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2015-08

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A Restatement

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DOI ou URL de l'article / DOI or URL of scientific paper : lien

Frontier Metatechnologies and Convexity: A Restatement*

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29th October 2015

Abstract

This contribution reconsiders the construction of metafrontiers based on underlying group frontiers using non-parametric technology specifications. We argue that the large majority of articles applying this popular methodology in fact assesses efficiency measures relative to a potentially poor approximation of the metafrontier. We develop a refined methodology for non-parametric specifications of technology yielding a proper non-convex metafrontier. Furthermore, this methodology is empirically applied on a secondary data set to verify the estimation of metatechnology ratios (as defined in O'Donnell, Rao, and Battese (2008)) as well as to illustrate the potential bias of using the currently established methods.

Keywords: Data Envelopment Analysis, Free Disposal Hull, Metafrontier.

PRELIMINARY VERSION. DO NOT CITE WITHOUT PRIOR CONSENT.

*We are grateful to participants of EWEPA 14 in Helsinki and in particular to Sverre Kittelsen and Mikulás Luptácik for most useful comments.

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1 Introduction

Organisations in different industries, regions and countries can face very different production options at different points in time. Differences in so-called production possibilities sets may be due to differences in available technologies (i.e., differences in the methods that are available to transform inputs into outputs) and/or to differences in production environments (e.g., geography, climate, economic infrastructure). This contribution is concerned with one particular method for accounting for this type of heterogeneity when estimating production relationships.

The problem of accounting for heterogeneity when estimating production relationships is quite old. One solution that was initiated by Hayami and Ruttan (1970a) involves estimating some type of metaproduction function. This metaproduction function concept has been empirically applied in several agricultural studies comparing mainly country-level data (e.g., Binswanger, Yang, Bowers, and Mundlak (1987) and Lau and Yotopoulos (1989), among others). An empirical survey to this literature is provided by Trueblood (1989).

Hayami and Ruttan (1970a, p. 898) state: "We may call the envelope of all known and potentially discoverable activities a secular or "meta-production function."" The secular period of production, which is distinct from the short and long run, describes a situation without constraints given by the available fund of technical knowledge and with access to all potentially discoverable knowledge. The basic hypothesis is that all producers have potential access to the same technology, but each may choose to operate on a different part of it depending on specific circumstances (quantities and qualities of natural endowments, relative input prices, basic economic environment, etc.). Thus, each country can produce a given level of output using different factor proportions. Adjustments in input mixes in response to changes in relative input prices represent movements along the isoquant of the metaproduction function which itself is composed of a series of less elastic isoquants. Trueblood (1989, Figure 1) and Hayami and Ruttan (1970b, Figure 5) contain figures depicting the idea of a metaproduction isoquant enveloping a series of less elastic isoquants. At least part of this literature allows for production below the production function, i.e., technical inefficiency (e.g., Lau and Yotopoulos (1989)).

More recently, these basic ideas have been refined and transposed into a frontier production framework by Battese and Rao (2002) and Battese, Rao, and O'Donnell (2004) using stochastic frontier estimators. The seminal article refining the loose ends in the methodology and finalising the formal framework for making efficiency comparisons across groups of firms using both stochastic frontiers and non-parametric, deterministic frontier analysis is clearly O'Donnell, Rao, and Battese (2008).

Thereafter, this metafrontier approach has been widely applied across sectors and even disciplines. Non-parametric metafrontier models have been estimated for sectors varying from agriculture (e.g., Chen and Song (2008)) over banking (e.g., Kontolaimou and Tsekouras (2010)), hotels (Assaf, Barros, and Josiassen (2012) and Huang, Ting, Lin, and Lin (2013), among others) and schools (e.g., Thieme, Prior, and Tortosa-Ausina (2013)) to water utilities (e.g., De Witte and Marques (2009)) and wastewater treatment technologies (e.g., Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)). Empirical metafrontier studies based on stochastic frontier analysis have been presented by Bos and Schmiedel (2007), Lee and Hwang (2011) and Moreira and Bravo-Ureta (2010), among others. Meanwhile, this basic metafrontier framework has been extended in several directions: one example is the transposition to a cost (rather than production) frontier framework (e.g., Huang and Fu (2013)); another example is the estimation of the popular Malmquist productivity indices relative to metafrontiers (see, e.g., Oh and Lee (2010) or Afsharian and Ahn (2015) for a primal index and Thanassoulis, Khanjani Shiraz, and Maniadakis (2015) for a dual approach); a final example is the introduction of more elaborate metafrontier efficiency decompositions (see Kounetas, Mourtos, and Tsekouras (2009)).[1] Note that some work in the literature does not refer explicitly to the metafrontier framework, but implicitly borrows the basic idea of an overall frontier defined as a union of different system or group technologies. Examples include Cooper, Seiford, and Tone (2007, Section 7.5) who talk about combining different systems and Kittelsen, Winsnes, Anthun, Goude, Hope, Häkkinen, Kalseth, Kilsmark, Medin, Rehnberg, and Rättö (2015) who pragmatically define a common frontier over several Nordic countries when comparing hospital productivity.

Reliable estimates of the metafrontier allow to compute reliable estimates of performance measures (e.g., technical efficiency, productivity change). In practice, it is common to use assumptions about production possibility sets to frame the estimation of the metafrontier. A key feature of the O'Donnell, Rao, and Battese (2008) framework is that the metafrontier is explicitly conceived as a union of the underlying group technologies. Basic non-parametric production models are most often underpinned by the assumption that input and output sets as well as the graph of the technology are convex. For instance, convexity of technology means that if two input vectors can produce two output vectors, then any linear combination of these input vectors can also produce some linear combination of these output vectors. When

[1] Sometimes this Malmquist productivity index, which is most frequently estimated within a frontier-based framework, has been combined with the more traditional metaproduction function approach: see, e.g., Fulginiti and Perrin (1998).

group technologies are convex, then the metafrontier as a union of these group technologies need not be convex (see O'Donnell, Rao, and Battese (2008)). However, O'Donnell, Rao, and Battese (2008) suggest estimating the metafrontier as a convex non-parametric technology based on the union of all observations over all groups. This convexification of a possibly non-convex metafrontier may yield a bias that hitherto has hardly ever been documented in the literature.

However, if this convexity assumption is not applicable, then non-parametric estimates of the boundary of the technology are biased. Therefore, associated measures of technical efficiency or productivity change will also be become unreliable. This could potentially undermine the credibility of policies (e.g., price cap (or RPI-X) regulation) where these associated performance measures are used.

Therefore, the main objectives of this contribution are three-fold. First, we clarify and restate the metafrontier framework as developed in O'Donnell, Rao, and Battese (2008) for convex non-parametric group technologies yielding a non-convex metafrontier. Furthermore, we document the potential bias in convexifying this non-convex metafrontier. Second, in case convexity is not suitable, we start from non-convex non-parametric group technologies (known as Free Disposal Hulls) to construct another non-convex metafrontier. Third, the similarities and differences of both convex and non-convex group technologies and the resulting different non-convex metafrontiers are empirically illustrated.

To achieve these objectives, this contribution is structured as follows. Section 2 defines basic concepts and measures of performance associated with the selection and use of technologies. Section 3 explains that these performance measures have the same mathematical structure, but not necessarily the same interpretation, when firms operate in different production environments, or, more generally, whenever firms can be classified into a partition of groups. Section 4 focuses on nonparametric representations of group frontiers and metafrontiers. The key methodological question here is whether the metafrontier can be assumed to be convex rather than non-convex. Section 5 contains an empirical illustration based on a secondary data set. Section 6 summarises the main contributions and outlines future research issues.

2 Technologies, Metatechnologies and Metatechnology Ratios

2.1 Technologies

In O'Donnell (2014, p. 4), a *technology* is defined as a "technique, process or method for transforming inputs into outputs." We adopt this definition here and we view a technology as a type of intellectual capital. For all practical intents and purposes, it is convenient to think of a technology as a book of instructions, or recipes.

Consider in period t an input vector $x^t = (x_1^t, \ldots, x_m^t) \in \mathbb{R}_+^m$, an output vector $y^t = (y_1^t, \ldots, y_n^t) \in \mathbb{R}_+^n$ and a set of S^t observations $A^t = \{(x_1^t, y_1^t), \ldots, (x_{S^t}^t, y_{S^t}^t)\} \in \mathbb{R}_+^{m+n}$.[2] We consider the case where this set of observations A^t in period t can be partitioned into G^t mutually exclusive groups A_g^t ($G^t > 1$ and $g = 1, \ldots, G^t$) each having a subset of S_g^t observations, where (i) $A_g^t \subset A^t$, (ii) $A_g^t \cap A_h^t = \varnothing$ for $g \neq h$, and (iii) $\cup_{g=1}^{G^t} A_g^t = A^t$. We also introduce the index set $J_g^t = \{k : (x_k^t, y_k^t) \in A_g^t\}$.

The set of all pairs of input and output vectors that are feasible using the technology determined by A_g^t in period t is:

$$T_g^t \equiv \{(x^t, y^t) \in \mathbb{R}_+^{m+n} : x^t \text{ can produce } y^t \text{ using the technology determined by } A_g^t\}. \quad (1)$$

This technology is referred to as *technology g* and its boundary as *technology g frontier*.

Equivalent representations of T_g^t include technology-specific output and input sets. Given our focus on input-oriented efficiency measurement later on, we concentrate on the input set. A technology-specific input set contains all input vectors that can produce a given amount of output y^t using a given technology g in period t:

$$L_g^t(y^t) \equiv \left\{x^t : (x^t, y^t) \in T_g^t\right\}. \quad (2)$$

Under weak regularity conditions, this set can also be represented using the following

[2] Drawing from the engineering economics literature, one can also complement observed data with so-called pseudo-data generated by engineering models to obtain larger sample sizes (see Griffin (1979)). This view is not without criticism (e.g., Maddala and Roberts (1981)).

technology-specific input distance function:

$$ID_g^t(x^t, y^t) \equiv \sup_{\lambda \in \mathbb{R}_+} \left\{ \lambda : x^t/\lambda \in L_g^t(y^t) \right\}. \tag{3}$$

This function is nonnegative, linearly homogeneous in inputs, and no less than unity for all $x^t \in L_g^t(y^t)$. An associated measure of residual input-oriented technical efficiency ($RITE$) is:

$$RITE_g^t(x^t, y^t) \equiv 1/ID_g^t(x^t, y^t). \tag{4}$$

This is a radial measure of efficiency that indicates the maximum equiproportionate reduction in x^t which still allows production of y^t using technology g in period t. The sense in which it is a residual measure is explained in Subsection 2.3.

2.2 Metatechnologies

If we view a technology as a recipe, then we can follow Caselli and Coleman (2006, p. 509) and talk about "a library, containing blueprints, or recipes to turn inputs into outputs". In this contribution, the set of technologies (or recipes) available in period t is referred to as the *period t metatechnology*.[3]

The period t *metatechnology* is the set of all input and output vector pairs that are feasible by at least one of the g technologies available:

$$MT^t \equiv \{(x^t, y^t) \in \mathbb{R}_+^{m+n} : \exists g \in \{1, \ldots, G^t\} \text{ such that } x^t \text{ can produce } y^t \text{ using technology } g\}. \tag{5}$$

An equivalent definition is:

$$MT^t \equiv T_1^t \cup T_2^t \cup \cdots \cup T_{G^t}^t. \tag{6}$$

The boundary of this metatechnology is referred to as the *period t metafrontier*. Equivalent representations of meta-"production possibilities sets" (or metatechnologies for short) include meta-output and meta-input correspondences. The meta-input correspondence, for example, contains all input vectors that can produce a given amount of output y^t using a given period t metatechnology:

$$ML^t(y^t) \equiv \left\{ x^t : (x^t, y^t) \in MT^t \right\}. \tag{7}$$

[3]Note that our notions of a technology and a metatechnology are very broad. Other concepts like General Purpose Technologies have a much more restricted definition: see, e.g., Bresnahan and Trajtenberg (2005) or Lipsey, Carlaw, and Bekar (2005) for details.

An equivalent definition is:

$$ML^t(y^t) \equiv L_1^t(y^t) \cup L_2^t(y^t) \cup \cdots \cup L_{G^t}^t(y^t). \tag{8}$$

Again, under weak regularity conditions, a meta-input correspondence can be represented using a period t input distance function:

$$ID^t(x^t, y^t) \equiv \sup_{\lambda \in \mathbb{R}_+} \left\{ \lambda : x^t/\lambda \in ML^t(y^t) \right\}. \tag{9}$$

An equivalent definition is:

$$ID^t(x^t, y^t) \equiv \max\{ID_1^t(x^t, y^t), ID_2^t(x^t, y^t), \ldots, ID_{G^t}^t(x^t, y^t)\}. \tag{10}$$

Again, $ID^t(x^t, y^t)$ is nonnegative, linearly homogeneous in inputs, and no less than unity for all $x^t \in ML^t(y^t)$. The reciprocal of $ID^t(x^t, y^t)$ is a common measure of input-oriented technical efficiency (ITE). To be precise, for a firm using inputs x^t to produce outputs y^t in period t, its ITE is defined as follows:

$$ITE^t(x^t, y^t) \equiv 1/ID^t(x^t, y^t). \tag{11}$$

An equivalent definition is

$$ITE^t(x^t, y^t) \equiv \min\{RITE_1^t(x^t, y^t), RITE_2^t(x^t, y^t), \ldots, RITE_{G^t}^t(x^t, y^t)\}. \tag{12}$$

Again, $ITE^t(x^t, y^t)$ is a radial measure of efficiency that lies in the closed unit interval. It indicates the maximum equiproportionate reduction in x^t which still allows production of y^t using the period t metatechnology. An equivalent output-oriented efficiency measure based on a similar enumeration over groups has recently been defined in Afsharian and Ahn (2015). Note that it can be undefined for some input-output combinations that are not contained in the technology metaset T^t (see Briec and Kerstens (2009) for more details on infeasibilities).

2.3 Metatechnology Ratios

A metatechnology ratio is a measure indicating whether a firm has chosen the most productive technology that is available. Depending on measurement-orientation, different types of metatechnology ratios are available. For example, an input-oriented metatechnology ratio (IMR) is defined using input distance functions. To be precise, for a firm using inputs x^t to produce outputs y^t using technology g in period t, its IMR is defined as:

$$IMR_g^t(x^t, y^t) \equiv ID_g^t(x^t, y^t)/ID^t(x^t, y^t). \tag{13}$$

Since $ID_g^t(x^t, y^t) \leq ID^t(x^t, y^t)$, $IMR_g^t(x^t, y^t) \leq 1$. An equivalent definition is:

$$IMR_g^t(x^t, y^t) \equiv ITE^t(x^t, y^t)/RITE_g^t(x^t, y^t). \tag{14}$$

This leads to the following convenient decomposition of the ITE for a particular observation:

$$ITE^t(x^t, y^t) = IMR_g^t(x^t, y^t) \times RITE_g^t(x^t, y^t). \tag{15}$$

Thus, technical efficiency measured with respect to the metafrontier can be decomposed into the product of a metatechnology ratio (measuring how close a technology-specific frontier is to the metafrontier) and a measure of residual technical efficiency (measuring how close a firm is operating to the group-specific frontier). Equation (15) is the time dependent input analogue of the output-oriented efficiency decomposition in O'Donnell, Rao, and Battese (2008, Eq. 10). Observe that $RITE$ is a *residual* measure in the sense that it is the component of ITE that remains after accounting for the IMR (i.e., for technology choice). It can be viewed as the component of ITE that is due to poor use of the chosen technology (i.e., failure to follow the recipe). Like measures of ITE and $RITE$, the IMR lies in the closed unit interval. This measure of performance is the time dependent input analogue of the output-oriented metatechnology ratio defined by O'Donnell, Rao, and Battese (2008, Eq. 9).[4]

3 Other Types of Metafrontiers

The metafrontier concept is relevant in other empirical contexts too. We first develop the case where technologies depend on the environment. Then, we discuss the role of the time dimension in the metafrontier framework. Thereafter, we establish some other links with the existing literature thereby potentially extending the scope for application of the metafrontier framework.

For a simple illustration, suppose there is only one agricultural technology available in

[4]If the metatechnology is time-invariant (i.e., if there is no technical change, or no new technologies are discovered and no existing technologies are lost), then our t notation can be suppressed and the measure defined by O'Donnell, Rao, and Battese (2008, Eq. 9) results.

period t, and that this technology works better in some environments than in others (e.g., there is only one method for growing a particular type of organism, and this method works better in a cool-dry environment than it does in a warm-humid environment). Suppose that the number of production environments (or states of nature) is finite. The set of all input and output vector pairs that are feasible in environment g in period t can be written as:

$$T_g^t \equiv \{(x^t, y^t) \in \mathbb{R}_+^{m+n} : x^t \text{ can produce } y^t \text{ in environment } g \text{ in period } t\}. \tag{16}$$

This environment-specific (or state-contingent) technology has exactly the same mathematical structure as the group-specific technology (1). Associated environment-specific input sets and distance functions are therefore given by (2) and (3). By extension, the set of all input and output vector pairs that are feasible in period t is $MT^t \equiv T_1^t \cup T_2^t \cup \cdots \cup T_{G^t}^t$ where G^t is now interpreted as the number of states of nature that can occur in period t. The associated meta-input correspondence, distance function and measure of ITE, for example, are still given by (7) to (11). Importantly, the boundary of MT^t is still a metafrontier.

The message to take from this discussion is that various functions and measures of performance associated with different production environments have the same mathematical structure, but not necessarily the same interpretation, as those associated with the selection and use of technologies. For example, the input-oriented meta-technology ratio should now be interpreted/described as an input-oriented meta-environment ratio.

From a mathematical viewpoint, the distinction between technologies and environments is immaterial. The distinguishing feature of metafrontier models is that firms can be partitioned into *groups*. In the remainder of this contribution, we avoid referring to sets and frontiers as technology-specific or environment-specific (state-contingent) sets and frontiers. Instead we refer to them as group-specific sets and frontiers (or simply group sets and frontiers). Thus, we will talk about group technologies, group distance functions, and group frontiers.

In a metafrontier framework, assumptions on the number of groups over time translate into well-known hypotheses about the nature of technological change (see, e.g., Tulkens and Vanden Eeckaut (1995)). One can distinguish three cases. First, if one assumes that the number of groups can both increase or decrease over time (e.g., since both technical progress and technical regress are possible), then the period t metatechnology is estimated using data points from this period t only. Such an contemporaneous approach is used when, for instance, computing Malmquist productivity indices using metafrontiers (e.g., Oh and Lee (2010)). Second, if the number of groups can increase over time but it can never decrease (e.g., since only technical progress is possible, but technical regress is excluded), then the period t metatechnology is estimated using data points from all periods up to and including period t. Such a sequential approach dates back to, e.g., Diewert (1980). Finally, if all groups exist in all time periods (e.g., if there is no technical change), then the period t metatechnology is estimated using data points from all periods. This is also known as the intertemporal approach. Note that if there is no technical change, then the "t" notation maintained so far can be suppressed and most definitions somewhat simplify.

Other links with the existing literature can be made that reveal the wide scope for application of the metafrontier framework as sketched so far. First, there is a limited literature that uses so-called cross-frontier analysis as initially proposed by Cummins, Weiss, and Zi (1999). Upon scrutiny, the formal analysis is structurally similar to the above metafrontier framework, but the analysis seems to be limited to two groups only. One can conjecture that if more than two groups would be involved, the metafrontier framework should be reinvented. Empirical applications along these lines seem mainly focusing on the insurance sector (e.g., Biener and Eling (2012), Brockett, Chang, Rousseau, Semple, and Yang (2004), Erhemjamts and Leverty (2010), among others).

Second, there is a wide variety of proposals to distinguish heterogeneity among production units in a frontier context using some multivariate statistical methodology to classify the units in the sample into either some natural number of groups (e.g., using some variant of cluster analysis) or some set of existing groups (e.g., using discriminant analysis and its variations). Examples include Samoilenko and Osei-Bryson (2010) and Llorca, Orea, and Pollitt (2014), among others. Along similar lines, frontier estimates have been used in a variety of ways to distinguish some strategic groups (e.g., Athanassopoulos (2003), Warning (2004), among others). As long as these groups form a partition, the metafrontier framework can be applied to compare these groups.

4 The Metafrontier is Non-Convex: An Illustration Using Non-Parametric Technology Specifications

How can group frontiers and metafrontiers be estimated using non-parametric methods? For the empirical application where we use an intertemporal frontier, we implicitly assume that the number of groups is constant over time such that the metatechnology is estimated using data points from all periods. Hence, while we could suppress the "t" notation, for the sake of consistency in the remainder this "t" notation is maintained.

This section starts from a discussion of traditional convex non-parametric group technologies and argues and illustrates that the associated metafrontier is almost inevitably non-convex. Thereafter, we question more systematically the convexity axiom at both the level of the group and metatechnologies by specifying Free Disposal Hulls (FDH) as well as the union of such FDH group frontiers.

4.1 Non-Parametric Group Frontiers

This section outlines the estimation of the group frontiers. For this purpose, it is convenient to introduce observation subscripts into the notation so that, for example, $x_i^t = (x_{1i}^t, \ldots, x_{mi}^t) \in \mathbb{R}_+^m$ now denotes the input vector of firm i in period t. Recall that A_g^t denotes the set of input-output combinations of those firms belonging to group g in period t having cardinality S_g^t. Thus, for example, if firms 1, 3 and 8 are the only firms that belong to group 4 in period t, then $A_4^t = \{(x_1^t, y_1^t), (x_3^t, y_3^t), (x_8^t, y_8^t)\}$ and $S_4^t = 3$. The corresponding index set $J_4^t = \{1, 3, 8\}$.

A non-parametric estimate of the group-g frontier can be obtained by enveloping the S_g^t data points in group g while maintaining some basic regularity assumptions (see Hackman (2008) or Ray (2004)). In the remainder of this contribution, we assume the group-g production possibility set or technology satisfies some combination of the following conventional assumptions:[5]

(T.1) $(0,0) \in T_g^t$ and $(0,y) \notin T_g^t$ if $y \geq 0$.

(T.2) T_g^t is a closed subset of $\mathbb{R}_+^{m+n}$.

(T.3) If $(x,y) \in T_g^t$ and $(x',y') \in \mathbb{R}_+^{m+n}$, then $(x',-y') \geq (x,-y) \Rightarrow (x',y') \in T_g^t$.

(T.4) T_g^t is convex.

These rather traditional axioms on the group technology state that: (i) inaction is feasible, and there is no free lunch, (ii) the set of feasible output input combinations contains all the points on its boundary (closedness), (iii) inputs and outputs are freely (or strongly) disposable, and (iv) the production possibilities set is convex (see, e.g., Hackman (2008) for details). This last assumption is not always maintained in this contribution.

[5]If there would only be one technology available (which is excluded a priori since $G^t > 1$), then the "g" notation can be suppressed to yield the axioms found in most textbooks.

The standard nonparametric estimator of a convex (C), strongly disposable group-g technology that exhibits variable returns to scale (VRS) is:

$$T_g^{t,C} = \left\{ (x^t, y^t) \in \mathbb{R}_+^{m+n} : \sum_{s \in J_g^t} \lambda_s x_s^t \leq x^t, \sum_{s \in J_g^t} \lambda_s y_s^t \geq y^t, \sum_{s \in J_g^t} \lambda_s = 1, \lambda_s \in \mathbb{R}_+, \forall s \in J_g^t \right\}. \tag{17}$$

This estimator satisfying (T.1)–(T.4) is commonly known under the label Data Envelopment Analysis (DEA). While the single output version of this estimator goes back to at least Afriat (1972), the general multi-output version appears to have been introduced to the literature by Banker, Charnes, and Cooper (1984) and Färe, Grosskopf, and Lovell (1983). This VRS DEA estimator (17) can be used to construct an estimator of the group-g input distance function (3) and the associated measure of $RITE$ (4). This requires solving for each evaluated observation a linear programming (LP) problem (see Hackman (2008) or Ray (2004)).

In case the convexity axiom (T.4) is disputed, a non-convex version of the above estimator satisfying (T.1)–(T.3) can be specified as follows:

$$T_g^{t,NC} = \left\{ (x^t, y^t) \in \mathbb{R}_+^{m+n} : \sum_{s \in J_g^t} \lambda_s x_s^t \leq x^t, \sum_{s \in J_g^t} \lambda_s y_s^t \geq y^t, \sum_{s \in J_g^t} \lambda_s = 1, \lambda_s \in \{0, 1\}, \forall s \in J_g^t \right\}. \tag{18}$$

This estimator is commonly known as the Free Disposal Hull (FDH) estimator. The single output version of this estimator also goes back to Afriat (1972). Again, this estimator can be used to construct an estimator of the group-g input distance function (3) and the associated measure of $RITE$ (4). This requires solving a mixed integer program for each evaluated observation. However, Leleu (2006) and Briec, Kerstens, and Vanden Eeckaut (2004) propose an LP solution and a closed form solution based on an implicit enumeration strategy, respectively.

Geometrically, the boundary of the set $T_g^{t,NC}$ is a simple monotonic hull of the S_g^t data points of A_g^t (denoted $mon(A_g^t)$), while the boundary of the set $T_g^{t,C}$ is a convexified version of this same monotonic hull (denoted $cm(A_g^t)$). Formally, the monotonic hull of a set $A_g^t \subset \mathbb{R}_+^{m+n}$ is defined as:

$$mon(A_g^t) \equiv (A_g^t + (\mathbb{R}_+^m \times \mathbb{R}_-^n)) \cap \mathbb{R}_+^{m+n}. \tag{19}$$

The convex monotonic hull of the same set is:

$$cm(A_g^t) \equiv mon(A_g^t) \cup con(A_g^t), \tag{20}$$

where the convex hull $con(A_g^t)$ is defined:

$$con(A_g^t) \equiv \{z \in \mathbb{R}_+^{m+n} : z = \alpha z_1 + (1-\alpha) z_2 \text{ for all } z_1, z_2 \in A_g^t, \alpha \in [0,1]\}. \quad (21)$$

Figure 1 illustrates these notions on a set A containing 32 observations in the single input-output case. The convex hull $con(A)$ is the closed region with dashed edge starting from observation 1 and continuing all the way round till observation 1 again. The monotonic hull $mon(A)$ consists of the region located below and to the right of the solid polyline starting vertical at the bottom towards observation 3 and then continuing to observations 2, 1, 23, 15, 31 and 16 using horizontal and vertical connections to end horizontally from observation 16 onwards. Unifying both these regions results in the convex monotonic hull $cm(A)$. This is the region below and to the right of the polyline starting with the vertical solid line to observation 2, then continuing via the dashed lines to observations 1, 15 and 16 to end with the horizontal solid line from observations 16 onwards.

Figure 1 about here

4.2 Non-Parametric Metafrontier

Turning to the specification and estimation of the metafrontier, since things are slightly more difficult, we prefer to start with a simple single-input single-output illustration. Figure 2 depicts a period t metatechnology consisting of the union of two convex group technologies (i.e., $MT^t = T_1^t \cup T_2^t$). In this figure, the convex technology T_1^t (resp. T_2^t) consists of all points between the horizontal axis and the frontier $A_1B_1F_1H_1I_1$ (resp. $A_2B_2F_2H_2I_2$). The period t metatechnology MT^t consists of all points between the horizontal axis and the frontier $A_1B_1PB_2F_2H_2I_2$. While each of the group technologies is convex, the period t metatechnology clearly is non-convex and no firm can produce at a point outside this set, e.g., in the region determined by $B_1PB_2F_2B_1$). The convexified version of this metatechnology is obtained by exactly adding this region. If one would add an additional technology in period t, then the period t metatechnology will only in very rare occasions become a convex set. While a third technology may fill up part of the region determined by $B_1PB_2F_2B_1$, depending on its range relative to both existing technologies it is likely to create further non-convexities to the left or to the right of the existing one.

Obviously, one must realise that in a multiple input and multiple output space (rather than in a single input single output case) a union of more than two group technologies is only by sheer coincidence gone end up yielding a convex metafrontier. The most likely outcome is simply that the resulting metafrontier is non-convex. Therefore, the basic question is to what extent convexity is justifiable as an axiom at the metafrontier level?

Figure 2 about here

While O'Donnell, Rao, and Battese (2008, p. 237-238) do recognise the importance of non-convexity of the period t metatechnology, they nevertheless propose an estimator that convexifies this set. Their estimator of the period t metatechnology is an estimator of the form:[6]

$$\widehat{MT}^{t,C} = \left\{ (x^t, y^t) \in \mathbb{R}_+^{m+n} : \sum_{s=1}^{S^t} \lambda_s x_s^t \leq x, \sum_{s=1}^{S^t} \lambda_s y_s^t \geq y, \sum_{s=1}^{S^t} \lambda_s = 1, \lambda_s \in \mathbb{R}_+, \forall s = 1, \ldots, S^t \right\}. \tag{22}$$

Except in restrictive special cases (e.g., there is only one group - which is excluded by default), $\widehat{MT}^{t,C} \supseteq MT^{t,C}$ with $MT^{t,C} = T_1^{t,C} \cup T_2^{t,C} \cup \cdots \cup T_{G^t}^{t,C}$. Thus, the estimator (22) is normally biased. In other words, the convex monotonic hull of S^t data points is larger than or equal to the union of G^t convex monotonic hulls of S_g^t data points:

$$cm(A^t) \supseteq \cup_{g=1}^{G^t} cm(A_g^t). \tag{23}$$

Except in restrictive special cases, the convex monotonic hull of the S^t observations is strictly larger than the union of the G^t convex monotonic hulls.[7] The basic question is then whether this convex monotonic hull (22) offers a good approximation to the true non-convex union of convex monotonic hulls.

This question has hardly been touched upon in the literature so far. Tiedemann, Francksen, and Latacz-Lohmann (2011) are probably the first to suggest an alternative estimator of the non-convex period t metatechnology and the associated measure of ITE (11). Their alternative VRS DEA estimator of the ITE of firm i in period t is exactly the definition (12) above.[8] This amounts to estimating input-oriented technical efficiency with respect to each of the group-specific frontiers, then taking the minimum of these efficiency estimates.

[6] O'Donnell, Rao, and Battese (2008) assume that all groups exist in all periods, and that the number of firms in each group is the same in each period (i.e., that the dataset is balanced). Thus, their estimator is slightly different from (22) (for a start, it uses data points from all firms in all periods).

[7] In the literature, one can find statements obviously violating this basic fact. For instance, Kounetas, Mourtos, and Tsekouras (2009, p. 211) state: "It is not difcult to see that the minimal, with respect to set inclusion, technology set satisfying the above plus the convexity condition (see related discussion in Rao et al., 2003) is the convex hull of the jointure of all technology sets".

[8] These authors describe this estimator, but offer no formal definition.

A formal definition of an equivalent output efficiency measure using a similar enumeration over groups is defined in Afsharian and Ahn (2015).

In practice, this estimation procedure is complicated by the fact that, except in restrictive special cases (e.g., there is only one group - which is excluded by default), only the measure of $RITE$ whereby an observation is projected with respect to its own group frontier is always well-defined, while all other projections of the same observation with respect to the other group-frontiers may potentially be undefined. In Figure 2, for example, firms in group 1 cannot operate at point F_2. Therefore, the input-oriented measure of technical efficiency with respect to the group-1 frontier is unity divided by zero, which is mathematically undefined.[9] The estimator (12) has rarely been applied so far (e.g., Huang, Ting, Lin, and Lin (2013) or Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)).

This computational issue disappears when using FDH estimators, because the monotonic hull of S^t observations now equals the union of G^t monotonic hulls of S^t_g observations:

$$mon(A^t) = \cup_{g=1}^{G^t} mon(A^t_g). \tag{24}$$

While the FDH estimator of the group-g technology is given by (18), the FDH estimator of the period t metatechnology is:

$$MT^{t,NC} \equiv \left\{ (x^t, y^t) \in \mathbb{R}^{m+n}_+ : \sum_{s=1}^{S^t} \lambda_s x^t_s \leq x, \sum_{s=1}^{S^t} \lambda_s y^t_s \geq y, \sum_{s=1}^{S^t} \lambda_s = 1, \right.$$
$$\left. \lambda_s \in \{0,1\}, \forall s = 1, \ldots, S^t \right\}. \tag{25}$$

The associated FDH estimator of the ITE of firm i in period t directly follows definition (11). Thus, by using FDH estimators one can avoid computing several measures of $RITE$, thereby bypassing any computational problems associated with infeasibilities.

[9] Huang, Ting, Lin, and Lin (2013, program (9)) provide a single mathematical program defined over the union of all group frontiers: but it risks breaking down when infeasibilities occur, in which case one must resort to optimisation per group technology. Afsharian and Ahn (2015) offer a similar solution in the context of defining a global Malmquist productivity index. A basic formulation of this same solution can already been found in Cooper, Seiford, and Tone (2007, Section 7.5).

4.3 Convexity Questioned

Obviously, the plausibility of the convexity axiom must be judged on its own and is independent of whether it is applied at the group frontier and/or metafrontier level. In general, the convexity axiom is justified by a time divisibility argument (see Hackman (2008, p. 39)). Indivisibilities, externalities, increasing returns to scale, etc. can be ignored as long as production processes are perfectly time divisible. Of course, perfect time divisibility seems like a strong assumption in, for instance, industrial processes, since it ignores the existence of positive setup times.

Furthermore, in the case of the metafrontier, a constructive argument is used to argue against the convexity of the period t metatechnology: any metatechnology constructed as the union of even convex group technologies is unlikely to be convex (unless by pure chance). However, the same reasoning can be applied to the notion of the group technology itself: if a group technology in, e.g., an industrial context, is constructed as a union of a series of underlying process technologies, some of these underlying sets being non-convex, then the resulting group technology is unlikely to be convex. Note that most industrial processes (e.g., cutting stock, scheduling, etc) are fraught with non-linearities and indivisibilities requiring non-linear and/or integer programming to obtain optimal solutions (it suffices to open any operations management handbook). Adding a managerial and administrative layer to these non-convex industrial processes is unlikely to yield a convex group technology. Thus, the upshot is that the convexity of the group technology need not be taken for granted, but ideally requires testing in itself.

Briec, Kerstens, and Vanden Eeckaut (2004) already illustrated how the measurement of technical and scale efficiencies is affected by the convexity axiom. Kerstens and Managi (2012) show how the Luenberger productivity indicator as well as its decomposition into technical change and efficiency change are affected by dispensing with convexity or not. Finally, Briec, Kerstens, and Vanden Eeckaut (2004) establish theoretically, and illustrate empirically, how the convexity axiom not only affects technologies, but also affects the level of the cost function derived from it (see also Balaguer-Coll, Prior, and Tortosa-Ausina (2007) for another empirical illustration).[10] Thus, convexity seems to matter both from a theoretical and empirical perspective.

Figure 3 again depicts a single-input-single-output metatechnology consisting of the union of two group technologies (i.e., $MT^t = T_1^t \cup T_2^t$). The data points in this figure

[10] In fact, WACM is implicitly based on an FDH technology: see Ray (2004).

are, in fact, the data points that were depicted earlier in Figure 2. However, the group-specific technologies are now assumed to be non-convex. In Figure 3, the non-convex group technology T_1^t (resp. T_2^t) consists of all points between the horizontal axis and the frontier $A_1B_1C_1D_1E_1F_1G_1H_1I_1$ (resp. $A_2B_2C_2D_2E_2F_2G_2H_2I_2$). The convexified group-1 (resp. group-2) technology is obtained by adding the region determined by $B_1C_1D_1E_1F_1$ and the triangle $F_1G_1H_1$ (resp. the region determined by $B_2C_2D_2E_2F_2$ and the triangle $F_2G_2H_2$). The period t metatechnology MT^t consists of all points between the horizontal axis and the frontier $A_1B_1C_1D_1B_2C_2D_2E_2F_2G_2H_2I_2$. The convexified version of MT^t is obtained by adding the regions determined by $B_1C_1D_1PB_2QF_1RD_2E_2F_2$ and $F_2G_2H_2$. If the underlying group technologies are non-convex, then adding these two regions yields a biased estimate of the metafrontier.

Figure 3 about here

Thus, one can question the convexity axiom at both the group frontier and metafrontier levels. Therefore, in the empirical section we illustrate three basic issues. (i) What is the difference between the non-convex metafrontiers resulting from the union of non-convex and convex group frontier technologies? Denoting the convex and non-convex estimate of ITE as defined in (12) by $ITE_g^{t,C}(x^t, y^t)$ and $ITE_g^{t,NC}(x^t, y^t)$ respectively, we introduce a convexity-related $CRITE$ as:

$$CRITE_g^t(x^t, y^t) \equiv \frac{ITE_g^{t,C}(x^t, y^t)}{ITE_g^{t,NC}(x^t, y^t)}. \tag{26}$$

Again, since $ITE_g^{t,C}(x^t, y^t) \leq ITE_g^{t,NC}(x^t, y^t)$, $CRITE_g^t(x^t, y^t) \leq 1$. This component $CRITE$ measures the inefficiency amount attributable to convexity operating at the non-convex metafrontier level due to its construction as a union of either non-convex or convex group frontiers.

(ii) What is the difference between the non-convex and convex group frontier technologies? Denoting the convex and non-convex estimate of $RITE$ as defined in (4) by $RITE_g^{t,C}(x^t, y^t)$ and $RITE_g^{t,NC}(x^t, y^t)$ respectively, we can following Briec, Kerstens, and Vanden Eeckaut (2004) introduce a convexity-related $CRRITE$ as:

$$CRRITE_g^t(x^t, y^t) \equiv \frac{RITE_g^{t,C}(x^t, y^t)}{RITE_g^{t,NC}(x^t, y^t)}. \tag{27}$$

Since $RITE_g^{t,C}(x^t, y^t) \leq RITE_g^{t,NC}(x^t, y^t)$, obviously $CRRITE_g^t(x^t, y^t) \leq 1$. This component $CRRITE$ indicates the amount of inefficiency that can be specifically attributed to the convexity axiom at the group frontier level. Note that metatechnology ratios, defined as ratios of the efficiency measures discussed in (i) and (ii), cannot be a priori related to one another in terms of the effect of convexity, though its component measures can be ranked.

(iii) What is the bias introduced by convexifying the metafrontier following the approach outlined in O'Donnell, Rao, and Battese (2008, p. 239)? This amounts to comparing the metafrontier resulting from the union of convex group frontier technologies $ITE_g^{t,C}(x^t, y^t)$ as defined in (12) and the estimate based on (22).

To the best of our knowledge, no single study has ever documented these issues simultaneously. Though some studies have covered some of these topics. For instance, De Witte and Marques (2009) and Thieme, Prior, and Tortosa-Ausina (2013) restrict attention to non-convex group and metatechnologies (and even robust order-m versions of both), but did not compare with their convex counterparts. Tiedemann, Francksen, and Latacz-Lohmann (2011) only compare convex group technologies to the correct non-convex metafrontier defined as the union of group technologies, but they ignore the bias issue (see also Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011)). The bias issue is only partly documented in Huang, Ting, Lin, and Lin (2013) and in the unpublished paper of Breustedt, Francksen, and Latacz-Lohmann (2007): the former article lists the units whose efficiency measure is different on the true non-convex compared to the biased convexified metafrontiers (see their Table 4), the latter study illustrates these same differences in metafrontier efficiency measures mainly graphically.[11] But, none of these studies reports any test statistic regarding these differences in metafrontier efficiency measures.

5 Empirical Illustration

In this section, we first present the data set used which has been adopted from an existing article including a data set. Then, we present the empirical results.

5.1 Secondary Data Set: Chilean Hydro Power

Our sample uses 16 hydro-electric power plants from Chile observed over several years with a monthly frequency (Atkinson and Dorfman (2009)). We limit the sample to the single year 1997, and we assume that the number of groups in this year is fixed. This allows specifying an inter-temporal frontier covering all months, resulting in 192 observations in total. The

[11] Both studies mention FDH as a possibility, but do not apply it in their empirical part.

single output is electricity generated. There is information on the quantities of three inputs: labor, capital, and water. Except for the input capital, all remaining flow variables are expressed in physical units. Descriptive statistics for the inputs and the single output are available in Atkinson and Dorfman (2009).

One can distinguish two groups of hydro-electric plants in this sample: a dam (impoundment) forming a reservoir, and run-of-river (diversion).[12] This partitions the sample of 192 observations into 60 dam plants and 132 run-of-river plants. It is well-known in the hydro-electric power industry that the often smaller run-of-river plants are always base load plants, while the dam plants can be employed as either base or peak load.

5.2 Empirical Results

Table 1 presents empirical results for decomposition (15) starting from convex and non-convex group frontiers yielding both different non-convex metafrontiers. This table is structured as follows. The first three columns report results based upon the convex group frontiers, while the last three columns list their non-convex counterpart results. The last column reports the number of infeasibilities when comparing observations to the alternative groups (in casu, there is just one cross-group comparison). The first row states the number of efficient observations while the next three rows report basic descriptive statistics related to the sample level: geometric average, standard deviation, and minimum.[13] The second and third part focuses separately on the descriptive statistics of the group of dam and the group of run-of-river plants, respectively.

Table 1 about here

Four basic observations can be made with regard to Table 1. First, convex group frontiers and the resulting metafrontier yield lower input efficiency measures compared to their non-convex counterparts, an observation in concordance with other studies comparing technical efficiency on convex and non-convex production frontiers (see supra). Non-convex group frontiers and the metafrontier have a manifold of efficient observations compared to the convex frontier counterparts.

Second, as a ratio of efficiency measures, the relation between convex and non-convex metatechnology ratios cannot a priori be signed (see also supra). This clearly shows up in

[12]Another type of hydropower called pumped storage (a potential third group) is not present in the sample.
[13]The use of an geometric average guarantees that the multiplicative decomposition (15) holds exactly.

the empirical results: while the convex MTR is lower than the non-convex MTR at the sample level and for dam plants, the reverse holds true for run-of-river plants. The number of efficient observations is now much closer to one another, whereby in the case of run-of-river plants the convex case outnumbers the non-convex case by just one.

Third, the convex MTR for run-of-river plants is on average unity. In fact, for the convex case all MTR scores are at unit level. For the non-convex MTR the average is very close to unity: just one observation is MTR inefficient. This indicates that these run-of-river plants could not benefit from having access to the metafrontier. Inversely, dam plants could benefit from having access to production plans that would make them resemble more closely to run-of-river plants. Thus, run-of-river plants seem to be an almost uniformly superior technology. Note that the determination of inferior and superior technologies is one of the interesting application areas that have relatively little been pursued using metafrontiers. For example, Sala-Garrido, Molinos-Senante, and Hernández-Sancho (2011) compare four wastewater treatment technologies and find one of these treatment technologies to dominate all others.

Fourth, 29 organisations obtain infeasible solutions when computing some of the distances to the convex group technologies in determining the measure (10). This amounts to about 15% of the sample.

Figure 4 displays the kernel densities of the distribution of the metafrontiers and group frontiers in parts (a) and (b), respectively, starting off from either non-convex or convex group frontiers.[14] For both distributions it seems clear that the densities between results starting from non-convex and convex group frontiers differ rather substantially. To formally assess the eventual differences between convex and non-convex frontiers, we employ a test statistic initially proposed by Li (1996) that is valid for both dependent and independent variables.[15] Note that dependency is characteristic for extremum or frontier estimators, since efficiency levels depend, among others, on sample size. The null hypothesis of this Li-test states that both convex and non-convex distributions for a given efficiency score and underlying specification of technology are equal. This test is only performed at the sample level and reported on the fifth row in Table 1. One can reject the null hypothesis of equal distributions for both the group frontiers and the metafrontiers. However, the equality of the convex and non-convex MTR distributions cannot be rejected.

[14] For reasons of comparability, a common Sheather and Jones plug-in bandwidth is used to compare both data series at stake (see, e.g., Sheather (2004)).

[15] See Fan and Ullah (1999) for a refinement of the same test.

Figure 4 about here

Table 2 reports convexity-related inefficiencies for ITE and $RITE$ as defined in (26) and (27) in the two first columns. Furthermore, the last six columns document the potential bias in estimating ITE and MTR when using convex group frontiers using the O'Donnell, Rao, and Battese (2008) approach. The first three of these six columns compare the metafrontier resulting from the union of convex group frontiers $ITE_g^{t,C}(x^t, y^t)$ as defined in (12) and the estimate based on (22) (denoted Unbiased and Biased, respectively), as well as the ratio of both estimates. The last three of these columns compare the potential bias when computing MTR on such results. Thus, the unbiased results reported in the third and sixth columns are in common with Table 1.

The key conclusions one can make from analysing Table 2 are the following. First, convexity-related ITE ($CRITE$) amounts to about 20% on average at the sample level. For dams this average inefficiency attributable to convexity is even higher (about 24%), while for run-of-river plants this average inefficiency is somewhat lower (about 18%). For 21 observations, there is no convexity-related inefficiency (about 11%). Among dams, only 1 observation is convexity-related efficient, while 20 run-of-river plants are convexity-related efficient. Second, at the sample level convexity-related inefficiency for the $RITE$ component ($CRRITE$) is about 17% on average. For dams this average inefficiency due to convexity is now lower (about 15%), while for run-of-river plants this average inefficiency is now slightly higher than the sample level. Note that for run-of-river plants the $CRITE$ and $CRRITE$ results are identical. Both these amounts of convexity-related inefficiencies in a certain sense just serve to document why the Li-test statistic reported above finds statistically significant differences.

Third, estimating ITE using the convexified approach of O'Donnell, Rao, and Battese (2008) rather than the union of convex group frontiers yields at the sample level on average just a 1.70% difference in efficiency level. Taking a ratio of both estimates reveals a difference of about 2.0% at the sample level. For dams this ratio is notably higher (about 5%), while for run-of-river plants this ratio is somewhat lower (just 1%). If we plug these slightly different estimates in the MTR, then we obtain at the sample level on average just a 2.10% difference in efficiency level. This difference in efficiency level increases to 4.20% for dam plants and decreases to just about 1.00% for run-of-river plants.

Figure 5 displays the kernel densities of the distribution of the metafrontiers starting off from either the convex group frontiers, or simply as a convex monotonic hull of the whole sample. For both distributions it may seem that the densities between both results differ rather substantially. To check whether these observed differences are statistically significant, we again report a Li-test in the fifth row of Table 2. However, the equality of the unbiased and biased efficiency distributions of the metafrontier cannot be rejected. By contrast, the null hypothesis of equal distributions for both the resulting MTR can be rejected. Thus, the convexification approach suggested by O'Donnell, Rao, and Battese (2008) may yield biases in MTR estimates.

Table 2 about here

Figure 5 about here

6 Conclusions

Starting from the seminal contribution of O'Donnell, Rao, and Battese (2008), we have restated the metafrontier approach while focusing on non-parametric specifications of group frontiers and metafrontiers. With respect to the three main objectives of this contribution as stated in the Introduction, we summarise results as follows. First, when starting from convex non-parametric group frontiers, the metafrontier -conceived as a union of group technologies- is normally non-convex. The convexification approach suggested in O'Donnell, Rao, and Battese (2008) can yield a potential bias. Second, if convexity is deemed unsuitable, non-convex non-parametric group frontiers yield another non-convex metafrontier. We have elaborated on the reasons why convexity of group frontiers and metafrontiers need not be assumed a priori, but should ideally be empirically tested.

Third, the similarities and differences of both convex and non-convex group frontiers and the resulting different non-convex metafrontiers have been empirically illustrated using secondary data. Comparing the decompositions of input efficiency estimated on a metafrontier into a residual group efficiency measure and a metatechnology ratio starting from either convex or non-convex group frontiers yields the following results. Both the input efficiency estimated on the metafrontier and the residual group efficiency measure are significantly different when estimated from convex or non-convex group frontiers. However, the resulting metatechnology ratios are not significantly different for our sample.

An additional perspective on these significant differences is obtained from computing convexity-related efficiency components: about 20% and 17% of these measured inefficiencies can be directly attributed to the convexity hypothesis in itself. Finally, when comparing input efficiency estimated on the metafrontier using the union of convex group frontiers or the convexification shortcut proposed in O'Donnell, Rao, and Battese (2008), we find a small bias that happens to be statistically insignificant. However, if we plug these estimates into the metatechnology ratio, we obtain a statistically significant difference.

Hence, the conclusion is that users of the metafrontier methodology should beware to correctly specify the non-convex metafrontier as a union of either convex or non-convex group frontiers. The shortcut suggested in the seminal contribution of O'Donnell, Rao, and Battese (2008) may lead to erroneous results.

Finally, it is clear that further research is welcome to verify how the wide variety of metafrontier applications hinted at in Section 1 are affected by our findings and how their corresponding methodologies eventually need to be refined. Some first steps seem to have been taken by Afsharian and Ahn (2015) when developing some variation on the primal Malmquist productivity index. Furthermore, we can briefly indicate whether and how this approach can be transposed to alternative frontier methodologies. While adopting this meticulous construction of a metafrontier should be rather straightforward in a deterministic parametric approach, its implications for the far more popular stochastic frontier model remain to be explored.

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Figure 1: Monotonic, Convex and Convex Monotonic Hulls of the Same Set

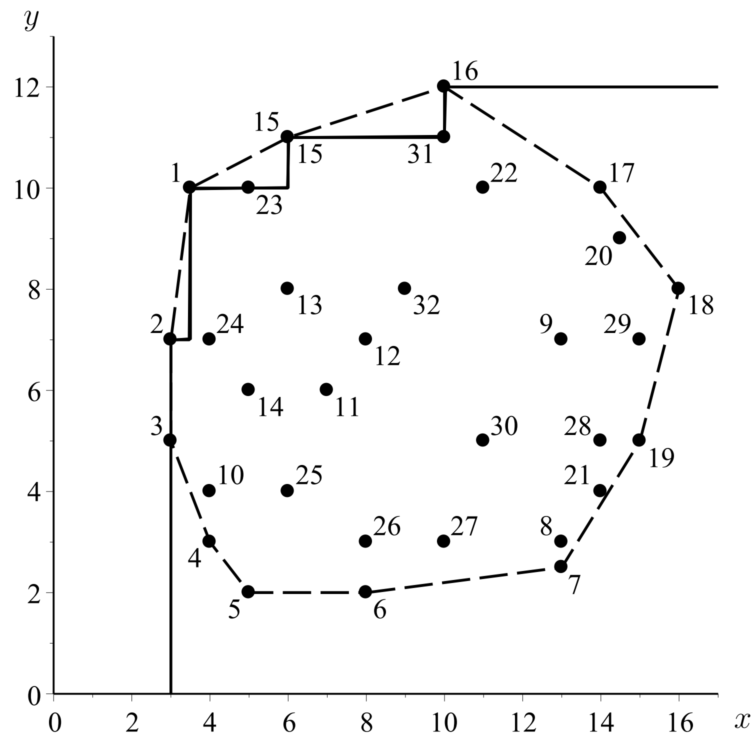


Figure 2: Convex Group-Specific Technologies and Non-Convex Metafrontier

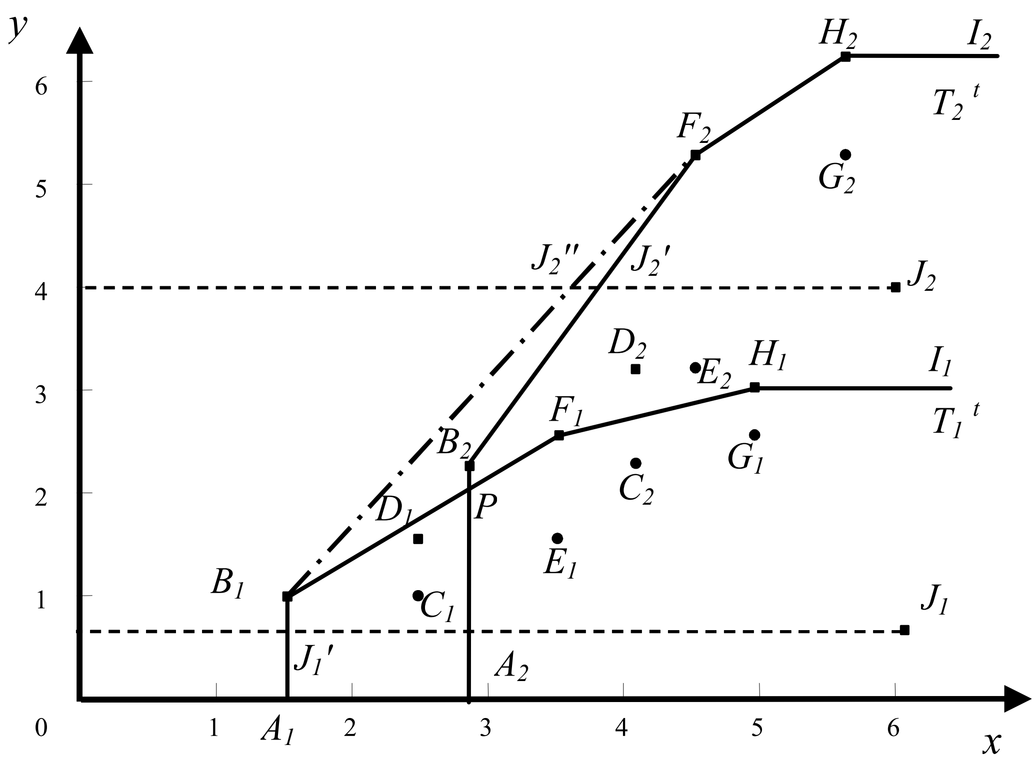


Figure 3: Non-Convex Group-Specific Technologies and Non-Convex Metafrontier

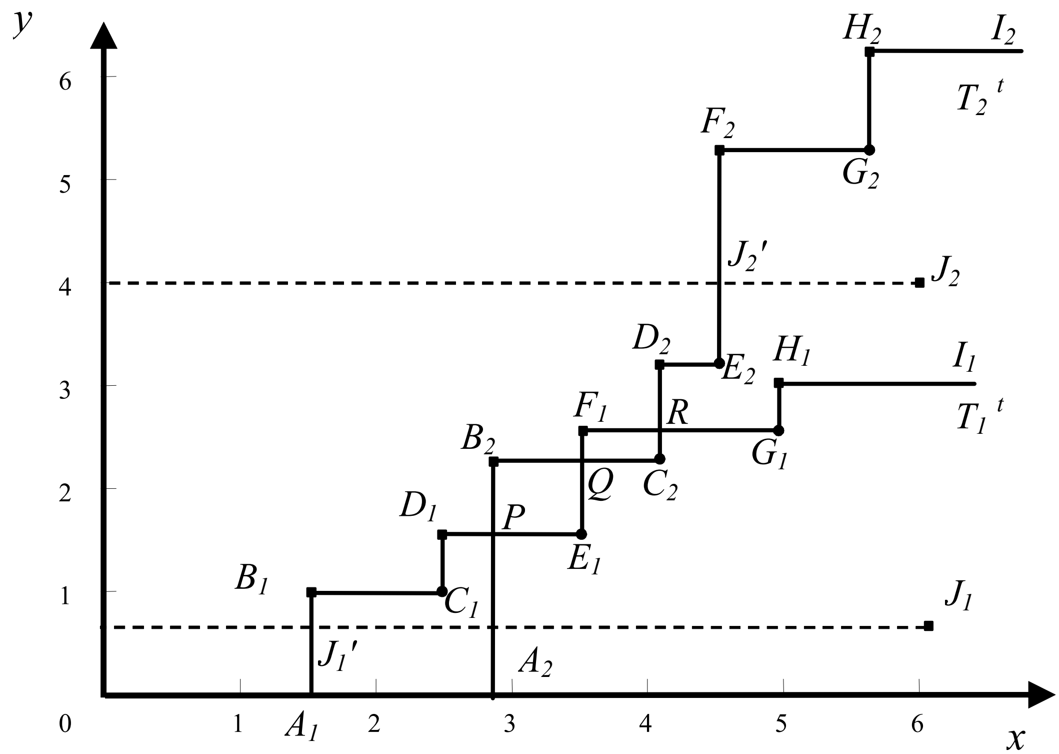
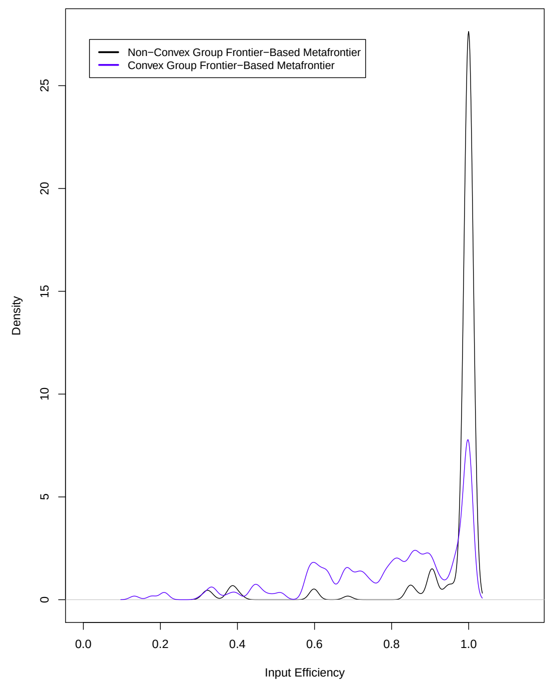
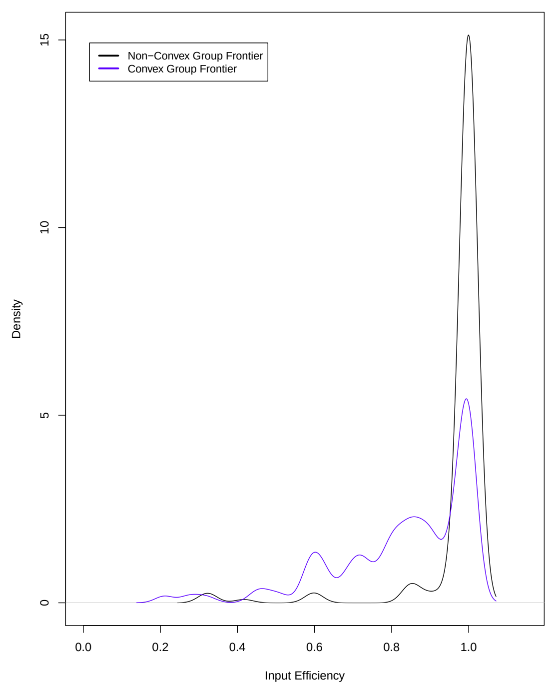


Figure 4: Kernel Density Estimates of (a) Metafrontier and (b) Group Frontier Estimates



(a)



(b)

Figure 5: Unbiased and Biased Metafrontiers Estimates

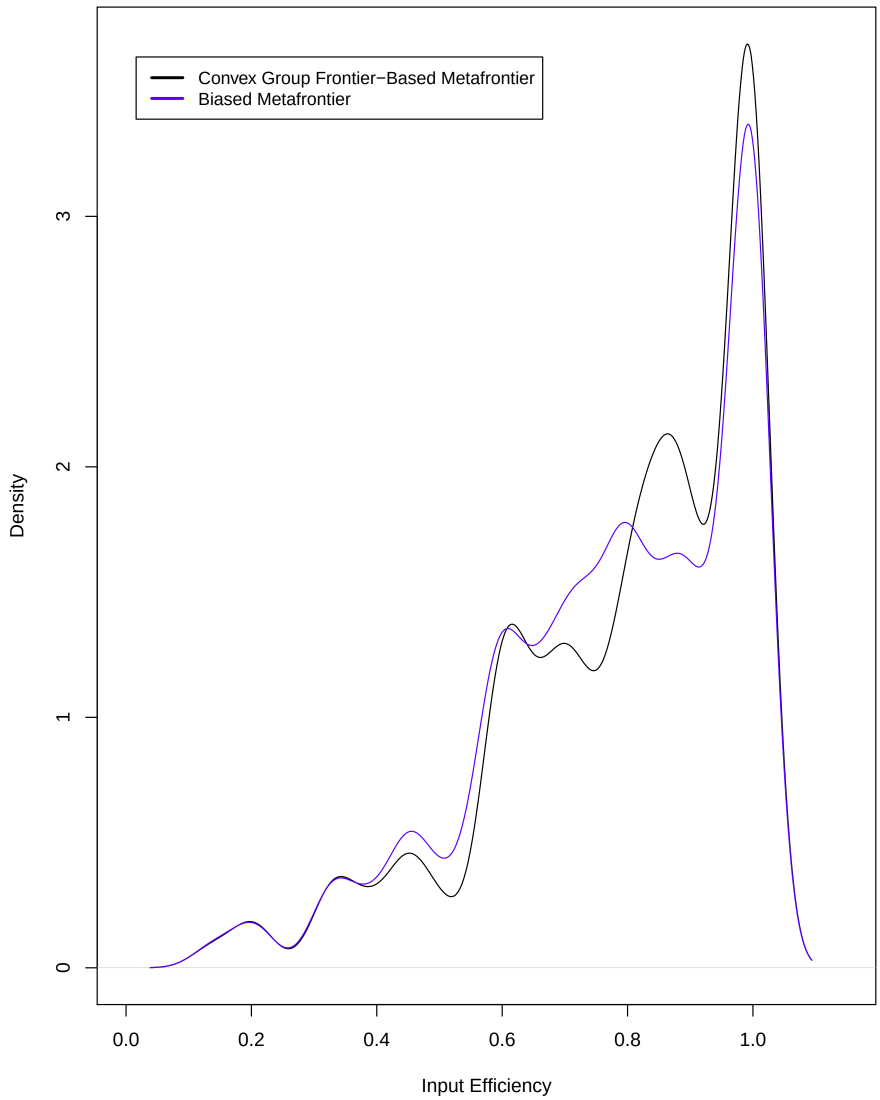


Table 1: Convex and Non-Convex Group Frontiers: ITE Decomposition (15)

		Convex Group Fr.			Non-convex Group Fr.			
		ITE	$RITE$	IMR	ITE	$RITE$	IMR	Inf.
Sample	#Eff. Obs.	21	24	163	154	168	175	
N=192	Geom. Mean	0.7582	0.8046	0.9424	0.9391	0.9629	0.9752	
	Stand. Dev.	0.2009	0.1791	0.1260	0.1378	0.1076	0.0878	
	Min.	0.1325	0.2094	0.3740	0.3154	0.3154	0.3866	
Li-test†		57.50***	57.67***	0.53				
Dam	#Eff. Obs.	1	4	31	39	52	44	
N=62	Geom. Mean	0.6751	0.8163	0.82705	0.8811	0.9547	0.9229	29
	Stand. Dev.	0.2141	0.1645	0.18961	0.1867	0.1181	0.1495	
	Min.	0.1325	0.2681	0.37400	0.3212	0.3212	0.3866	
Run	#Eff. Obs.	20	20	132	115	116	131	
-of-river	Geom. Mean	0.7994	0.7994	1.0000	0.9667	0.9667	1.0000	0
N=132	Stand. Dev.	0.1853	0.1853	0.0000	0.1024	0.1024	0.0004	
	Min.	0.2094	0.2094	1.0000	0.3154	0.3154	0.9953	

† Li test: critical values at 1% level = 2.33 (***); 5% level = 1.64 (**); 10% level = 1.28 (*).

Table 2: Convexity-Related Efficiencies and Bias in Convex ITE and MTR

				Convex ITE			Convex MTR		
		$CRITE$	$CRRITE$	Unbiased	Biased	Difference	Unbiased	Biased	Difference
Sample	# Eff. Obs.	21	24	21	23	67	163	71	88
N=192	Avg.	0.8269	0.8514	0.7940	0.7770	0.0170	0.9541	0.9335	0.0206
	Stand. Dev.	0.1644	0.1507	0.2009	0.2030	0.0470	0.1260	0.1317	0.0554
	Min.	0.3285	0.3285	0.1325	0.1325	0.0000	0.3740	0.3740	0.0000
Li-test†						0.39			10.67^{***}
Dam	# Eff. Obs.	1	4	1	2	68	31	1	18
N=60	Avg.	0.7850	0.8635	0.7205	0.6820	0.0385	0.8530	0.8080	0.0450
	Stand. Dev.	0.1576	0.1161	0.2141	0.2025	0.0760	0.1896	0.1776	0.0890
	Min.	0.3517	0.5223	0.1325	0.1325	0.0000	0.3740	0.3740	0.0000
Run	# Eff. Obs.	20	20	20	21	70	132	70	70
-of-river	Avg.	0.8460	0.8459	0.8274	0.8202	0.0072	1.0000	0.9905	0.0095
N=132	Stand. Dev.	0.1638	0.1638	0.1853	0.1879	0.0170	0.0000	0.0217	0.0217
	Min.	0.3285	0.3285	0.2094	0.2094	0.0000	1.0000	0.9036	0.0000

† Li test: for critical values see Table 1.