the budget for basic education increases the upward mobility to both $U1$ and $U2$ in the unified system, whereas it only increases the intakes in the university, but not in the GEs, in the GE system (Propositions 4 and 7).

3. In the long term, a rise in the budget of the highest level (G or $U2$) lessens mobility in the GE system and it raises the intakes of students in the highest level in the unified system. This makes a larger proportion of the population accede the upper SG in the unified system, whereas it favours the self-reproduction and insulation of elites in the GE system (Prop. 9).

4. The unified system results in higher human capital in the upper skill group and higher average human capital in the population that the GE system (Proposition 2).

In summary, the GE system lessens both mobility and human capital accumulation, and it tends to foster the self-reproduction and insulation of a narrow elite. This is typically not what was intended by its instigators who wished to promote meritocracy and personal aptitudes. The analysis developed here shows that this objective has not been reached and that the GE system has quite the opposite favoured the influence of family backgrounds. Moreover, these results are in line with the empirical evidence that stratified education systems and early selection reduces equality of opportunity (e.g., Horn, 2009).

It must however be emphasized that the pro-family backgrounds and anti-mobility orientation of the GE system has been diagnosed *in relation to* a benchmark system with one type of establishment only and a two-level tertiary education. Is this system representative of what is observed in other countries?

The difference between the GE system and the unified system is essentially twofold. First, the selective admission is made earlier, which favours the influence of family backgrounds. Second, the number of intakes is discretionarily decided and limited by the GEs themselves, whereas intakes are endogenously determined by the number of students with a human capital higher than $\tilde{h}$ at the end of U1 in the unified system. Hence, a general skill upgrading increases the intakes in the highest level in the unified system, but not in the GE system.

There is in fact a simple way to generate a GE system from an apparently unified system. This consists (i) in differentiating the universities in terms of their human capital requirement at the end of basic education ($\underset{\sim}{h}$ differs across universities) and (ii) in increasing the required level for the most prestigious establishments so as to maintain a limited number of intakes. This is what has been observed in the US and the UK. In the US, the admission to colleges depends on one's SAT[17] score at the end of high school. Hoxby (2009) shows that the SAT scores for the admission to colleges have increased in the most selective and decreased in the others from 1962 to 2007. In addition, Su et al. (2012) report that, contrary to the general move, the intake of students has not increased much in the selective colleges.

It can also be noted that the way to prevent the lower human capital attainment of the upper skill group in the GE system consists in increasing the per-student expenditure on the highest education level. This is what is observed in countries like France and the US. We have shown that this leads to an even lower mobility and to the self-reproduction of a narrow elite.

In contrast with the US and UK, higher education is closer to the unified system in Scandinavia and to a lesser extent in Germany. Even if there are differences in quality between universities, (i) these differences are not too large, (ii) the first level (bachelor) is quite comparable across establishments, (iii) the differentiation typically takes place in the superior levels (master or PhD level), (iv) the excellence is distributed across universities depending on the field of study, and (v) the divergence in terms of per-student expenditure is significantly lower than in France, the US and the UK.

From our diagnosis, it is finally possible to question the possible reform of the GE system. The pro-family background and anti-mobility specificity of this system is based on (i) the early selection to enter the GEs, (ii) their admission of a pre-determined number of students and (iii) their high per-student expenditure compared to the university. Hence, an increase in mobility could be achieved by relaxing these features. For instance, replacing the *classes préparatoires* by the university bachelor level, to which a special *concours* opened to the best students with a bachelor degree would be added, could postpone the admission to the GEs and foster the weight of personal aptitude to the detriment of family background, increasing thereby mobility and educational efficiency. By doing this, an increase in the funding on the bachelor level (U1 in our model) would also benefit the GEs' students, which permits then to

[17] Standardised test for college admission in the US implemented at the end of high school.

lessen the expenditure on the GEs. In fact, several steps towards a recruitment by other ways than the traditional *classes préparatoires* have already been opened, but those additional procedures remain limited in most GEs. Obviously, the most direct way would be to replace the GE system by a unified system, but a large part of the French deciders could oppose such a reform because they come from the GEs.

Finally, the scope of our findings can be extended to the analysis of higher education systems in general. The French *Grandes écoles* are typical examples of what Turner (1960) calls a *selective* education system and the GE system developed in this paper provides a synthetic model of such a structure. In addition our two-level unified system can be seen as a framework modelling Turner's *comprehensive* education system, provided that $\tilde{h}$ is not too high. In this respect, our model and results give credence to Kerckhoff's proposition that the impact of family backgrounds is magnified when the education system is highly stratified and selective (Kerckhoff, 1995). And, as usual, higher family backgrounds come with lower intergenerational mobility. Hence, the approach developed in this paper can be interpreted as modelling Turner's distinction and presenting an analytical demonstration of Kerckhoff's proposition.

References

Aaronson, D. and B. Mazumder. 2008. Intergenerational Economic Mobility in the U.S., 1940 to 2000. *Journal of Human Resources,* 43(1), 139-172.

Akyol, A. and K. Athreya. 2005. Risky higher education and subsidies. *Journal of Economic Dynamics and Control*. 29 (6), 979-1023.

Albouy V. and T. Wanecq. 2003. Les inégalités sociales d'accès aux grandes écoles (Social inequalities in the access to the Grandes écoles). *Economie et Statistiques,* 361, 27-52.

Arrow, K. 1973. Higher Education as a Filter. *Journal of Public Economics*, 2, 193–216.

Barham, V., R. Boadway, M. Marchand and P. Pestieau. 1995. Education and poverty trap. *European Economic Review*, 39(7), 1257-75.

Becker G.S. 1964. *Human capital*, Columbia University Press.

Becker, G. and N. Tomes. 1979. An equilibrium theory of the distribution of income and inter-generational mobility. *Journal of Political Economy,* 87(6), 1153-89.

Becker G.S. and N. Tomes. 1986. Human capital and the rise and fall of families. *Journal of Labor Economics*, 4, S1-S39.

Belley, P. and L. Lochner (2007). The Changing Role of Family Income and Ability in Determining Educational Achievement. *Journal of Human Capital*, 1(1), 37-89

Benabou, R. 1993. Workings of a city: location, education and production. *Quarterly Journal of Economics*, 108, 619-652.

Benabou, R. 1994. Human capital, inequality, and growth: A local perspective. *European Economic Review*, 38, 817-826.

Benabou, R. 1996. "Heterogeneity, stratification and growth: macroeconomic implications of community structure and school finance". *American Economic Review,* 86, 584-609.

Ben-Halima B., N. Chusseau and J. Hellier. 2014. Skill Premia and Intergenerational Education Mobility: The French Case. *Economics of Education Review*, 39, 50-64.

Ben-Porath Y. 1967. The Production of Human Capital and the Life Cycle of Earnings. *The Journal of Political Economy*, 75(4), 352-365.

Bertocchi, G. and M. Spagat. 2004. The evolution of modern educational systems. Technical versus general education, distributional conflict, and growth. *Journal of Development Economics*, 73, 559-582.

Björklund, A. and M. Jäntti. 2000. Intergenerational mobility of socio-economic status in comparative perspective. *Nordic Journal of Political Economy*, 26, 3-32.

Björklund, A. and M. Jäntti. 2009. Intergenerational income mobility and the role of family backgrounds, in: W. Salverda, B. Nolan and T.M. Smeeding, *The Oxford Handbook of Economic Inequality*, Oxford University Press, 491-521.

Björklund, A., J. Roine and D. Walderstrom. 2012. Intergenerational top income mobility in Sweden: Capitalist dynasties in the land of equal opportunity. *Journal of Public Economics,* 96, 474-84.

Blanden, J. 2013. Cross-country rankings in intergenerational mobility: A comparison of approaches from economics and sociology. *Journal of Economic Surveys*, 27(1), 38-73.

Blanden, J. and L. Macmillan. 2014. Education and Intergenerational Mobility/ Help or Hindrance? Institute of Education, University of London, Working Paper No 14-01.

Blanden, J., A. Goodman, P. Gregg and S. Machin. 2004. Changes in intergenerational mobility in Britain, in: M. Corak (ed.), *Generational Income Mobility in North America and Europe*, Cambridge: Cambridge University Press, 122-46.

Blanden, J., P. Gregg and L. Macmillan. 2007. Accounting for Intergenerational Income Persistence: Noncognitive Skills, Ability and Education. *The Economic Journal*, 117(519), C43-C60.

Blankenau, W. and X. Youderian. 2015. Early childhood education expenditures and the intergenerational persistence of income. Review of Economic Dynamics, 18, 334–349.

Black, S. and P. Devereux. 2011. Recent Developments in Intergenerational Mobility. In: O. Ashenfelter and D. Card (eds.) *Handbook of Labor Economics,* Elsevier, Amsterdam: North-Holland, 4B, 1487-541.

Brezis, E.S. and J. Hellier. 2013.Social mobility at the top: Why are elites self-reproducing?. ECINEQ working Paper series, 2013-312.

Caucutt, E. and K. Kumar. 2003. Higher Education Subsidies and Heterogeneity: A Dynamic Analysis. *Journal of Economic Dynamics and Control*, 27, 1459-1502.

Causa, O. and Å. Johansson. 2009. Intergenerational Social Mobility, *OECD Economics Department Working Papers*, No. 707, OECD Publishing.

Chetty, R., N. Hendren, P. Kline, E. Saez, and N. Turner. 2014a. Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational mobility. *American Economic Review: Paper & Proceedings*, 104(5), 141-147.

Chetty, R., N. Hendren, P. Kline, and E. Saez. 2014b. Where is the land of opportunity? The geography of intergenerational mobility in the United States. *Quarterly Journal of Economics*, 129(4), 1553-1623.

Chusseau, N. and J. Hellier. 2011. Educational systems, intergenerational mobility and social segmentation. *European Journal of Comparative Economics*, 8(2), 255-86.

Chusseau, N. and J. Hellier. 2013. Education, intergenerational mobility and inequality. In: J. Hellier and N. Chusseau (Eds), *Growing Income Inequalities, Economic Analyses*, Palgrave McMillan, 227-273.

Corak, M. 2013. Income inequality, Equality of Opportunity and Intergenerational Mobility. *Journal of Economic Perspectives*, 27(3), 79-102.

Das, M. 2007. Persistent inequality: an explanation based on limited parental altruism. *Journal of Development Economics*, 84, 251–270.

Driskill, R.A. and A. Horowitz. 2002. Investment in Hierarchical Human Capital. *Review of Development Economics*, 6(1), 48-58.

Dronkers J., R. van der Velden and A. Dunne. 2011. The effects of educational systems, school-composition, track-level, parental background and immigrants' origins on the achievement of 15-years old native and immigrant students. A reanalysis of PISA 2006, ROA Research Memorandum, 2011/6.

Dunne, A. 2010. *Dividing Lines: Examining the Relative Importance of between- and within-School Differentiation during Lower Secondary Education*. Ph.D. European University Institute (Florence).

Durlauf, S. 1994. Spillovers, stratification, and inequality. *European Economic Review*, 38(3-4), 836-845.

Durlauf, S. 1996. A Theory of Persistent Income Inequality. *Journal of Economic Growth*, 1, 75-93.

Galor, O. and D. Tsiddon. 1997. The distribution of human capital and economic growth. *Journal of Economic Growth*, 2, 93-124.

Galor, O. and J. Zeira. 1993. Income distribution and macroeconomics. *Review of Economic Studies*, 60, 35-52.

Gary-Bobo, R. and Trannoy, A. 2008.Efficient tuition fees and examinations. *Journal of the European Economic Association*, 6(6), 1211-1243.

Gilboa, Y. and M. Justman. 2005. Academic admission standards: Implications for output, distribution and mobility. *Journal of the European Economic Association,* 3(5), 1105–1133.

Gilboa, Y. and M. Justman. 2009. University tuition subsidies and student loans: a quantitative analysis, *Israel Economic Review* 7(1), 1–37.

Gradstein, M., Justman, M. and V. Meier. 2005. *The Political Economy of Education, Implications for Growth and Inequality*. The MIT Press, Cambridge, Mass. and London.

Hanushek, E.A. and L. Woessmann. 2006. Does Educational Tracking Affect Performance and Inequality? Differences-in-Differences Evidence across Countries. *Economic Journal*, 116, C63.

Herrington, C.M. 2015. Public education financing, earnings inequality, and intergenerational mobility. *Review of Economic Dynamics*, 18, 822–842.

Hopper, E. I. 1968. A Typology for the Classification of Educational Systems. *Sociology*, 2, 29-46.

Horn, D. 2009. Age of selection counts: a cross-country analysis of educational institutions. *Educational Research and Evaluation*, 15(4), 343-366.

Hoxby, C. 2009. The changing selectivity of American colleges. *Journal of Economic Perspective,* 23(4), 95-118

Kerckhoff, A.C. 1995. Institutional Arrangements and Stratification Processes in Industrial Societies. *Annual Review of Sociology*, 15, 323-347.

Lefranc, A. 2011. Educational expansion, earnings compression and changes in intergenerational economic mobility: Evidence from French cohorts, 1931-1976, Thema Working paper n°2011-11.

Lochner, L. 2004. Education, work and crime: A human capital approach. *International Economic Review*, 45(3), 811-843.

Loury, G. 1981. Intergenerational transfers and the distribution of earnings. *Econometrica*, 49(4), 843-867.

Maoz, Y.D. and O. Moav. 1999. Intergenerational mobility and the process of development. *The Economic Journal*, 109, 677-697.

Marks, G., J. Creswell and J. Ainley. 2006. Explaining Socioeconomic Inequalities in Student Achie-vement: The Role of Home and School Factors. *Education Research and Evaluation*, 12(2), 105-128.

Nicoletti, C. and J. Ermisch. 2007. Intergenerational earnings mobility: Changes across cohorts in Britain. *The B.E. Journal of Economic Analysis and Policy*, 7, Article 9.

Pfeffer, P.T. 2008. Persistent Inequality in Educational Attainment and its Institutional Context. *European Sociological Review*, 25(5), 543-565.

Raitano, M., C. Vittori and F. Vona 2015. The effects of parental background along the sons' earnings distribution: Does one model fit for all? *OFCE Working Paper* 2015-18.

Restuccia, D. and C. Urrutia. 2004. Intergenerational Persistence of Earnings: The Role of Early and College Education. *American Economic Review*, 94 (5), 1354-1378.

Solon, G. 1992. Intergenerational Income Mobility in the United States. *American Economic Review*, 82, 393-408.

Solon, Gary. 1999. Intergenerational mobility in the Labor Market. In: *Handbook of Labor Economics*. O.C. Ashenfelter and D. Card (Eds), Volume 3B. New York: Elsevier Science Press, 1762–1796.

Spence, A. M. 1973. Job Market Signaling. *Quarterly Journal of Economics*, 87, 355–374.

Stiglitz, J. E. 1975. The Theory of 'Screening,' Education, and the Distribution of Income. *American Economic Review*, 65, 283-300.

Su, X. 2004. The allocation of public funds in a hierarchical educational system. *Journal of Economic Dynamics and Control*, 28, 2485-2510.

Su, X. 2006. Endogenous Determination of Public Budget Allocation across Education Stages. *Journal of Development Economics*, 81, 438-456.

Su, X., M. Kaganovich and I. Schiopu. 2012. College expansion and curriculum choice. University of Alberta Working Paper No. 2012-25.

Turner, R.H. 1960. Sponsored and Contest Mobility and the School System. *American Sociological Review*, 25(6), 855-867.

Zimmerman, D. 1992. Regression Toward Mediocrity in Economic Stature. *American Economic Review*,82, 409-429.

Appendix A. Determination of the aptitude thresholds

a) The GE system

$$I_{it}^{B}=\int_{0}^{1} w_t e^{(\upsilon-r)\theta} h_{it}^{B} d\theta = w_t(\upsilon-r)^{-1} h_{it}^{B}\left[e^{(\upsilon-r)\theta}\right]_0^1 = w_t(\upsilon-r)^{-1} h_{it}^{B}\left(e^{(\upsilon-r)}-1\right)$$

$$I_{it}^{U}=\int_{\varphi_1}^{1} w_t e^{(\upsilon-r)\theta}(1+\delta_U {a_{it}}^{1-\beta}) h_{it}^{B} d\theta = \frac{w_t}{\upsilon-r}(1+\delta_U {a_{it}}^{1-\beta}) h_{it}^{B}\left[e^{(\upsilon-r)\theta}\right]_{\varphi_1}^1 = \frac{w_t}{\upsilon-r}(1+\delta_U {a_{it}}^{1-\beta}) h_{it}^{B}\left(e^{(\upsilon-r)}-e^{(\upsilon-r)\varphi_1}\right)$$

$$I_{it}^{G}=\int_{\varphi_2}^{1} w_t e^{(\upsilon-r)\theta}(1+\delta_G {a_{it}}^{1-\beta}) h_{it}^{B} d\theta = \frac{w_t}{\upsilon-r}(1+\delta_G {a_{it}}^{1-\beta}) h_{it}^{B}\left[e^{(\upsilon-r)\theta}\right]_{\varphi_2}^1 = \frac{w_t}{\upsilon-r}(1+\delta_G {a_{it}}^{1-\beta}) h_{it}^{B}\left(e^{(\upsilon-r)}-e^{(\upsilon-r)\varphi_2}\right)$$

By writing $E \equiv e^{(\upsilon - r)}$ and denoting '$\succ$' the relation 'is preferred to':

1) $U \succ B \Leftrightarrow I_{it}^{U} > I_{it}^{B} \Leftrightarrow a_{it} > a_{U} = \left(\frac{1}{\delta_U} \frac{E^{\varphi_1} - 1}{E - E^{\varphi_1}} \right)^{1/(1-\beta)}$

2) $G \succ U \Leftrightarrow I_{it}^{G} > I_{it}^{U} \Leftrightarrow a_{it} > a_{G} = \left(\frac{E^{\varphi_2} - E^{\varphi_1}}{\delta_G \left(E - E^{\varphi_2} \right) - \delta_U \left(E - E^{\varphi_1} \right)} \right)^{1/(1-\beta)}$

3) $G \succ B \Leftrightarrow I_{it}^{G} > I_{it}^{B} \Leftrightarrow a_{it} > a_{G/B} = \left(\frac{1}{\delta_G} \frac{E^{\varphi_2} - 1}{E - E^{\varphi_2}} \right)^{1/(1-\beta)}$

b) The unified system

$$I_{it}^{B} = \frac{w_t}{\upsilon - r} h_{it}^{B} \left[e^{(\upsilon - r)\theta} \right]_0^1 = \frac{w_t}{\upsilon - r} h_{it}^{B} \left(e^{(\upsilon - r)} - 1 \right) = \frac{w_t}{\upsilon - r} h_{it}^{B} (E - 1)$$

$$I_{it}^{U1} = \frac{w_t}{\upsilon - r} (1 + \delta_{U1} a_{it}^{1-\beta}) h_{it}^{B} \left[e^{(\upsilon - r)\theta} \right]_{\varphi_1}^1 = \frac{w_t}{\upsilon - r} (1 + \delta_{U1} a_{it}^{1-\beta}) h_{it}^{B} \left(e^{(\upsilon - r)} - e^{(\upsilon - r)\varphi_1} \right) \frac{w_t}{\upsilon - r} (1 + \delta_{U1} a_{it}^{1-\beta}) h_{it}^{B} \left(E - E^{\varphi_1} \right)$$

$$I_{it}^{U2} = \frac{w_t}{\upsilon - r} (1 + (\delta_{U1} + \delta_{U2}) a_{it}^{1-\beta}) h_{it}^{B} \left[e^{(\upsilon - r)\theta} \right]_{\varphi_2}^1 == \frac{w_t}{\upsilon - r} (1 + (\delta_{U1} + \delta_{U2}) a_{it}^{1-\beta}) h_{it}^{B} \left(E - E^{\varphi_2} \right)$$

1) $U1 \succ B \Leftrightarrow I_{it}^{U1} > I_{it}^{B} \Leftrightarrow a_{it} > a_{U1} = \left(\frac{1}{\delta_{U1}} \frac{E^{\varphi_1} - 1}{E - E^{\varphi_1}} \right)^{1/(1-\beta)}$

2) $U2 \succ U1 \Leftrightarrow I_{it}^{U2} > I_{it}^{U1} \Leftrightarrow a_{it} > a_{U2} = \left(\frac{E^{\varphi_2} - E^{\varphi_1}}{\delta_{U2} \left(E - E^{\varphi_2} \right) - \delta_{U1} \left(E^{\varphi_2} - E^{\varphi_1} \right)} \right)^{1/(1-\beta)}$

3) $U2 \succ B \Leftrightarrow I_{it}^{U2} > I_{it}^{B} \Leftrightarrow a_{it} > a_{U2/B} = \left(\frac{1}{\delta_{U1} + \delta_{U2}} \frac{E^{\varphi_2} - 1}{E - E^{\varphi_2}} \right)^{1/(1-\beta)}$

It can be easily verified that there are only two possible cases, $a_{U2} < a_{U2/B} < a_{U1}$ and $a_{U1} < a_{U2/B} < a_{U2}$, because the others violate transitivity.[18]

Appendix B. Personal aptitude vs. family background

Before demonstrating Propositions 1 and 2, we firstly build Figure B1 on which the proofs will be based. This figure will also be utilised in Appendix C and D.

---

[18] $a_{U1} < a_{U2} < a_{U2/B}$: $a_{U1} < a_{U2} < a_{it} < a_{U2/B} \Rightarrow U1 \succ B,\ U2 \succ U1$ *and* $B \succ U2 \Rightarrow$ *impossible*

$a_{U2} < a_{U1} < a_{U2/B}$: $a_{U2} < a_{U1} < a_{it} < a_{U2/B} \Rightarrow U2 \succ U1,\ U1 \succ B,$ *and* $B \succ U2 \Rightarrow$ *impossible*

$a_{U2/B} < a_{U1} < a_{U2}$: $a_{U2/B} < a_{it} < a_{U1} < a_{U2} \Rightarrow U2 \succ B, B \succ U1$ *and* $U1 \succ U2 \Rightarrow$ *impossible*

$a_{U2/B} < a_{U2} < a_{U1}$: $a_{U2/B} < a_{it} < a_{U2} < a_{U1} \Rightarrow U2 \succ B, U1 \succ U2,$ *and* $B \succ U1 \Rightarrow$ *impossible*

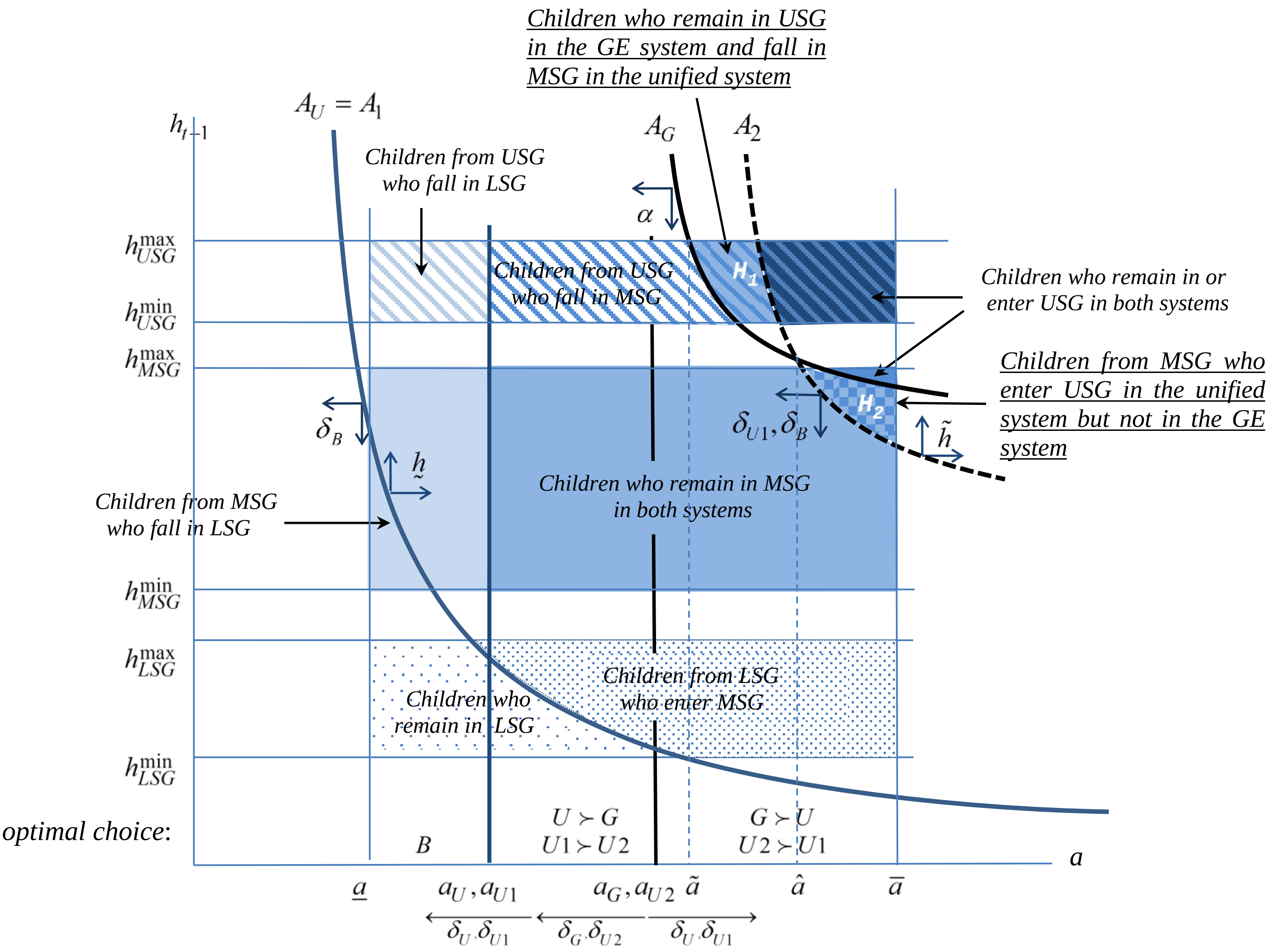

Figure B1. Determination of the individuals' skill groups in both systems

Figure B1 depicts the individuals defined by their two characteristics, personal aptitude a on the x-axis and family background h_{t-1} on the y-axis. The figure identifies the spaces corresponding to each skill group of origin (parents) and each skill group of destination (children).

Both education systems are inserted in Figure B1, with the assumptions indicated in Section 5: 1) similar admission tightness (same $\underset{\sim}{h}$ and same number of intakes in G and U2) in both systems; 2) similar expenditures (same $\delta_B, \delta_U = \delta_{U1}$ and $\delta_G = \delta_{U1} + \delta_{U2}$) in both systems; 3) identical cross-distribution of the attributes (a_{it}, h_{it-1}) across individuals.

The figure is constructed in the case $\underline{a} < a_U = a_{U1} < a_G = a_{U2} < \bar{a}$. Equalities $a_U = a_{U1}$ and $a_G = a_{U2}$ respectively derive from equalities $\delta_U = \delta_{U1}$ and $\delta_G = \delta_{U1} + \delta_{U2}$, given the values a_j, $j = U, U1, G, U2$ provided by relations (6), (7), (9) and (10). Inequality $\underline{a} < a_U = a_{U1}$ means that there are individuals who prefer basic education to the first level of tertiary education and inequality $a_G = a_{U2} < \bar{a}$ that there are individuals who prefer the second level of tertiary education to the first. The case $\underline{a} < a_G = a_{U2} < a_U = a_{U1} < \bar{a}$ is not treated here because it leads to similar results (available from the author upon request).

We suppose that both admission rules (for the entry in the first and second level of tertiary education) are effective (some children are not admitted) and that they do not prevent all the applicants to enter the related study.

The threshold values defining the individuals' optimal choices without admission constraints, a_j, $j = U, U1, G, U2$, are on the *x*-axis. The variation of the thresholds $a_j s$ in relation to the $\delta_j s$ are indicated below each a_j.

The limits corresponding to the parents' human capital (i.e., family backgrounds) and related to each skill group are on the *y*-axis. The parents' human capital are located inside the interval $\left[h_{j-SG}^{\min}, h_{j-SG}^{\max}\right]$ for the parents in the *j*-SG, *j* = low, middle, upper.

To enter the university (in the GE system) or U1 (in the unified system), an individual must be located in the right hand side of $a_U = a_{U1}$ and above the curve $A_U = A_1$. To enter the GE (in the GE system) or U2 (in the unified system), an individual must be located in the right hand side of $a_G = a_{U2}$ and above the curve A_G in the GE system, and A_2 (different from A_G) in the unified system, with $A_U(a) = A_1(a) = \left(\underset{\sim}{h} / \delta_B\right)^{1/\eta} a^{-\beta/\eta}$,

<u>*Proof of Proposition* 1</u>

Let $\hat{a} = \left(\tilde{h} / (1+\delta_{U1}) h_{\alpha,t}\right)^{1/(1-\beta)}$ be the aptitude at which $A_G(a)$ intersects the line $h_{t-1} = h_{USG}^{\max}$.[19]
Let $\mathcal{D}(S)$ be the number of intakes in study $S = U, G, U1, U2$. By assumption $\mathcal{D}(G) = \mathcal{D}(U2)$.
If $\hat{a} < \tilde{a}$, then $A_2(a) < A_G(a), \ \forall a \in \left[\tilde{a}, \bar{a}\right] \Rightarrow$ There are more intakes in *U*2 than in *G*.

If $\hat{a} > \bar{a}$, then $A_2(a) > A_G(a), \ \forall a \in \left[\tilde{a}, \bar{a}\right] \Rightarrow$ There are more intakes in *G* than in *U*2.

Hence, for $\mathcal{D}(G) = \mathcal{D}(U2)$, we must have: $\tilde{a} < \hat{a} < \bar{a}$.

In addition, for $\hat{a} \in \left[\tilde{a}, \bar{a}\right]$, it is straightforward that, for a given value of α and hence h_α, $\Delta(\hat{a}) = \mathcal{D}(U2) - \mathcal{D}(G)$ is a monotonically decreasing function of $\hat{a}$ (i.e., a monotonically decreasing function of $\tilde{h}$) with $\Delta(\tilde{a}) > 0$ and $\Delta(\bar{a}) < 0$. There is hence a unique $\tilde{a} = a^*$, and thereby a unique $\tilde{h}^*$, such that $\Delta(\hat{a}) = 0 \Leftrightarrow \mathcal{D}(U2) = \mathcal{D}(G)$.
The value $\tilde{h}^*$ is the admission threshold to U2 which ensures that $\mathcal{D}(G) = \mathcal{D}(U2)$.

---

[19] $h_{USG}^{\max} = \left(h_{\alpha,t} / \delta_B\right)^{1/\eta} a^{-\beta/\eta} \Rightarrow \tilde{a} = \left(h_{\alpha,t} / \left(\delta_B (h_{USG}^{\max})\right)^{\eta}\right)^{1/\beta}$.

From the admission curves $A_G(a)$ and $A_2(a)$, we define two sets of individuals:

1) Those situated between A_G and A_2 with an aptitude lower than $\hat{a}$ who are admitted to the highest level (*G*) in the GE system but are not admitted to the highest level (*U*2) in the unified system. Those individuals are in the set H1 in Figure B1.

2) Those situated between A_2 and A_G with an aptitude higher than $\hat{a}$ who are admitted to the highest level (*U*2) in the unified system but are not admitted to the highest level (*G*) in the GE system. Those individuals are in the set H_2 in Figure B1.

From Figure B1, it is clear that:

1) All the individuals admitted to *G* but not to *U*2 have a lower aptitude and a higher family background than all those admitted to *U*2 but not to *G*.

2) Corollary: all the individuals admitted to *U*2 but not to *G* have a higher aptitude and a lower family background than all those admitted to *G* and not to *U*2.

Hence, the unified system is strictly aptitude-biased compared to the GE system, and the GE system is strictly family background-biased compared to the unified system.

Appendix C. Human capital level

We show that:

1) *the individuals in the upper SG have a higher human capital in the unified system than in the GE system.*
2) *the total (and average) human capital level is higher in the unified system than in the GE system.*

1) Moving from the GE to the unified system makes a number of individuals who entered G not to enter U2 (individual inside the space H1 in Figure B1) and an equal number of individuals who did not enter G to enter U2 (individuals inside the space H2 in Fig. B1). To establish Proposition 2, we show that, for any pair of individuals $(j,i) \in H_1 \times H_2$, the human capital at the end of the top level of tertiary education is higher for individual *i* than for individual *j*.

Let us consider any individual (*i*,*t*) who enter *U*2 and not *G*, and any (*j*,*t*) who enters *G* and not *U*2. Then: $(1+\delta_{U1}a_{it})h_{it}^B > \tilde{h} > (1+\delta_{U1}a_{jt})h_{jt}^B$ and $h_{jt}^B > h_\alpha > h_{it}^B$, which implies $(1+\delta_{U1}a_{it})h_{it}^B > (1+\delta_{U1}a_{jt})h_{jt}^B$. Then:

$$(1+\delta_{U1}a_{it})h_{it}^B > (1+\delta_{U1}a_{jt})h_{jt}^B \Rightarrow \frac{1+\delta_{U1}a_{it}}{1+\delta_{U1}a_{jt}} > \frac{h_{jt}^B}{h_{it}^B}$$

$$a_{it} > a_{jt} \Rightarrow \frac{1+(\delta_{U1}+\delta_{U2})a_{it}}{1+(\delta_{U1}+\delta_{U2})a_{jt}} > \frac{1+\delta_{U1}a_{it}}{1+\delta_{U1}a_{jt}} > \frac{h_{jt}^B}{h_{it}^B}$$

$$\Rightarrow (1+(\delta_{U1}+\delta_{U2})a_{it})h_{it}^B > (1+(\delta_{U1}+\delta_{U2})a_{jt})h_{jt}^B \Rightarrow (1+(\delta_{U1}+\delta_{U2})a_{it})h_{it}^B > (1+\delta_G a_{jt})h_{jt}^B$$

Consequently, any individual (i,t) who enter $U2$ and not G has a higher human capital at the conclusion of $U2$ than any (j,t) who enters G and not $U2$ at the end of G. As the number of individual (i,t) who enter U2 and not G is equal to the number of those entering G and not $U2$, the human capital in the upper SG is higher in the unified system than in the GE system.

2) All individuals have the same human capital in both systems, except those in the spaces H_1 and H_2 (Fig. B1). Let K be the amount of human capital in both systems outside $H_1 \cup H_2$. Consider individual i in H_2 (she attains the top level in the unified system and not in the GE system). Her human capital at the conclusion of education is:

in the unified system: $(1+(\delta_{U1}+\delta_{U2})a_{it})h_{it}^B$

in the GE system: $(1+\delta_{U1}a_{it})h_{it}^B$

Consider individual j in H_1 (she attains the top level in the GE and not in the unified system). Her human capital at the conclusion of education is:

in the unified system: $(1+\delta_{U1}a_{jt})h_{jt}^B$

in the GE system: $(1+(\delta_{U1}+\delta_{U2})a_{jt})h_{jt}^B$

As $(1+\delta_{U1}a_{it})h_{it}^B > \tilde{h} > (1+\delta_{U1}a_{jt})h_{jt}^B$ and $h_{jt}^B > h_\alpha > h_{it}^B$, we have for any $(i,j) \in H_2 \times H_1$: $a_{it}h_{it}^B - a_{jt}h_{jt}^B > 0$,[20] and hence: $\sum_{i \in H_2} a_{it}h_{it}^B - \sum_{i \in H_2} a_{jt}h_{jt}^B > 0$.

The total human capital in the unified system is: $K + \sum_{i \in H_2}(1+(\delta_{U1}+\delta_{U2})a_{it})h_{it}^B + \sum_{j \in H_1}(1+\delta_{U1}a_{jt})h_{jt}^B$.

The total human capital in the GE system is: $K + \sum_{i \in H_2}(1+\delta_{U1}a_{it})h_{it}^B + \sum_{j \in H_1}(1+(\delta_{U1}+\delta_{U2})a_{jt})h_{jt}^B$.

The difference between the two is $\delta_{U2}\left(\sum_{i \in H_2} a_{it}h_{it}^B - \sum_{i \in H_2} a_{jt}h_{jt}^B\right) > 0$, which shows that the total human capital is higher in the unified than in the GE system.

[20] $(1+\delta_{U1}a_{it})h_{it}^B > (1+\delta_{U1}a_{jt})h_{jt}^B \Rightarrow \delta_{U1}(a_{it}h_{it}^B - a_{jt}h_{jt}^B) > h_{jt}^B - h_{it}^B > 0 \Rightarrow a_{it}h_{it}^B - a_{jt}h_{jt}^B > 0$.

Appendix D. Structural shifts in the short term

The short term analysis is based on the variations in thresholds a_j and in curves A_j in relation to δ_j, $j = U,\ U1,\ G,\ U2$ and to $\underset{\sim}{h}$, α and $\tilde{h}$.

<u>*Proof of Proposition 4*</u>. As depicted in Figure B1, an increase in δ_B:

1) displaces curves A_U and A_1 downwards ($\partial A_U / \partial \delta_B < 0,\ \partial A_1 / \partial \delta_B < 0$). These moves (i) increase the upward mobility of children from the low SG and shrink thereby the size of the low SG, (ii) reduce the downward mobility of children from the middle SG and expands thereby the size of the middle SG, and (iii) lessen the downward mobility to the low SG of children from the upper skill group and expand thereby the size of the middle SG.

2) displaces curve A_2 downwards ($\partial A_2 / \partial \delta_B < 0$), which increases the upward mobility of children from the middle SG and lessens the downward mobility of children from the upper SG in the unified system. This reduces the size of the middle SG and increases the size of the upper SG.

3) has no impact on the curve A_G. This is because (i) an increase in δ_B has no impact on the hierarchy of human capital across children at the end of basic education and (ii) the number of children admitted to the GEs (α) remains unchanged.

Note that all these moves are linked to the admission effect (the increase in δ_B makes the admission to *U*, *U*1 and *U*2 easier).

<u>*Proof of Proposition 5*</u>. An increase in the per-student expenditure on the first level of tertiary education (δ_U or δ_{U1}):

1) displaces a_U and a_{U1} to the left ($\partial a_U / \partial \delta_U < 0,\ \partial a_{U1} / \partial \delta_{U1} < 0$), which increases the upward mobility of children from the low SG and reduces the downward mobility of children from the middle SG, shrinking thereby the low SG and expanding the middle SG.

2) displaces a_G and a_{U2} to the right ($\partial a_G / \partial \delta_U > 0,\ \partial a_{U2} / \partial \delta_{U1} > 0$). This has no impact on the mobility between the middle and the upper skill groups in the GE system because all the admitted enter the GEs (the curve A_G is always at the right hand side of the vertical $a = a_G$ for $h < h_{USG}^{\max}$)[21]. In the unified system, this reduces the number of children who wish to enter the GEs from both the middle and upper SG.

---

[21] Note that without assuming that all the admitted students enter the GE, the increase in δ_U could foster the mobility to and from the upper SG through the incentive effect (because $\partial a_G / \partial \delta_U > 0$).

3) has no impact on A_G and displaces A_2 to the left. This latter move releases the admission constraint to enter U2, which tends to increase the upward mobility of children from the middle SG and to reduces the downward mobility of children from the upper SG.

In the unified system, the impact on mobility between the middle and the upper skill group depends on the opposite effects linker to impact of δ_{U1} on a_{U2} and on A_2.

Appendix E. Mobility and stratification in the long term

Consider the education functions related to each type of study (*B, U, G, U*1 and *U*2) and consider a given aptitude *a*. The education functions can be written:

$$h_{it}^{j} = \mathcal{A}_j(a)(h_{it-1})^{\eta}, \ \mathcal{A}_j(a) = \left(\delta_B a^{\beta} + \Delta_j a\right) \quad \text{(A1)}$$

with $\Delta_j = 0,\ \delta_U\delta_B,\ \delta_G\delta_B,\ \delta_{U1}\delta_B,\ (\delta_{U1}+,\delta_{U2})\delta_B,\quad j = B, U, G, U1, U2$.

The steady state of function (A1) is $\hat{h}_j(a) = \left(\mathcal{A}_j(a)\right)^{1/(1-\eta)}$ and it is stable since $\eta < 1$.

As $a \in [\underline{a}, \overline{a}]$, the <u>*j-steady segments*</u> $S_j = \left[\underset{\sim}{h}_j, \tilde{h}_j\right] = \left[\left(\mathcal{A}_j(\underline{a})\right)^{1/(1-\eta)}, \left(\mathcal{A}_j(\overline{a})\right)^{1/(1-\eta)}\right]$ are the locus of all the steady states corresponding to study *j*.[22]

As aptitudes are randomly distributed across individuals at each generation, one dynasty which perpetually remains in study *j* jumps from one curve $\mathcal{A}_j(a)(h_{it-1})^{\eta}$ to another depending on each generation's aptitude. All the dynasties within study *j* then enter sooner or later the *j*-steady segment and subsequently remain inside this segment.

However, the dynasties do not perpetually pursue the same study. They can go up or down the education ladder, depending on their wishes and on the admission constraints.

<u>1. Proof of Proposition 6</u>

We assume that that the admission constraints ($h_{it}^{B} \geq \underset{\sim}{h}$, $\mathcal{D}(G) = \alpha$ and $h_{it}^{U1} \geq \tilde{h}$) are effective and that there are children attending the *U*, *U*1, *G* and *U*2. Then, the long term steady segments corresponding to each study become:

[22] with: $S_B = \left[\underset{\sim}{h}_B, \tilde{h}_B\right] = \left[\left(\delta_B \underline{a}^{\beta}\right)^{\frac{1}{1-\eta}}, \left(\delta_B \overline{a}^{\beta}\right)^{\frac{1}{1-\eta}}\right]$, $S_U = \left[\underset{\sim}{h}_U, \tilde{h}_U\right] = \left[\left(\left(\underline{a}^{\beta} + \delta_U \underline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}, \left(\left(\overline{a}^{\beta} + \delta_U \overline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}\right]$,

$S_{U1} = \left[\underset{\sim}{h}_{U1}, \tilde{h}_{U1}\right] = \left[\left(\left(\underline{a}^{\beta} + \delta_{U1} \underline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}, \left(\left(\overline{a}^{\beta} + \delta_{U1} \overline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}\right]$, $S_G = \left[\underset{\sim}{h}_G, \tilde{h}_G\right] = \left[\left(\left(\underline{a}^{\beta} + \delta_G \underline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}, \left(\left(\overline{a}^{\beta} + \delta_G \overline{a}\right)\delta_B\right)^{\frac{1}{1-\eta}}\right]$,

$S_{U2} = \left[\underset{\sim}{h}_{U2}, \tilde{h}_{U2}\right] = \left[\left(\left(\underline{a}^{\beta} + (\delta_{U1} + \delta_{U2})\underline{a}\right)\delta_B\right)^{1/(1-\eta)}, \left(\left(\overline{a}^{\beta} + (\delta_{U1} + \delta_{U2})\overline{a}\right)\delta_B\right)^{1/(1-\eta)}\right]$

$$\bar{S}_B = \left[\underline{h}_B, \bar{h}_B\right[= \left[\left(\delta_B \underline{a}^{\beta}\right)^{1/(1-\eta)}, \max\left\{\underset{\sim}{h}, \left(\delta_B {a_U}^{\beta}\right)^{1/(1-\eta)}\right\}\right[$$

$$\bar{S}_U = \left[\underline{h}_U, \bar{h}_U\right[= \left[\left(\underline{a}^{\beta} + \delta_U \underline{a}\right)\delta_B \underset{\sim}{h}^{\eta}, \left(\left(\bar{a}^{\beta} + \delta_U \bar{a}\right)\delta_B\right)^{1/(1-\eta)}\right[$$

$$\bar{S}_{U1} = \left[\underline{h}_{U1}, \bar{h}_{U1}\right[= \left[\left(\underline{a}^{\beta} + \delta_{U1} \underline{a}\right)\delta_B \underset{\sim}{h}^{\eta}, \max\left\{\tilde{h}, \left(\left({a_{U2}}^{\beta} + \delta_{U1}\bar{a}_{U2}\right)\delta_B\right)^{1/(1-\eta)}\right\}\right[$$

$$\bar{S}_G = \left[\underline{h}_G, \bar{h}_G\right] = \left[\left(\left(\underline{a}^{\beta} + \delta_G \underline{a}\right)\delta_B\right)^{1/(1-\eta)}, \left(\left(\bar{a}^{\beta} + \delta_G \bar{a}\right)\delta_B\right)^{1/(1-\eta)}\right]$$

$$\bar{S}_{U2} = \left[\underline{h}_{U2}, \bar{h}_{U2}\right] = \left[\left(1 + \frac{\delta_{U2}}{\underline{a}^{\beta-1} + \delta_{U1}}\right)\tilde{h}, \left(\left(\bar{a}^{\beta} + (\delta_{U1} + \delta_{U2})\bar{a}\right)\delta_B\right)^{1/(1-\eta)}\right]$$

Note that the upper limit of segment $\bar{S}_B$ is $\bar{h}_B = \max\left\{\underset{\sim}{h}, \left(\delta_B {a_U}^{\beta}\right)^{1/(1-\eta)}\right\}$. When $\underset{\sim}{h}$ is large enough (in relation to a_U) so that all the children with a human capital $h_{it}^B > \underset{\sim}{h}$ enter the university, then $\bar{h}_B = \underset{\sim}{h}$. When there are children with a human capital $h_{it}^B > \underset{\sim}{h}$ who do not enter the university because their aptitude is lower than a_U, then $\bar{h}_B = \left(\delta_B {a_U}^{\beta}\right)^{1/(1-\eta)}$, where $\left(\delta_B {a_U}^{\beta}\right)^{1/(1-\eta)}$ is the steady state corresponding to the B-dynamics $h_{it} = \delta_B a^{\beta} {h_{it-1}}^{\eta}$ with $a = a_U$. The same explanation applies to the upper limit of $\bar{S}_{U1}$, $\bar{h}_{U1} = \max\left\{\tilde{h}, \left(\left({a_{U2}}^{\beta} + \delta_{U1}\bar{a}_{U2}\right)\delta_B\right)^{1/(1-\eta)}\right\}$.

2. Human capital at the end of basic education and at the end of U1[23]

The segments of human capital at the end of basic education in which are the children from the low SG (S_B^B), from the middle SG (S_U^B and S_{U1}^B) and from the upper SG (S_G^B and S_{U2}^B):

$$S_B^B = \left[\underline{h}_B^B, \bar{h}_B^B\right[= \left[\left(\delta_B \underline{a}^{\beta}\right)^{1/(1-\eta)}, \max\left\{\delta_B \bar{a}^{\beta} \underset{\sim}{h}^{\eta}, \delta_B \bar{a}^{\beta}\left(\delta_B {a_U}^{\beta}\right)^{\eta/(1-\eta)}\right\}\right[$$

$$S_U^B = \left[\underline{h}_U^B, \bar{h}_U^B\right[= \left[\underline{a}^{\beta}\left(\underline{a}^{\beta} + \delta_U \underline{a}\right)^{\eta} {\delta_B}^{1+\eta} \underset{\sim}{h}^{\eta^2}, \bar{a}^{\beta} {\delta_B}^{1/(1-\eta)}\left(\bar{a}^{\beta} + \delta_U \bar{a}\right)^{\eta/(1-\eta)}\right[$$

$$S_{U1}^B = \left[\underline{h}_{U1}^B, \bar{h}_{U1}^B\right[= \left[\underline{a}^{\beta}\left(\underline{a}^{\beta} + \delta_{U1}\underline{a}\right)^{\eta} {\delta_B}^{1+\eta} \underset{\sim}{h}^{\eta^2}, \delta_B \bar{a}^{\beta} \tilde{h}^{\eta}\right[$$

$$S_G^B = \left[\underline{h}_G^B, \bar{h}_G^B\right] = \left[\underline{a}^{\beta}\left(\underline{a}^{\beta} + \delta_G \underline{a}\right)^{\eta/(1-\eta)} {\delta_B}^{1/(1-\eta)}, \bar{a}^{\beta}\left(\bar{a}^{\beta} + \delta_G \bar{a}\right)^{\eta/(1-\eta)} {\delta_B}^{1/(1-\eta)}\right]$$

---

[23] Note that the individuals with $h_{it}^B > \tilde{h}$ and $a_{it} < a_{U2}$ do not enter U2. We suppose here that $\tilde{h}$ is large enough to prevent such cases. It can be easily seen from Figure B1 that, as we have assumed that $\tilde{a} > a_G$, the admission rule $h_{it}^B > \tilde{h}$ should be very lenient compared to the admission $\mathcal{D}(G) = \alpha$ to have $\bar{h}_{U1} > \tilde{h}$.

$$S_{U2}^{B}=\left[\underline{h}_{U2}^{B},\bar{h}_{U2}^{B}\right]=\left]\delta_B\underline{a}^{\beta}\left(1+\frac{\delta_{U2}}{\underline{a}^{\beta-1}+\delta_{U1}}\right)^{\eta}\tilde{h}^{\eta},\underline{a}^{\beta}\left(\bar{a}^{\beta}+(\delta_{U1}+\delta_{U2})\bar{a}\right)^{\eta/(1-\eta)}\delta_B^{1/(1-\eta)}\right]$$

with: $\frac{\partial \underline{h}_B^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_B^B}{\partial\delta_B}>0$, $\frac{\partial \underline{h}_U^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_U^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_U^B}{\partial\delta_U}>0$, $\frac{\partial \underline{h}_{U1}^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_{U1}^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_{U1}^B}{\partial\delta_{U1}}\geq 0$,
$\frac{\partial \underline{h}_G^B}{\partial\delta_B}>0$, $\frac{\partial \underline{h}_G^B}{\partial\delta_G}>0$, $\frac{\partial \bar{h}_G^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_G^B}{\partial\delta_G}>0$, $\frac{\partial \underline{h}_{U2}^B}{\partial\delta_B}>0$, $\frac{\partial \underline{h}_{U2}^B}{\partial\delta_{U2}}>0$, $\frac{\partial \underline{h}_{U2}^B}{\partial\delta_{U1}}<0$, $\frac{\partial \bar{h}_{U2}^B}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_{U2}^B}{\partial\delta_{U1}}>0$, $\frac{\partial \bar{h}_{U2}^B}{\partial\delta_{U2}}>0$

The segments of human capital at the end of $U1$ in which are the children from the low and middle SGs (S_{LM}^{U1}) and from the upper SG (S_{U2}^{U1}):

$$S_{LM}^{U1}=\left[\underline{h}_{LM}^{U1},\bar{h}_{LM}^{U1}\right[=\left[(1+\delta_{U1}\underline{a}^{1-\beta})\underset{\sim}{h},(1+\delta_{U1}\underline{a}^{1-\beta})\delta_B\bar{a}^{\beta}\tilde{h}^{\eta}\right[$$

$$\frac{\partial \underline{h}_{LM}^{U1}}{\partial\delta_{U1}}>0,\frac{\partial \underline{h}_{LM}^{U1}}{\partial\underset{\sim}{h}}>0\ ;\quad \frac{\partial \bar{h}_{LM}^{U1}}{\partial\delta_{U1}}>0,\frac{\partial \bar{h}_{LM}^{U1}}{\partial\delta_B}>0$$

$$S_{U2}^{U1}=\left[\underline{h}_{U2}^{U1},\bar{h}_{U2}^{U1}\right[=\left[(\underline{a}^{\beta}+\delta_{U1}\underline{a})\delta_B\left(1+\frac{\delta_{U2}}{\underline{a}^{\beta-1}+\delta_{U1}}\right)^{\eta}\tilde{h}^{\eta},(\bar{a}^{\beta}+\delta_{U1}\bar{a})\left(\bar{a}^{\beta}+(\delta_{U1}+\delta_{U2})\bar{a}\right)^{\eta/(1-\eta)}\delta_B^{1/(1-\eta)}\right[$$

with: $\frac{\partial \underline{h}_{U2}^{U1}}{\partial\delta_B}>0$, $\frac{\partial \underline{h}_{U2}^{U1}}{\partial\delta_{U2}}>0$, $\frac{\partial \underline{h}_{U2}^{U1}}{\partial\delta_{U1}}$ *ambiguous*; $\frac{\partial \bar{h}_{U2}^{U1}}{\partial\delta_B}>0$, $\frac{\partial \bar{h}_{U2}^{U1}}{\partial\delta_{U1}}>0$, $\frac{\partial \bar{h}_{U2}^{U1}}{\partial\delta_{U1}}>0$.

Proof of proposition 7

$\partial h_i^B/\partial\delta_B>0,\forall i$, and a_U, a_{U1} and $\underset{\sim}{h}$ are unchanged $\Rightarrow$ more children from the low, the middle and the upper SG can enter the university (and $U1$), and all the children who would have entered the university (and $U1$) without increase in δ_B can still do it. This establishes feature 1 of Proposition 7.

As regards the entry to GE, the rise in δ_B increases the human capital attainment at the end of basic education of all children without modifying their ranking. Hence, the α children with the highest human capital at the end of basic education are unchanged and, as α is constant, there is no modification in the GEs' intakes. This establishes feature 2 of Proposition 7.

As regards the entry in U2, $\frac{\partial h_i^{U1}}{\partial\delta_B}>0$, $\forall i$ entering $U1$, and a_{U2} and $\tilde{h}$ are unchanged $\Rightarrow$ more children from the middle and the upper SG enter $U2$ and all the children who would have entered U2 without increase in δ_B still do it. This establishes feature 3 of Proposition 7. The ambiguous effect on the size of the middle SG stems from the combination of its increase due to the larger number of intakes in $U1$ and its reduction due to those entering $U2$.

Proof of proposition 8

(a) $\frac{\partial a_U}{\partial \delta_U} < 0$ and $\frac{\partial a_{U1}}{\partial \delta_{U1}} < 0 \Rightarrow$ a rise in δ_U and δ_{U1} increases the upward mobility of children from the low SG, and decrease in the downward mobility of children from the middle SG in both systems. This entails a reduction in the size of the low SG. This establishes feature 1) of Proposition 8, which is reinforced by the increase in human capital off all the parents in the middle SG due to the rises in δ_U and δ_{U1}, which make less of their children to fall in the low SG because of the unchanged admission threshold $\underset{\sim}{h}$.

(b) $\partial a_G / \partial \delta_U > 0$, $\partial a_{U2} / \partial \delta_{U1} > 0 \Rightarrow$ a rise in δ_U and δ_{U1} increases the upward mobility of children from the middle SG in the unified system, but not in the GE system (we have assumed that all the children admitted to the GE enter the GE).

(c) In the GE system, the increase in δ_U moves upward the human capital of all the parents in the middle SG, which make some of their children overtake certain children from the upper SG at then of basic education (because of the increase in the family backgrounds of children from the middle SG) and make thereby some of them to enter the GE, and the same number of children from the upper SG to fail entering the GE, compared to the situation without increase in δ_U. This establishes feature 2 of Proposition 8.

(d) In the unified system, the increase in δ_{U1} moves upward the human capital of all the parents in the middle and the upper SG (the segment $\bar{S}_{U1}$ and $\bar{S}_{U2}$ move upwards) and, for a given $\underset{\sim}{h}$, increases the upward mobility of children from the middle SG and lessens the downward mobility of children from the upper SG, expanding thereby the size of the upper SG. This establishes feature 3 of Proposition 8.

Proof of proposition 9

An increase in δ_G augments the human capital level of all individuals who enter a GE, and thereby of parents in the upper SG. The segment $\bar{S}_G$ moves upwards. As the human capital level of parents from the upper SG raises compared to that of the middle SG, more children from the upper SG and less from the middle SG are admitted in the GE (since α is given), which signifies a decrease in the downward mobility of children from the upper skill group (feature 1 of Proposition 9) and a decrease in the upward mobility of children from the middle SG (feature 2 of proposition 9).

As $\partial a_{U2} / \partial \delta_{U2} < 0$, an increase in δ_{U2} lowers threshold , which incite more children born in the middle SG to entre $U2$ (incentive effect). The increase in δ_{U2} lessens the downward mobility of children from the upper SG because their parents have more human capital (which entails a higher gamily backgrounds). This establishes feature 1 of proposition 9. In addition, the decrease in a_{U2} incites more children born in the middle SG to enter $U2$.This expands the size of the upper SG. Both results establish feature 3 of Proposition 9.