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Global and Local Scale Characteristics in Convex and Nonconvex Nonparametric Technologies: A First Empirical Exploration*

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Abstract

The purpose of this contribution is to empirically implement and supplement the proposals made by Podinovski (2004b) to explore the nature of both global and local returns to scale in nonconvex nonparametric technologies. In particular, employing some secondary data sets, we investigate the frequency of the special case of global sub-constant returns to scale. Furthermore, we check how often global returns to scale yield concordant and conflicting information when evaluated relative to convex and nonconvex technologies. Finally, we explore local returns to scale in FDH by tracing the evolution of ray-average productivity for some typical individual observations and compare with DEA.

Keywords: Free Disposal Hull, Global Returns to Scale, Local Returns to Scale.

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1 Introduction

Thanks to the seminal article of Charnes, Cooper, and Rhodes (1978), the nonparametric approach to production theory has become one of the success stories in the operations research (OR) literature in terms of both methodological developments and empirical applications. While one of the early bibliographical overview article listed about 800 published articles and dissertations related to Data Envelopment Analysis (DEA) over the years 1978–1996 (see Seiford (1997)), one of the the more recent bibliography articles of Emrouznejad, Parker, and Tavares (2008) counted already 4000 research articles in journals or book chapters up to the year 2007.[1]

While the axiom of convexity is traditionally maintained in these nonparametric production models (see Afriat (1972), Banker, Charnes, and Cooper (1984), Charnes, Cooper, and Rhodes (1978), Diewert and Parkan (1983) or any of the early contributions in both economics and OR), Afriat (1972) was probably the first to mention a basic single output nonconvex technology imposing the assumptions of free disposal of inputs and outputs. Its multiple output extension has probably first been proposed in Deprins, Simar, and Tulkens (1984) and these authors introduced the moniker Free Disposal Hull (FDH).[2]

Convexity is justified for time divisible technologies (see Hackman (2008)), but becomes questionable when time indivisibilities compound all other reasons for spatial nonconvexities (e.g., indivisibilities, increasing returns to scale, economies of specialization, externalities, etc.). Shephard (1967, p. 215) puts things clearly when discussing the axiom of quasi-concavity of the production function in relation to convexity of the input level sets when formally defining the notion of a production function:

> The last one is effectively the only assumption which would appear to be restrictive, but even so it is essential if the production function is to represent the maximum output obtainable for time divisible processes. If the processes are not time divisible, the input $[(1-\theta)x+\theta y]$ is not evidently feasible. .... We exclude considerations of such technologies.

In addition to this general criticism, there are other more specific criticisms of convexity around in the literature. For instance, Emrouznejad and Amin (2009) indicate that the

[1] Including unpublished dissertations, working papers, and conference papers would have led to over 7000 entries.

[2] Tone and Sahoo (2003, p. 172) mention Scarf (1981a; 1981b) as an important but neglected predecessor of FDH, because he studied activity analysis models based on integer data.

traditional convexity axiom is problematic when some of the inputs and/or some of the outputs are ratio variables.

This basic FDH model has been extended in at least two directions. First, Kerstens and Vanden Eeckaut (1999) introduced constant, nonincreasing and nondecreasing returns to scale technologies complementary to the assumption of flexible or variable returns to scale embodied in the basic FDH model. Furthermore, these same authors proposed a new goodness-of-fit method to infer the characterization of global returns to scale for nonconvex technologies, since none of the existing methods (see, e.g., Seiford and Zhu (1999) for an early overview and Banker, Cooper, Seiford, Thrall, and Zhu (2004) for a more recent version) was suitable in this nonconvex setting. Second, this family of nonconvex technologies has been supplemented by nonconvex cost functions with corresponding returns to scale assumptions in Briec, Kerstens, and Vanden Eeckaut (2004).[3]

While these nonconvex technology and cost models are nowhere as popular as the convex DEA counterparts, the basic FDH model and its extensions have been regularly applied to assess performance-related research questions in a variety of sectors. We offer a limited selection of examples to provide some flavor of these results. Alam and Sickles (1998) study the evolution of technical efficiency in the US airline industry and analyze the news value of changes in frontier performance in relation to the stock market prices. Destefanis (2003) analyzes the macroeconomic relationship between the growth of output and the growth of productivity (known as Verdoorn's law) using nonconvex FDH models. Tone and Sahoo (2003) argue and illustrate that the nonconvex FDH model applied to a multi-stage production technology is capable to capture scale effects arising from process indivisibilities, whereas standard convex nonparametric technologies fail to exhibit such scale effects. Cummins and Zi (1998) contrast convex and nonconvex estimates of both technical and cost efficiency for US life insurers, while Balaguer-Coll, Prior, and Tortosa-Ausina (2007) document cost efficiency differences among Spanish municipalities.

An important point to note is that the results of these nonconvex technology and cost frontiers often yield different results compared to the convex ones. While it is true that nonconvex technology frontiers lead to higher efficiency levels and more efficient observations, the studies of Balaguer-Coll, Prior, and Tortosa-Ausina (2007) and Cummins and Zi (1998) document convincingly that convex cost frontier estimates may be substantially below the nonconvex ones under variable returns to scale.

[3] Ray (2004) shows that the nonconvex cost function based on flexible returns to scale FDH is the multiple output version of the cost function implicit in the Weak Axiom of Cost Minimisation of Varian (1984)).

Podinovski (2004a; 2004b) is the first to indicate that the goodness-of-fit method of Kerstens and Vanden Eeckaut (1999) to characterize global returns to scale for nonconvex technologies -which just like Färe, Grosskopf, and Lovell (1983) uses only scale efficiency measures- is incomplete. In particular, he argues that one must distinguish a fourth type of global sub-constant returns to scale case in addition to the three traditional cases (constant, decreasing and increasing returns to scale). This global sub-constant returns to scale case allows a unit to achieve its most productive scale size (see Banker, Charnes, and Cooper (1984)) by both reducing and increasing its scale of operations. This fourth type of global sub-constant returns to scale can never occur in traditional convex DEA technologies.

Independent of this contribution, there have been three articles that basically simplify the computations needed to implement the goodness-of-fit method of Kerstens and Vanden Eeckaut (1999) to characterize returns to scale: Soleimani-damaneh, Jahanshahloo, and Reshadi (2006), Soleimani-damaneh and Reshadi (2007), and Soleimani-damaneh and Mostafaee (2009). In fact, Soleimani-damaneh and Mostafaee (2009) furthermore offer some stability intervals to preserve the returns to scale classification via a polynomial time algorithm based on combining certain ratios of inputs and outputs. However, the classification procedure for global returns to scale proposed by these authors does not allow for the sub-constant returns to scale case. Therefore, we discuss how to amend their procedures for this purpose.

As far as the role of local returns to scale is concerned, Banker (1984) and especially Banker and Thrall (1992) show that in a convex technology global and local characterizations -based on scale efficiency and scale elasticity measures, respectively- coincide. The innovation of Podinovski (2004a; 2004b) is that he points out that this equivalence between global and local indicators breaks down for nonconvex technologies, due to the non-monotonic behavior of the ray average productivity (RAP) of a unit when expanding or contracting towards a point of most productive scale size.[4] However, he only provides an illustration of the RAP function in a single input and output fictitious FDH technology (see Podinovski 2004a, p. 234), while our aim is to depict this behavior and its consequences in an empirical multiple inputs and outputs setting.

This contribution intends to achieve several goals. First, we want to empirically determine the prevalence of the global sub-constant returns to scale case. Second, we want to establish some specific links between the Podinovski (2004a; 2004b) articles on the one hand, and the contributions made by Soleimani-damaneh, Jahanshahloo, and Reshadi (2006) and

[4]RAP indicates average productivity in a multiple inputs and output technology. See also Ray (2004, p. 63-64) for this RAP notion.

Soleimani-damaneh and Reshadi (2007) on the other hand. Third, we want to explore the differences between global returns to scale characterizations under the hypothesis of convexity or nonconvexity. Finally, we shed some light on the changes in local returns to scale in an empirical multiple inputs and outputs nonconvex technology by depicting the evolution of ray-average productivities for a selection of particular observations, and by comparing this evolution to its convex counterpart. To the best of our knowledge, this is the first contribution shedding some light on these issues.

For these purposes, this paper is structured as follows. Section 2 provides some basic definitions of the traditional convex and the less widely applied nonconvex technologies. Section 3 summarizes the known results to characterize returns to scale at both the global and local level. Then follows a Section 4 with some empirical illustrations based on secondary data sets. Section 5 concludes and outlines future research issues.

2 Nonparametric Technologies: A Unified Representation

Consider a set of K observations $A = \{(x_1, y_1), ..., (x_K, y_K)\} \in \mathbb{R}_+^{m+n}$. A production technology describes all available possibilities to transform input vectors $x = (x_1, \ldots, x_m) \in \mathbb{R}_+^m$ into output vectors $y = (y_1, \ldots, y_n) \in \mathbb{R}_+^n$. The production possibility set or technology S summarizes the set of all feasible input and output vectors: $S = \{(x, y) \in \mathbb{R}_+^{m+n} : x \text{ can produce } y\}$. Given our focus on input-oriented efficiency measurement later on, this technology can be represented by the input correspondence $L : \mathbb{R}_+^n \to 2^{\mathbb{R}_+^m}$ where $L(y)$ is the set of all input vectors that yield at least the output vector y:

$$L(y) = \{x : (x, y) \in S\}. \tag{1}$$

The radial input efficiency measure can be defined as:

$$E_i(x, y) = \min \{\lambda \ : \ \lambda \geq 0, \ \lambda x \in L(y)\}. \tag{2}$$

This Farrell efficiency measure, which is the inverse of the input distance function, indicates the minimum contraction of an input vector by a scalar λ while still remaining in the input correspondence. Obviously, the resulting input combination is located at the boundary of this input correspondence. For our purpose, the radial input efficiency has two key properties (see, e.g., Hackman (2008)). First, it is smaller or equal to unity ($0 < E_i(x,y) \leq 1$), whereby efficient production on the isoquant of $L(y)$ is represented by unity and $1 - E_i(x,y)$ indicates the amount of inefficiency. Second, it has a cost interpretation.

Non-parametric specifications of technology can be estimated by enveloping these K observations in the set A while maintaining some basic production axioms (see Hackman (2008) or Ray (2004)). We are interested in defining minimum extrapolation technologies satisfying strong disposability in the inputs and outputs, all four traditional returns to scale hypotheses (i.e., constant, nonincreasing, nondecreasing and variable (flexible) returns to scale), including those technologies that satisfy the assumption of convexity and those that do not

A unified algebraic representation of convex and nonconvex technologies under different returns to scale assumptions for a sample of K observations is found in Briec, Kerstens, and Vanden Eeckaut (2004):

$$S^{\Lambda,\Gamma} = \left\{ (x,y) \in \mathbb{R}^{m+n}_{+} : \ x \geq \sum_{k=1}^{K} x_k \delta \ z_k, \ y \leq \sum_{k=1}^{K} y_k \delta \ z_k, \ \sum_{k=1}^{K} z_k = 1, \ z_k \in \Lambda, \delta \in \Gamma \right\}, \quad (3)$$

where

$$\begin{aligned}
\text{(i) } \Gamma &\equiv \Gamma^{\text{CRS}} = \{\delta : \ \delta \geq 0\}; \\
\text{(ii) } \Gamma &\equiv \Gamma^{\text{NDRS}} = \{\delta : \ \delta \geq 1\}; \\
\text{(iii) } \Gamma &\equiv \Gamma^{\text{NIRS}} = \{\delta : \ 0 \leq \delta \leq 1\}; \\
\text{(iv) } \Gamma &\equiv \Gamma^{\text{VRS}} = \{\delta : \ \delta = 1\}; \ \text{ and} \\
\text{(i) } \Lambda &\equiv \Lambda^{\text{C}} = \{z_k \geq 0\}, \ \text{ and (ii) } \Lambda \equiv \Lambda^{\text{NC}} = \{z_k \in \{0,1\}\}.
\end{aligned}$$

First, there is the activity vector (z) operating subject to a convexity (C) or nonconvexity (NC) constraint. Second, there is a scaling parameter (δ) allowing for a particular scaling of all K observations spanning the technology. This scaling parameter is smaller than or equal to 1 or larger than or equal to 1 under nonincreasing returns to scale (NIRS) and nondecreasing returns to scale (NDRS) respectively, fixed at unity under variable returns to scale (VRS), and free under constant returns to scale (CRS).

Briefly discussing the computational methods for obtaining the radial input efficiency measure (2) for each evaluated observation relative to all technologies in (3), the convex case just requires solving a nonlinear programming problem (NLP): this is evidently simplified to the familiar linear programming (LP) problem found in the literature (see Hackman (2008) or

Ray (2004)) by substituting $w_k = \delta z_k$. For nonconvex technologies, nonlinear mixed integer programs must be solved in (3): however, Podinovski (2004c), Leleu (2006) and Briec, Kerstens, and Vanden Eeckaut (2004) propose mixed integer programs, LP problems, and closed form solutions derived from an implicit enumeration strategy, respectively. Kerstens and Van de Woestyne (2014) review all methods in this nonconvex case in more detail and empirically document that implicit enumeration is by far the fastest solution strategy.

3 Characterizing Returns to Scale

3.1 Global Returns to Scale

For a given input mix and given output mix a Most Productive Scale Size (MPSS) point refers to a scale size where the level of outputs produced 'per unit' of the inputs is maximized. Following Banker (1984), Banker, Charnes, and Cooper (1984, p. 37) and Banker and Thrall (1992, Definition 1)), the MPSS notion can be defined as follows.

Definition 3.1. A production possibility $(x_M, y_M) \in S^{\Lambda,VRS}$ represents an MPSS point if and only if for all production possibilities $(\delta x_M, \gamma y_M) \in S^{\Lambda,VRS}$ we have $\gamma/\delta \leq 1$.

This notion of MPSS is key in determining returns to scale for general technologies, since it does not require any differentiability assumption (in contrast to the scale elasticity notion). Note that Podinovski (2004a, Definition 2) defines MPSS as the inverse of the above ratio.

As a direct consequence of this definition, $(x_M, y_M) \in S^{\Lambda,VRS}$ represents an MPSS point if and only if $r^* = 1$ with

$$r^* = \max\left(\frac{\gamma}{\delta} : (\delta x_M, \gamma y_M) \in S^{\Lambda,VRS}, \delta, \gamma > 0\right). \tag{4}$$

This implies that at the optimum, $r^* = 1 \Leftrightarrow \gamma^* = \delta^*$, which reflects the familiar condition for proportional changes in inputs to equal proportional changes in outputs at the optimum.

Banker (1984) shows that in a convex technology each scale-efficient point (i.e., CRS efficient) is an MPSS and also the reverse (see Banker (1984, Proposition 2)), while each scale-inefficient point locally exhibits either decreasing or increasing returns to scale according to the sign of the divergence between their actual scale size and their MPSS (see Banker (1984, Corollary 1)). Thus, a classification method can exclusively rely on the "global" comparison between a unit and its MPSS (i.e., its scale efficiency), without depending explicitly on the quantitative information supplied by the "local" scale elasticity measure.

In the literature, several methods are available to obtain qualitative information regarding global returns to scale (see Seiford and Zhu (1999)). Since none of these existing methods are suitable for nonconvex technologies, Kerstens and Vanden Eeckaut (1999, Proposition 2) generalize the existing goodness-of-fit method proposed by Färe, Grosskopf, and Lovell (1983) in a convex setting such that it becomes perfectly general.

Proposition 3.1. *Using $E_i(x, y|.)$ and conditional on the optimal projection point, technology $S^{\Lambda,VRS}$ is globally characterized by:*

(a) $GCRS \Leftrightarrow E_i(x, y|CRS) = \max\{E_i(x, y|CRS), E_i(x, y|NIRS), E_i(x, y|NDRS)\}$;

(b) $GIRS \Leftrightarrow E_i(x, y|NDRS) = \text{strict max}\{E_i(x, y|CRS), E_i(x, y|NIRS), E_i(x, y|NDRS)\}$;

(c) $GDRS \Leftrightarrow E_i(x, y|NIRS) = \text{strict max}\{E_i(x, y|CRS), E_i(x, y|NIRS), E_i(x, y|NDRS)\}$.

where GCRS, GIRS and GDRS stand for globally constant, increasing and decreasing returns to scale respectively. Following Soleimani-damaneh, Jahanshahloo, and Reshadi (2006, p. 1056: Note 1), we introduce a "strict max" expression defined as: $a = \text{strict max}\{a, b, c\}$ if and only if $a > b$ and $a > c$.

As noted by Podinovski (2004b, p. 173), following Briec, Kerstens, Leleu, and Vanden Eeckaut (2000, Proposition 5) one can simplify the above result for general technologies.

Proposition 3.2. *Using $E_i(x, y|.)$ and conditional on the optimal projection point, technology $S^{\Lambda,VRS}$ is globally characterized by:*

(a) $GCRS \Leftrightarrow E_i(x, y|NIRS) = E_i(x, y|NDRS) = \max\{E_i(x, y|NIRS), E_i(x, y|NDRS)\}$;

(b) $GIRS \Leftrightarrow E_i(x, y|NDRS) = \text{strict max}\{E_i(x, y|NIRS), E_i(x, y|NDRS)\}$;

(c) $GDRS \Leftrightarrow E_i(x, y|NIRS) = \text{strict max}\{E_i(x, y|NIRS), E_i(x, y|NDRS)\}$.[5]

This result is qualified by Podinovski (2004a, Theorem 3) and Podinovski (2004b, Theorem 2) in that he adds a fourth case of global sub-constant returns to scale case that is only relevant for nonconvex technologies.

[5]This proposition qualifies Briec, Kerstens, Leleu, and Vanden Eeckaut (2000, Proposition 4): as an implication of their Proposition 5, since a CRS technology is always the union of NIRS and NDRS hulls, the goodness-of-fit test in their Proposition 2 always simplifies (not just for convex technologies).

Proposition 3.3. *Using $E_i(x, y|.)$ and conditional on the optimal projection point, technology $S^{\Lambda,VRS}$ is globally characterized by:*

(a) $GCRS \Leftrightarrow E_i(x, y|NIRS) = E_i(x, y|NDRS) = E_i(x, y|VRS)$;

(b) $GIRS \Leftrightarrow E_i(x, y|NIRS) < E_i(x, y|NDRS) \leq E_i(x, y|VRS)$;

(c) $GDRS \Leftrightarrow E_i(x, y|NDRS) < E_i(x, y|NIRS) \leq E_i(x, y|VRS)$;

(d) $GSCRS \Leftrightarrow E_i(x, y|NIRS) = E_i(x, y|NDRS) < E_i(x, y|VRS)$.

As stressed in Podinovski (2004a; 2004b), this case of global sub-constant returns to scale cannot occur in convex technologies. Instead of solving for these three efficiency measures using any of the solution methods listed above, we follow a specific theorem in Soleimani-damaneh, Jahanshahloo, and Reshadi (2006, p. 1057) that proposes a simple enumeration algorithm valid for nonconvex technologies solely to guarantee a maximal computational advantage:

Proposition 3.4. *For a given observation (x_o, y_o), let $\lambda^{jo} = \max\{\frac{y_{ro}}{y_{rj}} : 1 \leq r \leq n, y_{ro} + y_{rj} > 0\}$ and $\theta^{jo} = \max\{\frac{x_{ij}\lambda^{jo}}{x_{io}} : 1 \leq i \leq m, x_{io} + x_{ij} > 0\}$ for $j = 1, \ldots, K$. Let $E_i^{NC}(x_o, y_o|CRS) = \min\{\theta^{jo} : j = 1, \ldots K\}$. Now denote the set $A_o = \{k \in \{1, \ldots, K\} : \theta^{ko} = E_i^{NC}(x_o, y_o|CRS)\}$. Assuming that (x_o, y_o) is an FDH-efficient point, then the following conditions identify the situation of RTS at this point:*

(a) *There exists $k \in A_o$ such that $\lambda^{ko} = 1 \Rightarrow GCRS$;*

(b) *$\lambda^{ko} < 1$ for each $k \in A_o \Rightarrow GIRS$;*

(c) *$\lambda^{ko} > 1$ for each $k \in A_o \Rightarrow GDRS$;*

(d) *There exist $k, k' \in A_o$ such that $\lambda^{ko} > 1$ and $\lambda^{k'o} < 1 \Rightarrow GSCRS$.*

Since the latter authors fail to consider the possibility of global sub-constant returns to scale, which corresponds to their case (d), we have amended this proposition and labeled the outcome with $GSCRS$, because of the presence of scale inefficiency in the DMU under evaluation (inefficiency which the authors fail to consider). Exactly the same criticism applies to Soleimani-damaneh and Reshadi (2007, Theorem 1) and Soleimani-damaneh and Mostafaee (2009, Theorem 1).

To the best of our knowledge, no article ever reported any empirical evidence on the incidence of the global sub-constant returns to scale in relation to the other cases.

3.2 Local Returns to Scale

The exact relation between scale efficiency and scale elasticity has first been elaborated in convex nonparametric production frontiers in the seminal analysis of Banker and Thrall (1992). These authors prove explicitly the equivalence between the local method based on the values of scale elasticity and the global method relying on the sign of the difference between actual and most productive scale sizes (see Banker and Thrall (1992, Propositions 3 and 4, resp.)). Other contributions on this topic are, among others, those of Førsund and Hjalmarsson (2004) and Førsund, Hjalmarsson, Krivonozhko, and Utkin (2007). The potential empirical differences between both these concepts have been illustrated in, for instance, Evanoff and Israilevich (1995).

However, as noted by Podinovski (2004a, p. 228)): "in a general non-convex technology the RTS classes no longer play the role of global indicators", because local maxima of the RAP function are not necessarily global maxima. In other words, even for a differentiable non-convex technology, global analysis of returns to scale must be separated from local analysis (i.e. "RTS classes"). With regard to the latter, Podinovski (2004b, p. 176-177) clearly points out that the use of the traditional notion of scale elasticity is only possible for smooth differentiable technologies, but that this notion is undefined for FDH because of its lack of smoothness. Instead, Podinovski (2004b, p. 176) suggests that the "type of local RTS indicates whether the function of average productivity is increasing, decreasing or stationary". This motivates the approach adopted in this contribution, where the local classification is based on the detection of relative optima (i.e., points with RAP greater than one) in a neighborhood of the DMU under examination.

The reconstruction of production frontiers has been analyzed in a few contributions (see, e.g., Hackman (2008, Ch. 10) for a brief review). Since the convex technologies in (3) are convex polyhedra, facets can be enumerated so as to reconstruct the boundaries of the technology. A two-dimensional projection is then defined relative to a particular point of the technology. For example, Krivonozhko, Utkin, Volodin, Sablin, and Patrin (2004) offers parametric optimization tools to reconstruct an intersection of the multidimensional convex production frontier with a two-dimensional plane determined by any pair of given directions. We simply adapt this same idea to a nonconvex technology. Moreover, to the best of our knowledge, this is the first time that a computed section of an FDH technology is ever displayed.

While for the nonconvex case we follow this same basic setup, we employ a specific enumeration algorithm. Indeed, as Podinovski (2004a, p. 233) indicates, MPSS points can be determined by solving either for the MPSS definition (3.1) relative to a VRS technology ($S^{\Lambda,VRS}$), or a radial efficiency measure relative to a CRS technology ($S^{\Lambda,CRS}$) (see also Banker (1984, Proposition 1)). Following Soleimani-damaneh and Reshadi (2007, Lemma 1), the former solution is equivalent to the specific enumeration algorithm developed in Soleimani-damaneh, Jahanshahloo, and Reshadi (2006, p. 1057) and Soleimani-damaneh and Reshadi (2007, p. 2172-2173) for nonconvex technologies and it is based on the notion of RAP.

It is important to point out that average productivity under convexity may be higher or equal to average productivity under nonconvexity. To develop this intuition, one can look at the two Figures 1 and 2 .

From a small numerical example we reconstructed in Figure 1 both a convex (part (a)) and nonconvex (part (b)) CRS technology in a two inputs single output space. In the convex case, only two points span the three faces of the convex cone. In the nonconvex case, three observations span the ridge lines emanating from the origin and determining the non-convex cone because these observations operate under CRS. Based on these 3-D figures one may infer that the convex cone contains the nonconvex cone.

This is clearly made visible by the section with a vertical plane along a ray through the origin and along the single output depicted in the same Figure 1. Figure 2 depicts this latter section in just two dimensions by a projection into the X_1Y-plane: it is clear that average productivity under convexity is higher than under nonconvexity along this particular section of Figure 1.

FIGURES 1 AND 2 ABOUT HERE

4 Empirical Illustrations

In this section, we first present the data sets adopted from existing studies. Then, we present empirical results on global returns to scale. Thereafter, we turn to a selection of results focusing on local returns to scale.

4.1 Secondary Data Sets Employed

To empirically illustrate these developments, we employ several existing data sets. Table 1 summarizes some key features of each data set: sample size, number of inputs and outputs, and the sector. There is one small unbalanced panel (Färe, Grosskopf, and Logan (1983)) and four cross sections (Cesaroni (2011), Fan, Li, and Weersink (1996), Färe, Grosskopf, Logan, and Lovell (1985), and Haag, Jaska, and Semple (1992)).

The main points to note are the following. There are three single output samples, and two multiple-output samples. Sample sizes vary from very small to rather big. The data sets have been sorted in Table 1 according to their sample size. In the other tables we maintain this same order.

TABLE 1 ABOUT HERE

4.2 Global Returns to Scale

Turning to the determination of global returns to scale, we set ourselves two goals. First, we want to document any eventual differences between convex and nonconvex technologies in terms of the nature of returns to scale for individual observations. This has to the best of our knowledge nowhere been reported. Second, it is important to evaluate the incidence of the global sub-constant returns to scale case developed by Podinovski (2004a; 2004b).

Table 2 reports the basic decomposition of overall technical efficiency (OTE) into a scale efficiency (SCE) and a technical efficiency (TE) component. This amounts to comparing efficiency relative to CRS and VRS technologies. In particular, $OTE = E_i(x, y|CRS)$, $TE = E_i(x, y|VRS)$ and $SCE = E_i(x, y|CRS)/E_i(x, y|VRS)$. The first and second parts of Table 2 report this decomposition for the convex and nonconvex family of technologies. For each data set, there are three lines per efficiency component in a column: (i) the number of efficient observations, (ii) the average efficiency, and (iii) the Li (1996) test statistic. We comment on each of these three elements in turn.

TABLE 2 ABOUT HERE

For any efficiency component, it is well-known that the number of efficient observations is higher or equal under nonconvexity compared to the convex case. This number turns out to be equal for the OTE and SCE components in two data sets: Färe, Grosskopf, Logan, and Lovell (1985) and Färe, Grosskopf, and Logan (1983). Average efficiency is also known to be higher or equal under nonconvexity, except for the SCE component since it is a ratio derived from the other two components.[6] This average turns out to be equal for the OTE component in just one data set: Färe, Grosskopf, Logan, and Lovell (1985).

One can assess the differences between convex and nonconvex efficiency estimates by using a test statistic initially proposed by Li (1996) that is valid for both dependent and independent variables.[7] The null hypothesis of this Li-test states that both convex and nonconvex distributions for a given efficiency measure are equal. One can reject the null hypothesis of equal distributions for all components for the Fan, Li, and Weersink (1996) data set and for at least two components for all remaining data sets.[8] Thus, it seems rather safe to conclude that convex and nonconvex efficiency estimates differ for most components and data sets.

The last column of Table 2 reports both the number of CRS efficient observations under nonconvexity that are CRS inefficient under convexity, and the average amount of convexity-related OTE $(= E_i^C(x, y|CRS)/E_i^{NC}(x, y|CRS))$ for these same observations.[9] On the one hand, this is the net gain in the number of MPSS points due to dropping convexity. It varies between 0 and 42 observations among the data sets analysed. On the other hand, convexity-related OTE indicates the amount of overall technical efficiency that can be attributed to the convexity axiom. Not surprisingly, this convexity-related OTE equals zero in two data sets: Färe, Grosskopf, Logan, and Lovell (1985) and Färe, Grosskopf, and Logan (1983). On average, this amount varies between 0.856 and 0.972 when computed relative to the concerned observations: thus, the convex estimates suggest further gains in overall technical efficiency varying between 2.8% and 14.4%. Recall that Figure 2 represents the section shown in both convex and nonconvex technologies depicted in Figure 1: it clearly illustrates these cases where the nonconvex CRS technology is situated below the convex one. Thus, convex CRS technologies may well overestimate potential gains in average productivity.

FIGURE 2 ABOUT HERE

[6]The multiplicative decomposition of OTE need not hold exactly at the sample level, since arithmetic rather than geometric averages are reported.

[7]Dependency is a basic characteristic of extremum or frontier estimators, since efficiency measures depend, among others, on sample size. Note that Fan and Ullah (1999) refine this same test.

[8]Note that this Li-test cannot be computed for the nonconvex TE component of Haag, Jaska, and Semple (1992), since all observations are technically efficient and hence the kernel density cannot be estimated.

[9]The notion of convexity-related efficiency is introduced by Briec, Kerstens, and Vanden Eeckaut (2004): for any input-oriented efficiency component it is the convex efficiency measure divided by the nonconvex one.

Tables 3 and 4 each have two major parts. The first and second parts of Table 3 report on the percentage of observations relative to the sample size operating under increasing (IRS), constant (CRS) and decreasing returns to scale (DRS) for the convex and nonconvex technology respectively. Table 4 again has two major parts. The first part lists the efficient observations on both technologies that share a common characterization of returns to scale for each of the three cases. The second part focuses on conflicting cases: switches from IRS to DRS (denoted IRS-DRS), from CRS to IRS (CRS-IRS), from CRS to DRS (CRS-DRS), and the total percentage of these conflicts relative to the sample size.

One can draw the following conclusions. First, the amount of common efficient observations spanning both technologies is quite modest. Obviously, the amount of common CRS observations is low because few observations are CRS efficient in the convex case in the first place. While the percentage of common IRS observations is low, especially the DRS part of technology is built on strikingly little common ground: almost no observations are in common. Second, apart from the first study with the smallest sample size, all other samples yield some minimal to moderate conflict in classification between convex and nonconvex technologies. This conflict varies from a modest about 7% for the Färe, Grosskopf, and Logan (1983) sample to a quite substantial about 40 % for the Haag, Jaska, and Semple (1992) case, all three cases confounded. Third, the detailed sources of conflict in classification vary a lot among the different samples. While for the Haag, Jaska, and Semple (1992) study the CRS-IRS conflict dominates for about 20% of observations, for Cesaroni (2011), Färe, Grosskopf, and Logan (1983) and Fan, Li, and Weersink (1996) the IRS-DRS case is dominant: for a small about 7% for the first two cases to a substantial about 14% of observations for the third sample.

TABLES 3 AND 4 ABOUT HERE

On the empirical evaluation of the incidence of the global sub-constant returns to scale case we can be very brief. We found none in any of the five samples investigated. This explains why this notion is not reported in any of the tables so far. It remains an open question which conditions determine the existence as well as the empirical incidence of this global sub-constant returns to scale case.

4.3 Local Returns to Scale

To contrast the above results to scale with some preliminary results on local returns to scale, we have chosen to depict some typical observations selected from the Cesaroni (2011) sample.

In particular, we have selected observation 40 because it is efficient in both the convex and nonconvex CRS technologies. Then, we depict three observations that each represent one type of conflict in the classification: observations 44, 63 and 82 stand for the conflict between IRS-DRS, CRS-IRS and CRS-DRS respectively. To avoid repetition, the discussion of these last two observations is available in the Appendix.

For each of these cases we show a pair of figures: the above represents the optimal (δ, γ)-combinations of a section from the origin through the observation in input-output space; the below depicts the evolution of RAP along the same radial section. The observations under scrutiny are situated at the coordinates (1,1) in both the upper and lower parts of the figures. The convex (nonconvex) case is shown as a dashed (continuous) line. Note that for the RAP figure one must distinguish between points where RAP is smaller and larger than unity: only the latter points indicate improvements with respect to the observation under evaluation and are candidates for optima. RAP points smaller than unity form at best a local optimum or sub-optimum for themselves in that RAP may be stationary at such points. But, these points can never be optimal since the RAP level is below that of the observation under examination.

We first comment on observation 40 depicted in Figure 3. Being a unique optimal MPSS point labeled A, there is a close to optimal point labeled B to the right where RAP is close to constant under convexity but varies a lot under nonconvexity. Beyond this point B to the right RAP declines monotonously under convexity and more rapidly and close to monotonously except for the end of the empirical range under nonconvexity. To the left of the MPSS point A, RAP declines monotonously, albeit more rapidly again under nonconvexity.

FIGURE 3 ABOUT HERE

Next, we comment on observation 44 shown in Figure 4. As can be noticed from the upper part, this observation is situated under DRS (IRS) under convexity (nonconvexity). In the lower part, it is clearly visible that the MPSS point under nonconvexity labeled A is situated to the right of unity, while the MPSS point under convexity labeled B is positioned slightly to the left of unity and suggests a higher RAP than the nonconvex case. This is a perfect illustration of the phenomenon depicted by a numerical example in Figure 2. To both the right and especially the left from the nonconvex MPSS point A, there is quite some variation: there are three local optima of RAP to the right and at least six local optima of RAP to its left. Under convexity, the RAP curve suggests a smooth rise and decline around its optimal point B.

This case of observation 44 also illustrates the application of the FDH local classification-criterion as proposed in Section 3.2. In a small neighborhood of this observation 44 (the region $1 \pm 6\%$ in the lower part of Figure 4), at $\delta = 1.06$, this is the point labeled C, there is only one point ensuring a RAP greater than one (1.001), such that the observation is classified as increasing returns to scale (even locally). However, we can notice that if the size of the interval under consideration were to be enlarged to include the relative maxima on the left (situated at $\delta = 0.73$), this interval would also include another relative maxima on the right ($\delta = 1.19$), then we would have a case of local sub-constant returns to scale: i.e., a situation in which average productivity can be locally increased by both reducing and increasing the DMU's scale size. Such outcome is impossible in a convex technology.

FIGURE 4 ABOUT HERE

Summarizing these local results, one can draw two preliminary conclusions. First, the evolution of RAP under nonconvexity is not smooth at all and reveals a variety of local optima that remain hidden in the smooth increase and subsequently decrease of RAP in the convex case. Remedying issues of suboptimal scale size is rather straightforward under convexity. Any diagnosis of IRS or DRS leads to an unambiguous recommendation to either increase or decrease the scale of operations, whereby any step in the right direction monotonously increases or decreases RAP respectively. Under nonconvexity remedying the scale of operations is much harder and depends on choosing the right step size to either increase or decrease the scaling of the unit under evaluation. In empirical applications, there seem to be many areas where the lack of data is filled up by the convexity axiom, while the nonconvex approach clearly reveal the gaps in the empirical range of the data and our ensuing lack of knowledge about the technology. Of course, it cannot be excluded that sector specialists (managers, engineers, regulators, etc) may have an a priori understanding on which ranges of operation are actually feasible even though these are currently not supported by the empirical range of the data.

Second, these local returns to scale results are hard to summarize neatly using standard descriptive statistics. While their local nature lends itself excellently for depiction, this approach carries the risk that an empirical analysis becomes somehow casuistic and does not allow to draw any general conclusions at the sample level. Lacking standards to report the results of nonconvex analysis in economics, this problem cannot be easily solved in the short run.

5 Conclusions

Starting from the seminal contributions of Podinovski (2004a; 2004b) who characterizes both the notions of global and local returns to scale for nonconvex technologies, this contribution leads to three main conclusions.

First, we have clearly empirically established that the characterization of returns to scale on convex and nonconvex technologies may yield conflicting advice for substantial parts of samples. This confirms that Podinovski (2004a; 2004b) was certainly right in further scrutinizing the notion of returns to scale for nonconvex nonparametric technologies.

Second, while Podinovski (2004a; 2004b) convincingly argued for the existence a fourth type of global sub-constant returns to scale case complementing the three traditional cases (constant, decreasing and increasing returns to scale), our empirical tests reveal that none of the five secondary data sets analyzed contains a single observation that experiences such global sub-constant returns to scale. Which conditions determine the existence as well as the empirical incidence of this global sub-constant returns to scale case remains a question for future research.

Third, we have made a start to explore the differences between global and local returns to scale characterizations on FDH models. Especially the local results are revealing in that these clearly show how RAP evolves nonsmoothly and nonmonotonously under nonconvexity, while it is smooth and monotonous for convex nonparametric technologies. As spelled out earlier, this makes remedying scale deficiencies much harder under nonconvexity.

To the best of our knowledge, this is the first contribution that has managed to shed some light on all these issues. Of course, much more remains to be done. For instance, outliers are an issue for all nonparametric technology specifications and it could be interesting to evaluate how these affect the empirical differences as to returns to scale observed between convex and nonconvex technologies. As another example, a more refined definition of local returns to scale, including the case of sub-constant returns to scale, could prove insightful when analyzing nonconvex nonparametric technologies. Finally, while this research has been confined to analysing changes along a radial section in input-output space, keeping in mind that some managers may well prefer mimicking actual observations (e.g., Halme, Korhonen, and Eskelinen (2014)), it could be interesting to also develop an average productivity notion along a non-radial rather than a radial path.[10]

[10]See, e.g., Chambers and Mitchell (2001) for other examples on the importance of non-radial changes.

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Appendix: Empirical Illustrations on Local Returns to Scale (Section 4.3)

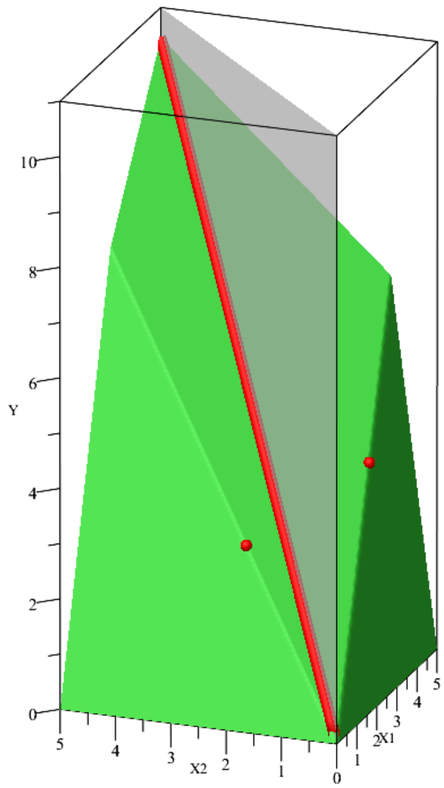
Turning to observation 63 shown in Figure 5, one can observe from the upper part that this observation labeled A is situated under IRS (CRS) under convexity (nonconvexity). Being a unique optimal MPSS point under nonconvexity, note that RAP is smaller than unity both to the left and right of this observation. The MPSS point under convexity labeled B is situated slightly to the right of the nonconvex case and is a bit higher than unity. This suggests that RAP under convexity can slightly increase to the right of the nonconvex optimum. To both the left and right from the nonconvex MPSS point A, there is some variation: there is one local optimum of RAP to the left and three local optima of RAP to its right. The convex case again suggests a smooth rise and decline of RAP around its optimal point B.

FIGURE 5 ABOUT HERE

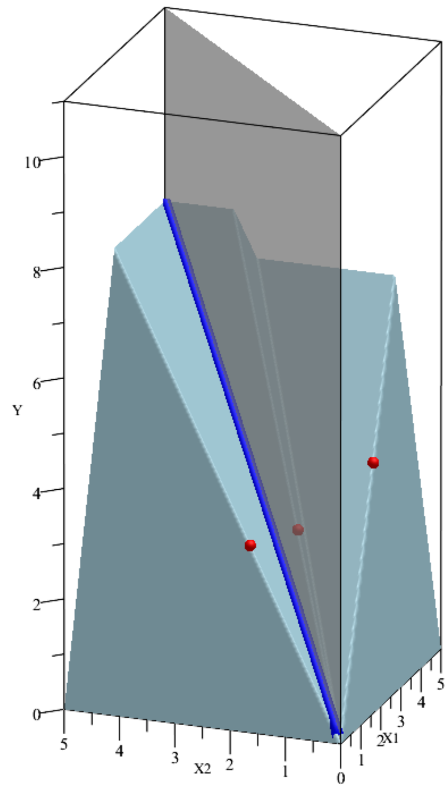
Finally, observation 82 is displayed in Figure 6. From the upper part, one can verify that this observation labeled A is situated under DRS (CRS) under convexity (nonconvexity). Again being a unique optimal MPSS point under nonconvexity, RAP is smaller than unity both to the left and right of this observation. The corresponding MPSS point under convexity labeled B is now situated slightly to the left of the nonconvex case and is just a little bit higher than unity, suggesting that RAP under convexity can slightly increase to the left of the nonconvex MPSS point A. Again, to both the left and right from the nonconvex MPSS point A, there is some variation, while the convex case just suggests a smooth increase and decrease of RAP around its optimal point B.

FIGURE 6 ABOUT HERE

Figure 1: Two Inputs Single Output (a) Convex and (b) Nonconvex CRS Technology



(a)



(b)

Figure 2: Single Input Single Output Representation of Section of Convex and Nonconvex CRS Technologies

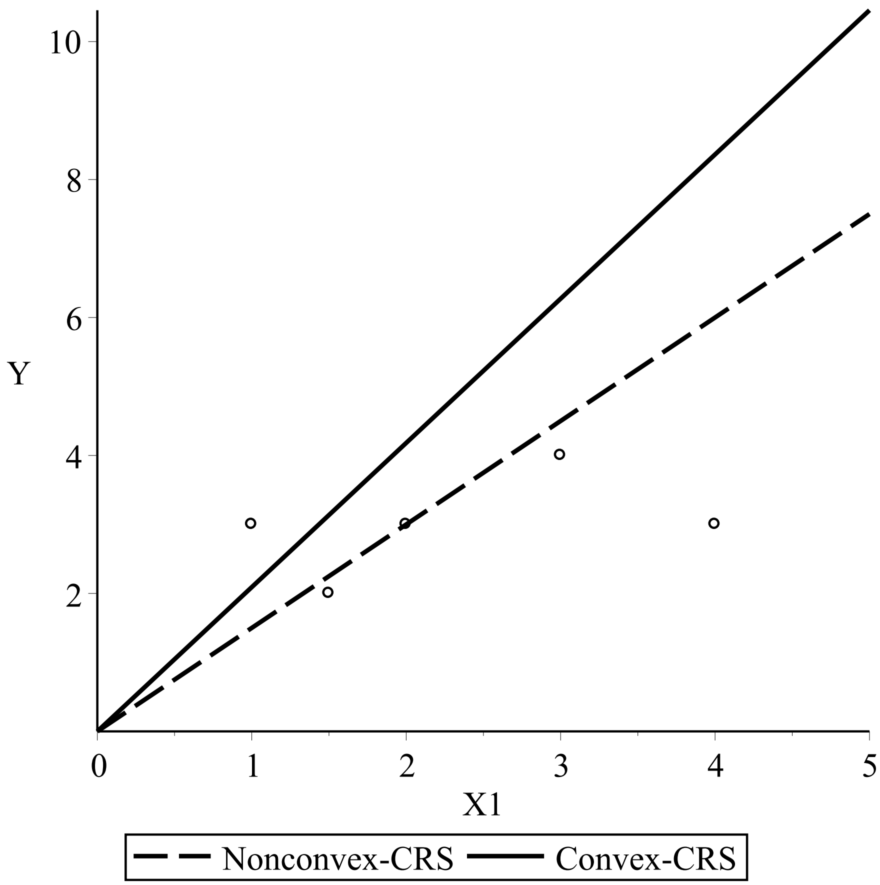
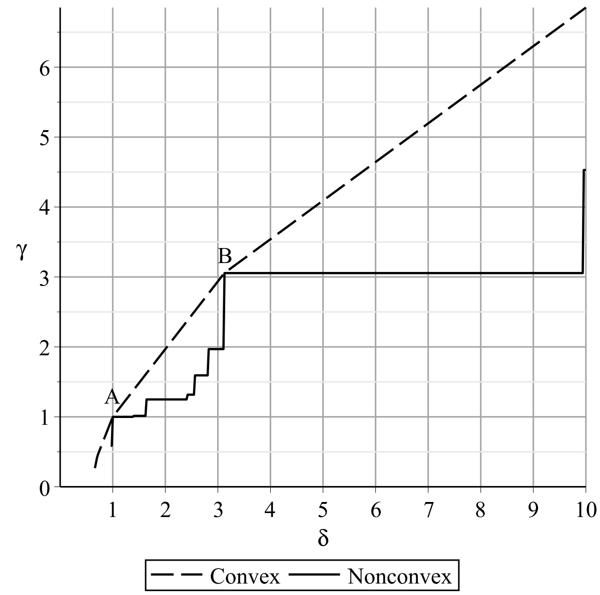
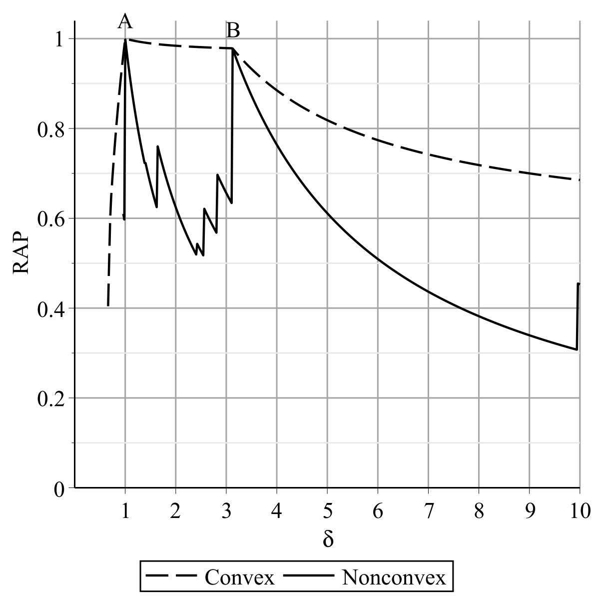


Figure 3: Representation of Radial Section for Observation 40 in Input-Output Space (a) and Evolution of RAP (b)

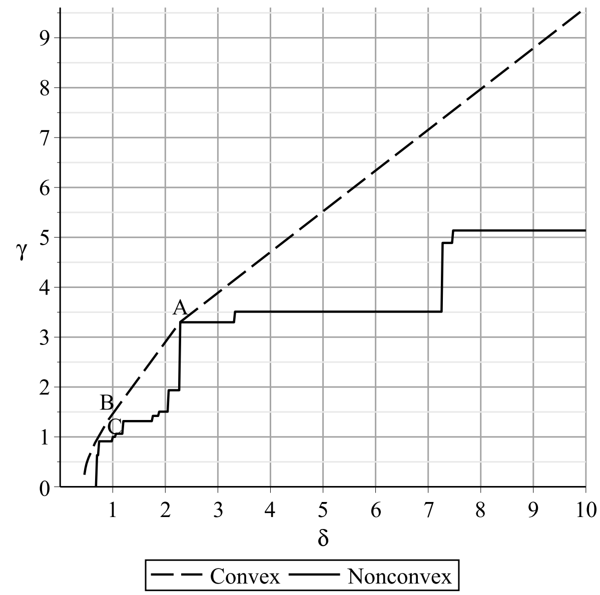


(a)

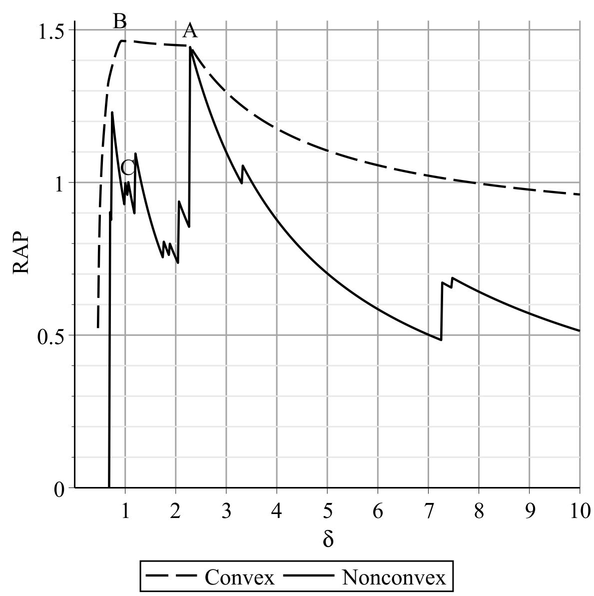


(b)

Figure 4: Representation of Radial Section for Observation 44 in Input-Output Space (a) and Evolution of RAP (b)

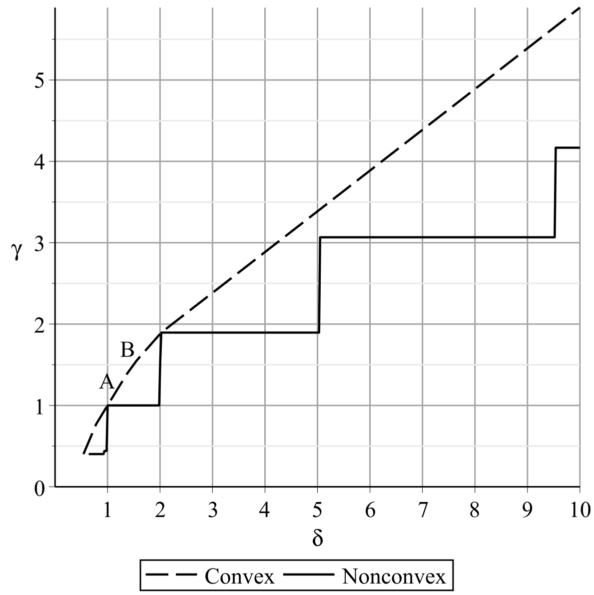


(a)

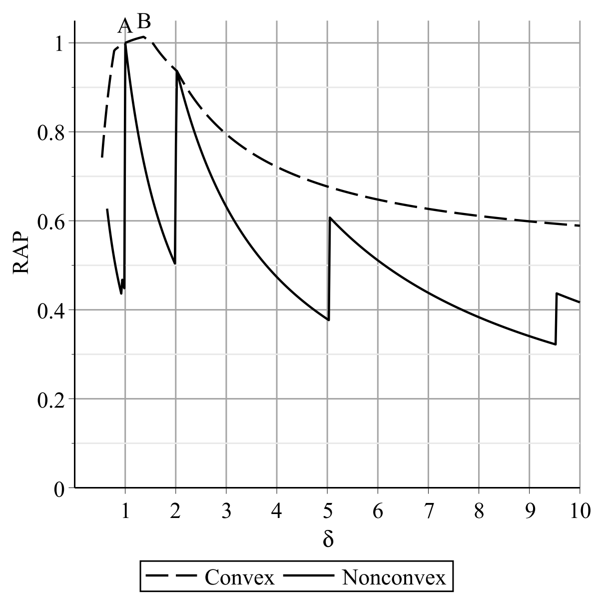


(b)

Figure 5: Representation of Radial Section for Observation 63 in Input-Output Space (a) and Evolution of RAP (b)

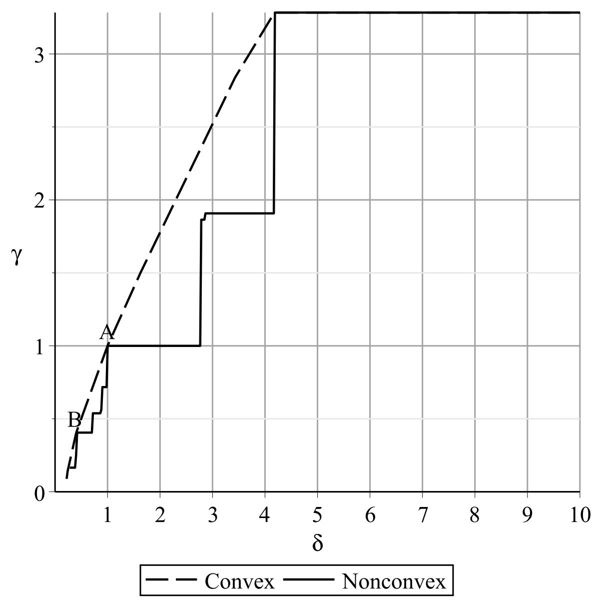


(a)

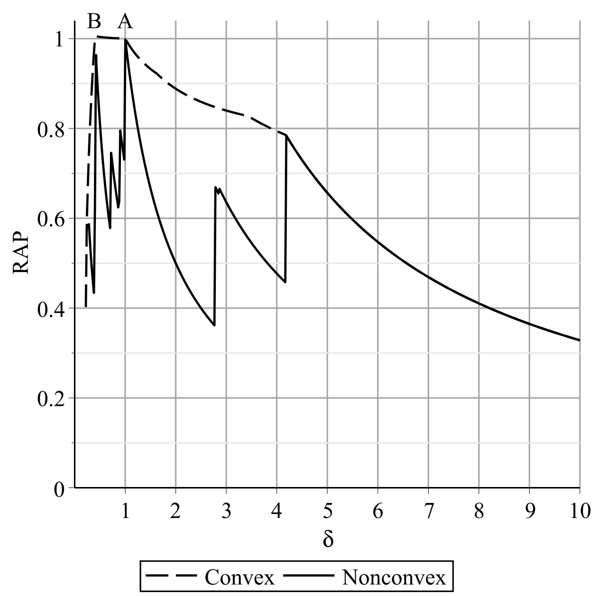


(b)

Figure 6: Representation of Radial Section for Observation 82 in Input-Output Space (a) and Evolution of RAP (b)



(a)



(b)

Article	Sample	# Inp.	# Outp.	Sector	Remarks
Färe et al (85)	32	3	1	Electricity	
Haag et al (92)	41	4	2	Agriculture	
Färe et al (83)	86	3	1	Electricity	Unbalanced (N=20 & T=5)
Cesaroni (11)	92	2	5	Car registration	
Fan et al (96)	471	3	1	Agriculture	

Table 1: Sources of Empirical Data

Sample		Convexity			Nonconvexity			OTE^{NC}&
		OTE	SCE	TE	OTE	SCE	TE	$\neg OTE^{C}$
Färe et al (85)	#Eff. obs.	2	2	9	2	2	29	0
	Mean	0.905	0.952	0.951	0.905	0.906	0.998	0.000
	Li-test†				0.000	6.887***	8.533***	
Haag et al (92)	#Eff. obs.	8	8	10	20	20	41	12
	Mean	0.841	0.959	0.880	0.923	0.923	1.000	0.856
	Li-test†				4.208***	2.270**	na	
Färe et al (83)	#Eff. obs.	4	4	18	4	4	68	0
	Mean	0.897	0.966	0.930	0.898	0.907	0.990	0.000
	Li-test†				0.000	21.926***	26.032***	
Cesaroni (11)	#Eff. obs.	9	9	15	12	12	56	3
	Mean	0.652	0.876	0.733	0.702	0.761	0.911	0.972
	Li-test†				0.313	5.547***	21.123***	
Fan et al (96)	#Eff. obs.	18	18	49	60	60	164	42
	Mean	0.765	0.945	0.811	0.841	0.921	0.913	0.924
	Li-test†				18.459***	19.999***	52.878***	

† Li test: critical values at 1% level = 2.33 (***); 5% level = 1.64 (**); 10% level = 1.28 (*).

Table 2: Decomposition of Overall Technical Efficiency: Convex and Nonconvex

Sample	Convexity			Nonconvexity		
	IRS	CRS	DRS	IRS	CRS	DRS
Färe et al (85)	78.13%	6.25%	15.63%	78.13%	6.25%	15.63%
Haag et al (92)	53.66%	19.51%	26.83%	43.90%	48.78%	7.32%
Färe et al (83)	66.28%	4.65%	29.07%	73.26%	4.65%	22.09%
Cesaroni (11)	83.70%	9.78%	6.52%	78.26%	13.04%	8.70%
Fan et al (96)	52.44%	3.82%	43.74%	52.23%	12.74%	35.03%

Table 3: Returns to Scale on Convex and Nonconvex Technologies: Basic Results

Sample	# Common effic. obs.			Conflicting cases			
	IRS	CRS	DRS	IRS-DRS	CRS-IRS	CRS-DRS	Total conflicts
Färe et al (85)	15.63%	6.25%	6.25%	0.00%	0.00%	0.00%	0.00%
Haag et al (92)	4.88%	19.51%	0.00%	9.76%	19.51%	9.76%	39.02%
Färe et al (83)	16.28%	4.65%	0.00%	6.98%	0.00%	0.00%	6.98%
Cesaroni (11)	2.17%	9.78%	1.09%	7.61%	2.17%	1.09%	10.87%
Fan et al (96)	1.70%	3.82%	1.91%	13.80%	3.40%	5.52%	22.72%

Table 4: Returns to Scale on Convex and Nonconvex Technologies: Common Efficient Observations and Conflicts