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# Could a rise in household savings boost output growth in the euro area?

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### Abstract

In some euro area countries, households have started to save more. There are several possible causes of such behaviour: households may want to increase their savings in order to repay their debts, rebuild their wealth, maintain a desired level of future consumption or pay future higher expected taxes. Here, we are more interested in the consequences of their behaviour: would a rising household saving rate hamper or help economic recovery in the euro area? We study the effects of such a shock in one country of a monetary union using an open-economy dynamic stochastic general equilibrium (DSGE) model. We find that the effects on output growth both in the country hit by the shock and in the rest of union could be positive in the short term as long as net exports to the rest of the world could increase ("export channel") and if fiscal policy were not restrictive. We also show that the ratio of public debt to GDP could be lower than its steady-state level in both countries. Hence, this kind of shock could boost economic growth and help fiscal consolidation.

Keywords: open monetary union, spillovers, saving, public debt

**JEL Classification:** H31, E21, E44, E62, F41

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# 1. Introduction

During the recession of 2009, the household gross saving rate rose in the Euro Area (EA) countries (except in Italy). Then, it started to decrease in most EA countries. Between 2009 and 2012, it fell from 10.3% of GDP to 8.6% of GDP in the EA.<sup>4</sup> As many households were facing a fall in income, they could be tempted to use their savings in order to pay their expenses in goods and services. However, six years after the start of the crisis, economic recovery is still fragile in several EA countries. In this context, precautionary saving might pile up. As a matter of fact, the ratio of household gross saving to GDP rose again in some countries, in 2012 (such as in Portugal, Belgium, Austria) or in 2013 (in Slovenia, Ireland, Italy, Latvia, Estonia, the Netherlands, and still in Portugal).<sup>5</sup> There are different possible reasons why households might want to reduce consumption and increase saving: they might want to repay their debts – in what is usually called a deleveraging process – (Bauer and Nash, 2012), to rebuild the wealth lost during the financial crisis (Cooper, 2012), to save in order to smooth consumption over time in the case of lower expected future income or to pay higher expected future taxes (Ricardian behaviour in a context of rising public indebtedness).

We want to study the consequences – and not the causes – of a higher saving rate (meaning a decline in household consumption). Would this hamper or boost economic growth? What would be the impact on the public debt (knowing that EA governments have recently been much concerned about fiscal consolidation)? To answer these questions, we build a micro-founded New Keynesian open-economy dynamic stochastic general equilibrium (DSGE) model which describes a monetary union between two countries. The union is open to the rest of the world (RoW). We solve, calibrate and simulate the model in order to study a positive asymmetric shock on household savings in one country of the union.<sup>6</sup> We are interested in the effects of the shock on output growth and public debt in both countries of the monetary union.

There are a few recent papers which encompass themes close to the issue of a rise in household savings and its effect on economic recovery. These papers are mostly related to deleveraging and focused on the channel of falling asset prices. From an empirical point of view, Bauer and Nash (2012) believe that U.S. household deleveraging plays a significant role in the sluggish recovery along with the decrease in household net worth (due to falling house prices) and uncertain future income (due to weak employment growth and persistently high unemployment). On the theoretical side, Eggertsson and Krugman (2012) propose a closed-economy New-Keynesian model where aggregate demand is composed of consumption by impatient agents (net borrowers) and consumption by patient agents (savers). They show that the extent of deleveraging can be so large that it induces the so-called debt-deflation vicious circle stated by Irving Fisher. In other respects, Cuerpo et al. (2013) use a DSGE model – a three-region version of the QUEST model developed by the European Commission – in order to study the impact of household sector deleveraging on economic activity. In their model, there are two kinds of households: Ricardian households are net lenders while credit-constrained households are net borrowers. They consider a combination of two shocks, namely a fall in access to

<sup>&</sup>lt;sup>4</sup> Source: AMECO database of the European Commission.

<sup>&</sup>lt;sup>5</sup> It was also the case in the United Kingdom in 2009-2010 and in 2012, as well as in the United States in 2009 and in Japan in 2009 and 2011.

<sup>&</sup>lt;sup>6</sup> We did not study a common shock, because the recent rise in household savings has not been seen in all EA countries. Yet, we will give some hints about what could be the effects of a common shock when we will discuss the results.

credit and a fall in house prices in order to account for household deleveraging. They also assume an exogenous increase in Ricardian households' desire to hold foreign assets so that the net financial asset (NFA) position of the nation improves. They find that deleveraging leads to a "marked contraction" in output.

In our paper, we are more interested in studying the most general case of an increase in household savings than explicitly describing a specific source of such a shock, as far as the causes are diverse, especially across individual EA countries. Thus, the saving shock is simply equivalent to a shock on consumer preferences in our model. The increase in savings results from a sudden household decision to postpone consumption. We assume that it is an asymmetric shock as long as it occurs in one country of the monetary union but not in the rest of the union (RoU).

Since we are interested in the effects of the shock not only on output growth but also on the public debt/GDP ratio, we pay special attention to the modelling of public sector finance. We propose an original approach in this regard. Specifically, each government finances public expenditures (consumption, transfers and interest payments) by levying taxes (on consumption, labor income and capital income) and issuing debt. Each one adjusts the fiscal instrument (public consumption in our model) with reaction to output growth (in a counter-cyclical way in the baseline simulation) and to the debt/GDP ratio (in deviation with a non-zero steady-state level). In each country of the union, banks use household deposits in order to lend funds. Banking activity is segmented into financing public debts and financing private investment in such a way that the evolution of public indebtedness has an impact on the cost of borrowing for the private sector. The model also allows for financial frictions via a financial accelerator mechanism. Another specific feature of our model is to allow each government to borrow from domestic banks and foreign banks in the RoU (by selling sovereign bonds). With such a characteristic, our model can account for a home bias in banking loans to governments. Furthermore, the cost of public borrowing may differ whether the government borrows from domestic banks or foreign banks. Indeed, it is determined by two elements: the banking lending benchmark rate which may differ across countries and the sovereign risk premium which depends on the gap between the ratio of public debt to GDP and its non-zero steady-state level. In addition, we assume that there are internationally traded bonds that allow households to borrow from the RoW or lend to the RoW. There is a risk premium on internationally traded bonds which depends on the net aggregate foreign financial position of households, as in the new area-wide model (NAWM) of the euro area described by Coenen, McAdam and Straub (2008).<sup>7</sup> Moreover, since monetary union is open with the RoW, we can take into account both intra and extra-zone net exports of each country and underline the importance of trade openness.

We find that when households start to reduce consumption and save more, the effects on output in both countries can be positive as long as net exports to the rest of the world can increase and fiscal policy is not restrictive. Moreover, the public debt/GDP ratio can decrease in both countries thanks to an improvement in the primary budget balance. As a matter of fact, in a closed economy, the shock would lead to slower output growth in the short term: higher private savings would lead to an increase in private investment, but output would be negatively affected by the decline in private consumption

<sup>&</sup>lt;sup>7</sup> However, the original version of the NAWM cannot be used for studying spillovers across EA member countries, since the euro area is modeled as a single big country. A four-country version of the NAWM was developed by Gomes, Jacquinot and Pisani (2010), but in this version, public debt is issued only on domestic financial markets and the financial sector is not explicitly modeled.

(given the higher relative weight of the latter in GDP)<sup>8</sup>. The negative effect on output would be stronger if the central bank raised its interest rate in response to the initial increase in prices and output.<sup>9</sup> In contrast to the effects in a closed economy, there is an "export channel" in our model of an open-economy monetary union: the positive shock on household savings can support output growth in member countries of the union as long as an adjustment of the real exchange rate (depreciation) boosts their net exports of goods to the rest of the world.<sup>10</sup> In addition, we show that the stance of fiscal policy can also influence the results: if public consumption were procyclical and decreased in response to an initial decline in output growth induced by lower private consumption, investment would increase less than if public consumption were counter-cyclical, because entrepreneurs would expect lower future aggregate demand.

In the following, we describe the structure of the model (section 2) and the calibration (section 3). We then explain the results under the baseline scenario (section 4) and under different macro-policy scenarios (section 5). We also do some robustness checks (section 6), and finally conclude (section 7).

# 2. Model

We consider two symmetric countries of equal size, home(h) and foreign(f). They are members of a monetary union and are open to the rest of the world (w) which is fully exogenous (for simplicity). The model contains monopolistic competition in final goods market, price stickiness, capital adjustment costs, financial market frictions and fiscal policy instruments.

Each economy is populated by households, banks, government and three types of firms: entrepreneurs, capital producers, and retailers. There is a common monetary authority that sets the unique nominal risk-free interest rate for both countries. Capital producers build new capital and sell it to the entrepreneurs. Entrepreneurs produce wholesale goods and sell them to domestic goods retailers. Retailers set nominal prices of final goods  $\dot{a}$  la Calvo (1983). Banks convert households' deposits in loans to finance the government deficit in both countries and the entrepreneurial purchase of capital. Each government decides upon fiscal policy.

<sup>&</sup>lt;sup>8</sup> The extent of this short-term negative effect would differ among EA countries depending on the ratio of private final consumption expenditure to GDP. In 2013, this ratio varied from 31% in Belgium to 63% in Finland (AMECO database).

<sup>&</sup>lt;sup>9</sup> Higher output and inflation could initially be induced by the additional demand for capital goods in the economy.

<sup>&</sup>lt;sup>10</sup> Our positive effect derived from the "export channel" cannot be easily opposed to the negative effects induced by some financial channels in Cuerpo et al. (2013), because the nature of the shock is not the same: they do not study household savings per se but deleveraging by assuming two simultaneous shocks, one is a fall in access to credit, and the other is a fall in house prices. Admittedly, the negative effects could be predominant if the extent of deleveraging was large (depending on the initial level of private indebtedness) and if the degree of openness to the RoW was low.

#### 2.1. Households

Each country  $i \in \{h, f\}$  is populated by a continuum of unit mass households with infinite life. The representative household of country *i* maximizes the following expected discounted sum of utilities:

$$E_{t} \sum_{t=0}^{\infty} \beta^{t} \left( \varepsilon_{t}^{i} \frac{(C_{t-1}^{i})^{1-\sigma}}{1-\sigma} - \frac{(N_{t}^{i})^{1+\eta}}{1+\eta} \right)$$
(1)

where  $C_t^i$  is aggregate consumption and  $N_t^i$  denotes the number of hours worked.  $E_t$  is the conditional expectation operator. The parameters  $0 < \beta < 1$ ,  $\sigma > 0$ ,  $\eta > 0$  and 0 < h < 1 are, respectively, the subjective discount factor, the inverse intertemporal elasticity of substitution, the inverse of the Frisch elasticity of labour supply, and the parameter that controls habit persistence. The variable  $\varepsilon_t^i$  represents a saving shock and follows a stationary first order autoregressive process. A negative  $\varepsilon_t^i$  shock implies that agents wish to postpone consumption over time, and will thus increase their desired savings.

The household's period-by-period budget constraint is defined by:

$$(1 + \tau_{c,t}^{i})C_{t}^{i} + \frac{D_{t}^{i}}{P_{t}^{i}} + \frac{S_{t}B_{w,t}^{i}}{P_{t}^{i}}$$

$$= (1 - \tau_{w,t}^{i})\frac{W_{t}^{i}}{P_{t}^{i}}N_{t}^{i} + R_{t-1}\frac{D_{t-1}^{i}}{P_{t}^{i}} + R_{w,t-1}\Psi_{b,t-1}^{i}(b_{t-1}^{i}, Z_{t-1}^{i})\frac{S_{t}B_{w,t-1}^{i}}{P_{t}^{i}} + \frac{TR_{t}^{i}}{P_{t}^{i}}$$

$$+ \Lambda_{t}^{i}$$

$$(2)$$

where  $P_t^i$  is the consumer price index (CPI),  $W_t^i$  the nominal wage,  $D_t^i$  nominal deposits that pay gross nominal interest rate  $R_t$  and  $B_{w,t}^i$  nominal internationally traded bonds, denominated in rest of the world currency, that pay a gross nominal interest rate  $R_{w,t-1}\Psi_{b,t-1}^i$ .  $S_t$  is the nominal exchange rate (expressed in terms of units of home currency per unit of foreign currency).  $\tau_{c,t}^i$ ,  $\tau_{w,t}^i$ ,  $TR_t^i$  and  $\Lambda_t^i$  are, respectively, tax rate on consumption, tax rate on wages, government transfers and real profits from the monopolistic sector. Finally,  $\Psi_{b,t}^i$  represents a risk premium that is a function of the household's real level of net foreign financial asset position in percentage of output, as follows:

$$\Psi_{b,t}^{i}\left(b_{t}^{i}, Z_{t}^{i}\right) = exp\left(-\psi_{b}^{i}\left(\frac{S_{t}B_{w,t}^{i}}{Y_{t}^{i}P_{t}^{i}}\right)\right)$$
(3)

where  $b_t^i \equiv \frac{s_t B_{w,t}^i}{Y_t^i p_t^i}$  is the household's real aggregate net foreign financial position in percentage of output (household is a net borrower when  $b_t^i < 0$ );  $\psi_b^i > 0$  is a measure of the elasticity of the risk premium with respect to household's net financial assets position. The term  $\Psi_{b,t}^i(b_t^i)$  is assumed to be strictly decreasing in  $b_t^i$  and satisfies  $\Psi_b^i(0,0) = 1$ . It captures imperfect integration in the international financial markets and ensures a well-defined steady-state in the model (Schmitt-Grohé and Uribe, 2003). The variable *Zt* represents an exogenous shock that is an unexplained part of the country-risk premium.

Households choose the paths for  $\{C_t^i, N_t^i, D_t^i, B_{w,t}^i\}_0^\infty$  in order to maximize (1) subject to the budget constraint in (2). The following optimality conditions hold:

$$\varepsilon_{t}^{i}(C_{t}^{i} - hC_{t-1}^{i})^{-\sigma} - \beta h \, E_{t}\varepsilon_{t+1}^{i}(C_{t+1}^{i} - hC_{t}^{i})^{-\sigma} = \lambda_{t}^{i}(1 + \tau_{c,t}^{i})$$
(4)

$$\lambda_t^i \frac{(1 - \tau_{w,t}^i) W_t^i}{P_t^i} = (N_t^i)^\eta$$
(5)

$$-\frac{\lambda_t^i S_t}{P_t^i} + E_t \beta \frac{\lambda_{t+1}^i S_{t+1}}{P_{t+1}^i} R_{w,t} \Psi_{b,t}^i \left( b_t^i, Z_t^i \right) = 0$$
(6)

$$-\frac{\lambda_{t}^{i}}{P_{t}^{i}} + E_{t}\beta \frac{\lambda_{t+1}^{i}}{P_{t+1}^{i}}R_{t} = 0$$
(7)

 $\lambda_t^i$  is the Lagrangian multiplier in (4), (5), (6) and (7).

The final good,  $X_t^i$ , is allocated to consumption,  $C_t^i$ , investment,  $I_t^i$ , and public spending,  $G_t^i$ . It is an aggregate function of goods produced in the home country,  $X_{i,t}^i$ , in the RoU,  $X_{k,t}^i$ , and in the RoW,

$$X_{w,t}^{i}: \qquad X_{t}^{i} = \left[ \left( 1 - a_{1}^{i} - a_{2}^{i} \right)^{\frac{1}{\theta}} (X_{i,t}^{i})^{\frac{\theta-1}{\theta}} + (a_{1}^{i})^{\frac{1}{\theta}} (X_{k,t}^{i})^{\frac{\theta-1}{\theta}} + (a_{1}^{i})^{\frac{\theta}{\theta}} (X_{k,t}^{i})^{\frac{\theta}{\theta}} + (a_{1}^{i})^{\frac$$

for  $X = \{C, I, G\}$ ;  $i, k \in \{h, f\}$  and  $i \neq k$ .

The parameters  $\theta > 1$ ,  $a_1^i$ , and  $a_2^i$  are, respectively, the elasticity of substitution between the three types of goods, the share of imported goods from the RoU and the share of imported goods from the RoW. We suppose that these shares are identical reciprocally between each country  $i \in \{h, f\}$  of the union and the RoW. Therefore, the fraction $(1 - a_1^i - a_2^i)$  is the degree of home bias in consumption, investment and public goods.

The price index (CPI) associated to (8) is given by:

$$P_t^i = \left[ \left( 1 - a_1^i - a_2^i \right) \left( P_{i,t}^i \right)^{1-\theta} + a_1^i \left( P_{k,t}^i \right)^{1-\theta} + a_2^i \left( P_{w,t}^i \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$
(9)

where  $P_{i,t}^i P_{k,t}^i$  and  $P_{w,t}^i$  are, respectively, the domestic price of home goods, the domestic price of imported goods from the RoU and the domestic price of imported goods from the RoW.

We define 
$$X_{i,t}^{i} \equiv \left(\int_{0}^{1} X_{i,t}^{i}(j)^{\frac{\chi-1}{\chi}} d_{j}\right)^{\frac{\chi}{\chi-1}}$$
,  $X_{k,t}^{i} \equiv \left(\int_{0}^{1} X_{k,t}^{i}(j)^{\frac{\chi-1}{\chi}} d_{j}\right)^{\frac{\chi}{\chi-1}}$  and  $X_{w,t}^{i} \equiv \left(\int_{0}^{1} X_{w,t}^{i}(j)^{\frac{\chi-1}{\chi}} d_{j}\right)^{\frac{\chi}{\chi-1}}$ 

as the composite aggregates of differentiated varieties produced domestically, inside and outside the monetary union, respectively, with  $\chi$  being the elasticity of substitution between varieties originating in the same country;  $X_{i,t}^i(j)$ ,  $X_{k,t}^i(j)$  and  $X_{w,t}^i(j)$  being a typical variety *j* of domestic goods, imported goods from the RoU and imported goods from the RoW, respectively. The corresponding prices are derived easily and are given by, respectively:

$$P_{i,t}^{i} = \left(\int_{0}^{1} P_{i,t}^{i}(j)^{1-\chi} d_{j}\right)^{\frac{1}{1-\chi}}, \quad P_{k,t}^{i} = \left(\int_{0}^{1} P_{k,t}^{i}(j)^{1-\chi} d_{j}\right)^{\frac{1}{1-\chi}}, \quad P_{w,t}^{i} = \left(\int_{0}^{1} P_{w,t}^{i}(j)^{1-\chi} d_{j}\right)^{\frac{1}{1-\chi}},$$

where  $P_{i,t}^{i}(j)$  (respectively  $P_{k,t}^{i}(j)$  and  $P_{w,t}^{i}(j)$ ) is the price of a typical variety *j* produced in the home country (respectively imported prices from the RoU and the RoW).

We assume that the law of one price holds, thus:  $P_{k,t}^i = P_{k,t}^k$  and  $P_{w,t}^i = S_t P_{w,t}^w$ . By assuming that the RoW is fully exogenous, we can write the following identity  $P_{w,t}^w = P_t^w$ .

The optimal demands for domestic, RoU and RoW goods, are derived from expenditure minimization<sup>11</sup>:

$$X_{i,t}^{i} = \left(1 - a_{1}^{i} - a_{2}^{i}\right) \left(\frac{P_{i,t}^{i}}{P_{t}^{i}}\right)^{-\theta} X_{t}^{i}$$
(10)

$$X_{k,t}^{i} = a_{1}^{i} \left(\frac{P_{k,t}^{k}}{P_{t}^{i}}\right)^{-\theta} X_{t}^{i}$$

$$(11)$$

$$X_{w,t}^{i} = a_{2}^{i} \left(\frac{S_{t} P_{w,t}^{w}}{P_{t}^{i}}\right)^{-\theta} X_{t}^{i}$$
(12)

 $\forall i, k \in \{h, f\} \text{ and } i \neq k.$ 

### 2.2. Banks

Banks are in charge of the financial intermediation. We explicitly specify their activity in order to let fiscal and monetary policies influence the economy via the bank-lending channel. Banks are assumed to have two segments in their activity: a segment where they finance less-risky borrowers (public debt segment) and a segment where they finance riskier borrowers (corporate banking segment).

At the beginning of each period t, the representative bank takes deposits,  $D_t^i$ , from the representative household at the (gross nominal) risk-free rate  $R_{t.}^{12}$  It lends not only to domestic entrepreneurs *via* the corporate banking segment, but also to domestic government and foreign government (in the RoU) *via* the public debts segment.

Bank total loans are defined by  $L_t^i$  such as:

$$D_t^i = L_t^i = L_{G,t}^i + L_{E,t}^i$$
(13)

where  $L_{G,t}^{i}$  and  $L_{E,t}^{i}$  denotes banking loans provided respectively for financing public debts of both governments (domestic and foreign) and for financing purchases of capital by domestic entrepreneurs.

<sup>11</sup> The optimization program is  $\min_{\substack{C_{i,t}^i, C_{k,t}^i, C_{w,t}^i, C_t^i}} P_{i,t}^i C_{i,t}^i + P_{k,t}^i C_{k,t}^i + P_{w,t}^i C_{w,t}^i = P_t^i C_t^i$  subject to the following constraint:  $C_t^i = \sum_{\substack{c_{i,t}^i, C_{k,t}^i, C_{w,t}^i, C_t^i}} P_{i,t}^i C_{k,t}^i + P_{k,t}^i C_{k,t}^i + P_{w,t}^i C_{w,t}^i = P_t^i C_t^i$ 

$$\left[ (1 - a_1 - a_2)^{\frac{1}{\theta}} (C_{i,t}^i)^{\frac{\theta - 1}{\theta}} + (a_1)^{\frac{1}{\theta}} (C_{k,t}^i)^{\frac{\theta - 1}{\theta}} + (a_2)^{\frac{1}{\theta}} (C_{w,t}^i)^{\frac{\theta - 1}{\theta}} \right]^{\frac{\theta}{\theta - 1}}.$$

<sup>&</sup>lt;sup>12</sup> That is possible under the assumption that banking activity is risk-free for depositors, since the banks' assets portfolio is perfectly diversified.

The share of loans provided by the representative bank in each country to domestic government and foreign government are given respectively by  $\zeta^i L_{G,t}^i$  and  $(1 - \zeta^i) L_{G,t}^i$ . The parameter  $\zeta^i$  measures the "home bias" in the banking loans provided to governments. Furthermore, we suppose that this banking activity is undertaken in a competitive market so that the representative bank sets interest rates on loans to governments under a zero profit hypothesis.

Governments are less risky than entrepreneurs. Nevertheless, they may face a sovereign risk premium in their cost of borrowing, depending on the level of their indebtedness. We thus introduce risk premia  $\Psi_{l,t}^i(l_t^i, l_t^k)$  and  $\Psi_{l,t}^k(l_t^i, l_t^k)$  that are a function of the government debt/GDP ratios.<sup>13</sup> In setting the interest rate on the loans to governments, the representative bank adds the sovereign risk premium to a nominal benchmark interest rate,  $R_{L,t}^i$ , which would be applied to loans to a hypothetical borrower (without a specific risk premium).

The profit function of the bank in the public debt segment is given by:

 $R_{L,t}^{i}\Psi_{l,t}^{i}(l_{t}^{i}, l_{t}^{k})\zeta^{i}L_{G,t}^{i} + R_{L,t}^{i}\Psi_{l,t}^{k}(l_{t}^{i}, l_{t}^{k})(1-\zeta^{i})L_{G,t}^{i} - R_{t}D_{G,t}^{i}, \quad \forall i, k \in \{h, f\} \text{ and } i \neq k$ (14) where  $D_{G,t}^{i}$  represents household deposits that finance debt of domestic and foreign governments.

The zero profit condition guarantees that:

$$R_{L,t}^{i} = \frac{R_{t}}{\left[\zeta^{i}\Psi_{l,t}^{i} + (1-\zeta^{i})\Psi_{l,t}^{k}\right]}$$
(15)

 $\forall i, k \in \{h, f\} \text{ and } i \neq k.$ 

Equation (15) shows that the benchmark interest rate that is applied to the hypothetical borrower decreases when risk premia of relative risky agents increase, and thus when their debts increase. Thus, the risk of lending to governments is analysed as a relative risk compared to the reference agent (risk-free hypothetical borrower). Therefore, banks provide loans in a way consistent with a portfolio management goal. Specifically, the cost of borrowing for a given government depends on its relative risk compared to the risk of other borrowers. For instance, if government *f* becomes riskier than government *h* ( $\Psi_{l,t}^f(l_t^h, l_t^f)$  goes up), *ceteris paribus*, the benchmark rate  $R_{L,t}^h$  goes down and this leads to a decrease of the cost of borrowing for government *h* for a given risk premium of government *h* (and hence for a given indebtedness of the latter).<sup>14</sup>

In the riskier business segment (corporate banking segment), the setting of interest rates on loans to firms is determined as in the traditional financial accelerator. Accordingly, the bank opportunity cost would be the rate that it could obtain by financing the public debt instead of lending to entrepreneurs. Thus, the nominal rate applied to loans to entrepreneurs (their external financing cost) is given by:  $R_{E,t}^{i} = R_{L,t}^{i} \Psi_{l,t}^{i} (l_{t}^{i}, l_{t}^{k}) \Psi_{E,t}^{i} (\cdot)$  (16)

 $\forall i, k \in \{h, f\}$  and  $i \neq k$ , where  $\Psi_{E,t}^i(\cdot)$  is entrepreneur's (specific) external finance premium at the beginning of each period *t* (see section 2.3 below).

<sup>&</sup>lt;sup>13</sup> Sovereign risk premia are defined in section 2.4 below.

<sup>&</sup>lt;sup>14</sup> The Government (*h*) costs of borrowing in domestic and foreign market are  $R_{L,t}^h \Psi_{l,t}^h (l_t^h, l_t^f)$  and  $R_{L,t}^f \Psi_{l,t}^h (l_t^h, l_t^f)$ , respectively.

#### 2.3. Production sector

#### 2.3.1. Entrepreneurs

The behaviour of entrepreneurs is specified in order to introduce the financial accelerator mechanism. As in Bernanke et *al.* (1999), there is a continuum of firms  $j \in [0,1]$  that produce wholesale (intermediate) goods in a perfectly competitive market, using  $K_t^i$  units of capital and  $N_t^i$  units of labour, according to the following technology:

$$Y_t^i(j) = A_t^i K_t^i(j)^{\alpha} N_t^i(j)^{1-\alpha}$$
(17)

where  $A_t^i$  is a technological (productivity) shock that is common to all firms and follows a stationary first-order autoregressive process;  $\alpha \in [0,1]$  is the share of capital in the production technology.

The representative firm maximizes its profit by choosing  $K_t^i$  and  $N_t^i$  subject to the production function (17). The first-order conditions for this optimization problem are:

$$w_t^i = (1 - \alpha)mc_t^i \frac{Y_t^i}{N_t^i} \tag{18}$$

$$mpc_t^i = \alpha \, mc_t^i \frac{Y_t^i}{K_t^i} \tag{19}$$

where  $mc_t^i$  is the Lagrangian multiplier associated with the production function (17) and denotes the real marginal cost;  $w_t^i$  is the real wage; and  $mpc_t^i$  is the real marginal productivity of capital.

Entrepreneurs are risk neutral and borrow in order to finance a share of capital used in the production process. Following Bernanke *et al.* (1999), we assume that entrepreneurs have a finite expected horizon, so that they do not accumulate enough funds to fully self-finance their activity. In each period *t*, entrepreneurs face a constant probability (1 - v) of leaving the economy. We also follow Christensen and Dib (2008) in allowing newly entering entrepreneurs to inherit a fraction of the net worth of those firms which exit from the business. This assumption is made in order to ensure that new entrepreneurs start out with a positive net worth.<sup>15</sup> At the end of each period, entrepreneurs purchase capital,  $K_{t+1}^i$ , that will be used in the next period at the real price  $q_t^i$ . Thus, the total funding needed by an entrepreneur to purchase capital is  $q_t^i K_{t+1}^i - NW_{t+1}^i$ , from a financial intermediary. Financial intermediaries (banks) obtain their funds from household deposits (section 2.2). In optimum, the entrepreneur's aggregate demand for capital in the economy depends on the expected marginal return and on the expected marginal financing cost at t + 1. Thus, the capital demand must satisfy the following differentiation between the *ex post* marginal return on capital,  $E_t(R_{k,t+1}^i)$ , and the marginal productivity of capital at t + 1,  $mpc_t^i$ , which is the rental rate of capital:

$$E_t(R_{K,t+1}^i) = E_t\left[\frac{(1-\tau_{K,t}^i) mpc_{t+1}^i + (1-\delta)q_{t+1}^i}{q_t^i}\right]$$
(20)

<sup>&</sup>lt;sup>15</sup>In contrast, Bernanke *et al.* (1999) ensure this by assuming that entrepreneurs also work. This difference does not affect the results.

where  $\delta$  is the capital depreciation rate,  $\tau_{K,t}^i$  is the tax rate on capital-income (whose introduction here is a specific feature of our model) and  $(1 - \delta)q_{t+1}^i$  is the value of one unit of capital used in t + 1. According to the optimal financial contract between borrower and lender, the entrepreneur's demand for capital satisfies, optimally, the equality between expected return on capital and gross premium for external finance plus the gross real opportunity costs equivalent to the gross real interest rate on loans<sup>16</sup>:

$$E_t(R_{K,t+1}^i) = E_t\left[\Psi_{E,t+1}^i(\cdot) \frac{R_{L,t}^i \Psi_{l,t}^i(l_t^i, l_t^k)}{\pi_{t+1}^i}\right]$$
(21)

where  $R_{L,t}^i$  is the gross nominal interest rate on banks loans.  $\Psi_{E,t+1}^i(\cdot)$  is the function that describes how the external finance premium depends on the financial position of the firm and is given by:  $\Psi_{E,t+1}^i(\cdot) = \left(\frac{NW_{t+1}^i}{q_t^i K_{t+1}^i}\right)^{-\gamma}$  with  $\left(\Psi_{E,t+1}^i(\cdot)\right)' < 0$ ,  $\Psi_E^i(1) = 1$  and  $\gamma$  is the elasticity of the external finance premium with respect to firm's leverage ratio. Thus, the external finance premium is an equilibrium inverse function of the aggregate financial position in the economy, expressed by the leverage ratio. Equation (21) provides the basis for the financial accelerator. If entrepreneur's net worth goes up, the external finance premium falls, the cost of borrowing falls and firms get cheaper access to credit.

Aggregate entrepreneurial net worth accumulation of the economy depends on profits earned in previous periods plus the bequest,  $\Omega_t^i$ , that newly entering entrepreneurs receive from entrepreneurs who leave the economy, and evolves according to:

$$NW_{t+1}^{i} = \nu \left[ R_{K,t}^{i} q_{t-1}^{i} K_{t}^{i} - \frac{R_{L,t-1}^{i}}{\pi_{t}^{i}} \left( \frac{NW_{t}^{i}}{q_{t-1}^{i} K_{t}^{i}} \right)^{-\gamma} \left( q_{t-1}^{i} K_{t}^{i} - NW_{t}^{i} \right) \right] + (1 - \nu) \Omega_{t}^{i}$$
(22)

#### 2.3.2. Capital producers

Competitive capital producers use a linear technology to produce new capital  $K_{t+1}^i$  from final investment goods  $I_t^i$  and existing capital stock leasing from entrepreneurs without costs. When producing capital, they are subject to quadratic capital adjustment costs specified as

$$\frac{\psi_I}{2} \left( \frac{I_t^i}{K_t^i} - \delta \right)^2 K_t^i.$$

The aggregate capital stock used by producers in each economy *i* evolves as follow:

$$K_{t+1}^{i} = \left[\frac{I_{t}^{i}}{K_{t}^{i}} - \frac{\psi_{I}}{2} \left(\frac{I_{t}^{i}}{K_{t}^{i}} - \delta\right)^{2}\right] K_{t}^{i} + (1 - \delta) K_{t}^{i}$$
(23)

where  $\psi_I > 0$  is the parameter that measures the adjustment costs elasticity.

<sup>&</sup>lt;sup>16</sup> For details, see Bernanke et al. (1999).

Capital producers face an optimization problem which consists, in real terms, in choosing the level of investment that maximizes their profits:

$$\max_{I_t^i} \left\{ q_t^i I_t^i - I_t^i - \frac{\psi_I}{2} \left( \frac{I_t^i}{K_t^i} - \delta \right)^2 K_t^i \right\}$$
(24)

The following equilibrium condition holds:

$$q_t^i - \psi_I \left( \frac{I_t^i}{K_t^i} - \delta \right) = 1 \tag{25}$$

which is the standard Tobin's Q equation that links the price of capital to the marginal adjustment costs.

### 2.3.3. Retailers: price and inflation dynamics

The existence of retailers provides the source of nominal stickiness in the economy. Retailers take wholesale goods as inputs, repackage the latter costlessly, and sell them in a monopolistically competitive market. Following Calvo (1983), we assume that retailers set nominal prices on a staggered basis: at each period, a fraction  $(1 - \phi^i)$  of retailers are randomly selected to set new prices while the remaining fraction  $\phi^i$  of retailers keep their prices unchanged. In each country  $i, k \in \{h, f\}$ , home goods retailers purchase the wholesale goods from entrepreneurs at a price equal to the entrepreneurs' nominal marginal cost. Each retailer j who sets prices at t will choose the optimal price,  $\tilde{P}_{i,t}^i$ , that maximizes the expected profits for s periods, so that:

$$\max_{\widetilde{P}_{i,t}^{i}(j)} E_t \left\{ \sum_{s=0}^{\infty} \left( \beta \phi^i \right)^s \frac{\lambda_{t+s}^{i}}{\lambda_t^{i}} \left[ Y_{i,t+s}^{i}(j) \left( \widetilde{P}_{i,t}^{i}(j) - P_{i,t+s}^{i} m c_{t+s}^{i} \right) \right] \right\}$$
(26)

subject to the demand function,  $Y_{i,t+s}^{i}(j) = \left(\frac{\tilde{P}_{i,t+s}^{i}(j)}{P_{i,t+s}^{i}}\right)^{-\chi} Y_{i,t+s}^{i}$ , where  $\frac{\lambda_{t+s}^{i}}{\lambda_{t}^{i}}$  is the households' marginal utilities ratio between t + s and t.

The first-order condition for this problem yields,

$$\widetilde{P}_{i,t}^{i}(j) = \frac{\chi}{\chi - 1} \frac{E_t \{\sum_{s=0}^{\infty} (\beta \phi^i)^s \lambda_{t+s}^{i} Y_{i,t+s}^{i}(j) P_{i,t+s}^{i} m c_{t+s}^{i}\}}{E_t \{\sum_{s=0}^{\infty} (\beta \phi^i)^s \lambda_{t+s}^{i} Y_{i,t+s}^{i}(j)\}}$$
(27)

Aggregating across all retailers, the price index for domestically produced goods is given by,

$$P_{i,t}^{i} = \left[ \left(1 - \phi^{i}\right) \left(\tilde{P}_{i,t}^{i}\right)^{1-\chi} + \phi^{i} \left(P_{i,t-1}^{i}\right)^{1-\chi} \right]^{\frac{1}{1-\chi}}$$
(28)

Combining log-linearized versions of equations (27) and (28) yields an expression of the inflation rate for domestically produced goods, defined by the following New Keynesian Phillips curve:

$$\hat{\pi}_{i,t}^{i} = \beta E_t \hat{\pi}_{i,t+1}^{i} + \frac{(1-\phi^i)(1-\beta\phi^i)}{\phi^i} \widehat{mc}_t^{i}$$
(29)

where  $mc_t^i$  is the real marginal cost,  $\pi_{i,t}^i = \left(\frac{P_{i,t}^i}{P_{i,t-1}^i}\right)$  is domestic inflation and variables with hats are log deviations from their steady-state values.

Finally, from equation (29) and since firms do not segment markets by country, CPI inflation  $\hat{\pi}_t^i$  is a composite of domestic, foreign and world goods prices variation, such that:

$$\hat{\pi}_{t}^{i} = (1 - a_{1}^{i} - a_{2}^{i})\hat{\pi}_{i,t}^{i} + a_{1}^{i}\hat{\pi}_{k,t}^{k} + a_{2}^{i}\hat{\pi}_{w,t}^{w}\Delta S_{t}$$

$$\forall i, k \in \{h, f\} \text{and } i \neq k.$$
(30)

# 2.4. Government Budget Constraint and Fiscal policy

In each economy, government spends in purchases of aggregate goods  $G_t^i$  and transfers to households  $TR_t^i$ . The government finances its purchases by collecting tax revenues on consumption, wages and capital income, and borrowing funds from domestic and foreign banks ( $\zeta^i L_{G,t}^i$  and  $(1 - \zeta^k) L_{G,t}^k$  respectively).

The government budget constraint is given by:

$$\zeta^{i}L_{G,t}^{i} + (1 - \zeta^{k})L_{G,t}^{k} = R_{L,t-1}^{i}\Psi_{l,t-1}^{i}(l_{t-1}^{i}, l_{t-1}^{k})\zeta^{i}L_{G,t-1}^{i} + R_{L,t-1}^{k}\Psi_{l,t-1}^{i}(l_{t-1}^{i}, l_{t-1}^{k})(1 - \zeta^{k})L_{G,t-1}^{k} + PD_{t}^{i}$$
(31)

In equation (31),  $PD_t^i$  is the nominal primary budget balance and is expressed by:

$$PD_{t}^{i} = P_{t}^{i}G_{t}^{i} + TR_{t}^{i} - \tau_{c,t}^{i}P_{t}^{i}C_{t}^{i} - \tau_{w,t}^{i}w_{t}^{i}N_{t}^{i}P_{t}^{i} - \tau_{K,t}^{i}mpc_{t}^{i}K_{t}^{i}P_{t}^{i}$$
(32)

The term  $\Psi_{l,t}^{i}(l_{t}^{i}, l_{t}^{k})$  is the government's risk premium:

$$\Psi_{l,t}^{i}\left(l_{t}^{i}, l_{t}^{k}\right) \equiv exp\left(\psi_{l}^{i}\left(\frac{\zeta^{i}L_{G,t}^{i} + (1-\zeta^{k})L_{G,t}^{k}}{Y_{t}^{i}P_{t}^{i}}\right)\right)$$
(33)

where  $\psi_l^i$  is the elasticity of the risk premium with respect to government debt<sup>17</sup>;  $l_t^i \equiv \frac{L_{Gt}^i}{Y_t^i P_t^i}$  and  $l_t^k \equiv \frac{L_{Gt}^k}{Y_t^k P_t^k}$  are respectively the total of real loans/GDP made by banks in economy *i* and *k*.

#### Fiscal policy instruments

The government needs to adjust tax revenues or expenditure to stabilize its deficit and debt. We choose public consumption as the fiscal policy instrument. Primary public consumption is adjusted in response to cyclical fluctuations according to the following fiscal rule:

<sup>&</sup>lt;sup>17</sup> A higher public debt level can raise the sovereign risk premium because it can raise the probability of sovereign default (see Bi, 2012).

$$\log\left(\frac{G_t^i}{G^i}\right) = \rho_g \log\left(\frac{G_{t-1}^i}{G^i}\right) + (1 - \rho_g)\rho_{gy} \log\left(\frac{Y_t^i}{Y^i}\right) - (1 - \rho_g)\rho_{gl} \log\left(\frac{DY_t^i}{DY^i}\right)$$
(34)

where  $\rho_g, \rho_{gy}, \rho_{gl} \in [0,1]$  capture, respectively, the degree of public consumption smoothing, fiscal reaction to output deviation and fiscal reaction to debt/GDP ratio  $(DY_t^i \equiv \frac{\zeta^i L_{G,t}^i + (1-\zeta^k) L_{G,t}^k}{Y_t^i P_t^i})$ .

The parameter  $\rho_g$  represents some inertia in the implementation of spending programs due to institutional constraints (voting procedures for instance) or some irreversibility in some public expenditures (social benefits for instance). The parameter  $\rho_{gy}$  measures the extent of the reaction of public consumption to the business cycle (output gap) and its sign captures the cyclical behaviour of public consumption: if it is negative (resp. positive), public consumption is counter-cyclical (resp. procyclical) in the sense that public consumption is lower (resp. stronger) than its steady-state level when output is stronger (resp. lower) than its steady-state level. We also assume that public consumption is adjusted in response to the public debt/GDP ratio: primary public consumption is lowered if the public debt/GDP ratio is higher than its steady-state level. Thus, governments care more or less – according to the value of the parameter  $\rho_{gl}$  – about debt sustainability

As for the other fiscal instruments – namely government transfers  $(TR_t^i)$ , and tax rates on consumption  $(\tau_{c,t}^i)$ , wages  $(\tau_{w,t}^i)$  and capital income  $(\tau_{K,t}^i)$  – they are exogenous.

#### 2.5. Monetary authority

In the monetary union, the common central bank sets the nominal interest rate according to the following Taylor-type interest rate rule:

$$\log\left(\frac{R_t}{R}\right) = \beta_0 \log\left(\frac{R_{t-1}}{R}\right) + (1 - \beta_0) \left[\beta_1 \log\left(\frac{E_t \pi_{t+1}^{um}}{\pi^{um}}\right) + \beta_2 \log\left(\frac{Y_t^{um}}{Y^{um}}\right)\right]$$
(35)

R,  $\pi^{um}$  and  $Y^{um}$  are the steady-state values of  $R_t$ ,  $\pi^{um}_t$  and  $Y^{um}_t$ , that are, respectively, the nominal interest rate, the inflation rate and output of the union. The variables  $\pi^{um}_t$  and  $Y^{um}_t$  are the average values of inflation and output of the two equal-size countries:

$$\pi_t^{um} = \frac{1}{2} \left( \pi_t^h + \pi_t^f \right) \text{ and } Y_t^{um} = \frac{1}{2} \left( Y_t^h + Y_t^f \right)$$
(36)

 $\beta_1 > 1$  and  $\beta_2 < 1$  are coefficients that measure central bank responses to expected inflation and output deviations. The parameter  $0 < \beta_0 < 1$  captures the degree of interest rate smoothing.

#### 2.6. General equilibrium conditions

In equilibrium, the factor markets, the final goods market, the loan market and the international traded bonds market must clear in each country  $i \in \{h, f\}$ .

Equilibrium in factor markets requires:

$$N_t^i = \int_0^1 N_t^i(j) dj \text{ and } K_t^i = \int_0^1 K_t^i(j) dj$$
(37)

The loan market clears when the household deposits equalize the total funds lent to entrepreneurs, domestic government and government in the RoU (see equation 13 supra).

Let  $Y_t^i \equiv \left(\int_0^1 Y_t^i(j)^{\frac{\chi-1}{\chi}} d_j\right)^{\frac{\chi}{\chi-1}}$  denote aggregate output. Thus, the goods market clearing condition satisfies:

$$Y_{t}^{i} = C_{i,t}^{i} + I_{i,t}^{i} + G_{i,t}^{i} + EX_{t}^{i}$$
(38)  
where  $EX_{t}^{i} = a_{1}^{i} \left(\frac{P_{i,t}^{i}}{P_{t}^{k}}\right)^{-\theta} AB_{t}^{k} + a_{2}^{i} \left(\frac{P_{i,t}^{i}}{S_{t}P_{t}^{W}}\right)^{-\theta} AB_{t}^{W}$ 

The variable  $EX_t^i$  represents total exports and  $AB_t^i$  stands for absorption.

Then the domestic economy's aggregate resource constraint can be rewritten as:

$$Y_{t}^{i} = \left(\frac{P_{i,t}^{i}}{P_{t}^{i}}\right)^{-\theta} \left[ \left(1 - a_{1}^{i} - a_{2}^{i}\right)AB_{t}^{i} + a_{1}^{i} \left(\frac{P_{t}^{i}}{P_{t}^{k}}\right)^{-\theta} AB_{t}^{k} + a_{2}^{i} \left(\frac{P_{t}^{i}}{S_{t}P_{t}^{w}}\right)^{-\theta} AB_{t}^{w} \right]$$
(39)

where  $AB_t^i$ ,  $AB_t^k$  and  $AB_t^w$  are, respectively, absorption in the domestic economy, RoU and the RoW. We have:

$$AB_t^i = C_t^i + I_t^i + G_t^i \tag{40}$$

$$AB_t^k = C_t^k + I_t^k + G_t^k \tag{41}$$

and  $AB_t^w$  is an exogenous process.

The internationally traded bonds market is in equilibrium when the positions of the export and importing firms vis-à-vis the RoW equals the households' choice of internationally traded bonds holdings. The evolution of net foreign assets NFA (net cross-border loans plus households' internationally traded bonds holdings) at the aggregate level can be expressed as:

$$S_{t}B_{w,t}^{i} + (1 - \zeta^{i})L_{G,t}^{i} - (1 - \zeta^{k})L_{G,t}^{k}$$

$$= S_{t}R_{w,t-1}\Psi_{b,t-1}^{i}(b_{t-1}^{i}, Z_{t-1}^{i})B_{w,t-1}^{i} + R_{L,t-1}^{i}\Psi_{l,t-1}^{k}(l_{t-1}^{i}, l_{t-1}^{k})(1 - \zeta^{i})L_{G,t-1}^{i}$$

$$- R_{L,t-1}^{k}\Psi_{l,t-1}^{i}(l_{t-1}^{i}, l_{t-1}^{k})(1 - \zeta^{k})L_{G,t-1}^{k} + EX_{t}^{i} - (IM_{k,t}^{i} + IM_{w,t}^{i})$$

$$(42)$$

where  $IM_{k,t}^{i}$  and  $IM_{w,t}^{i}$  are imports of country *i* originating from country *k* (of the RoU) and from the RoW, respectively.

Noting that the definitions of  $b_t^i$ ,  $l_t^i$  and  $l_t^k$  are  $b_t^i \equiv \frac{S_t B_{w,t}^i}{Y_t^i P_t^i}$ ,  $l_t^i \equiv \frac{L_{Gt}^i}{Y_t^i P_t^i}$  and  $l_t^k \equiv \frac{L_{Gt}^k}{Y_t^k P_t^k}$ , we can rewrite the evolution of total real NFA position in percentage of steady-state output as:

$$b_{t}^{i} + (1 - \zeta^{i})l_{t}^{i} - (1 - \zeta^{k})l_{t}^{k}\frac{P_{t}^{k}Y_{t}^{k}}{P_{t}^{i}Y_{t}^{i}} = \frac{R_{w,t-1}\Psi_{b,t-1}^{i}}{\pi_{t}^{i}}b_{t-1}^{i} + \frac{R_{L,t-1}^{i}\Psi_{l,t-1}^{k}}{\pi_{t}^{i}}(1 - \zeta^{i})l_{t-1}^{i}\frac{Y_{t-1}^{i}}{Y_{t}^{i}} - \frac{R_{L,t-1}^{k}\Psi_{l,t-1}^{i}}{\pi_{t}^{i}}(1 - \zeta^{k})l_{t-1}^{k}\frac{P_{t-1}^{k}Y_{t-1}^{k}}{P_{t-1}^{i}Y_{t}^{i}} + \left(\frac{P_{i,t}^{i}}{P_{t}^{i}} - \frac{C_{t}^{i}}{Y_{t}^{i}} - \frac{I_{t}^{i}}{Y_{t}^{i}} - \frac{G_{t}^{i}}{Y_{t}^{i}}\right)$$
(43)

 $\forall i, k \in \{h, f\}$  and  $i \neq k$ .

#### 3. Calibration

The equations of the model are log-linearized around the steady state. For simulations, we used the software Dynare. We calibrated the model (see Table 1 in appendix) by following the literature on DSGE models applied to the euro area or by doing our own computations based on data from the European Commission (EC) - in particular the AMECO database -, the Organization for Economic Co-operation and Development (OECD) or the European Central Bank (ECB) for the euro area as a whole. Amongst the structural parameters, we computed the shares of imported goods from the RoU and from the RoW ( $a_1^i = 0.21$  and  $a_2^i = 0.11$ ) by taking the average of intra EU exports and imports of goods and that of extra EU exports and imports of goods as a percentage of total exports and imports respectively, for the EA17 (the euro area composed of 17 countries) over the 1999-2012 period. We deduced the capital contribution to production ( $\alpha = 0.36$ ) from the average labour income share ratio of the EA17 over 1999-2010. The "home bias" in the banking loans provided to governments ( $\zeta^{i} = 0.67$ ) is computed using the share of securities issued by EA governments in total securities held by EA MFIs (Monetary and Financial Institutions) in the last quarter of 2012. As for the main steady-state ratios, we took average figures for the EA17 over the period 1999-2012: the ratio of private final consumption expenditure to GDP is 57% and the ratio of final consumption expenditure of general government to GDP is 21% and the gross fixed capital formation of the private sector is 19% of GDP.

In the baseline calibration, we follow Kollmann et al. (2013). The parameters of the monetary policy rule were set in order to represent the preferences of the ECB concerning the priority given to price stability ( $\beta_1 = 2.2$  and  $\beta_2 = 1$  for the response coefficients to inflation and output respectively). A gradual adjustment of the key interest rates to changes in policy variables is considered, because the instrument variability is perceived as undesirable/costly by central bankers (in the literature on monetary policy rules, the coefficient of instrument smoothing ( $\beta_0$ ) is estimated to be rather high – at least 0.90 – for the EA). For robustness checks, we then use Blattner et Margaritov (2010) and consider different reasonable values for the coefficients of the monetary rule for the EA in order to understand the role of the monetary policy for the transmission of saving shocks. As regards the public expenditure rule, we base the calibrated parameters on the estimation results of fiscal policy reaction found by Kollmann & al. (2013) for the EA. We set  $\rho_g = 0.5$  (public spending smoothing) and  $\rho_{gl} = 0.01$  (public spending reaction to debt/GDP ratio). In the baseline simulations, we assume

that public consumption is acyclical ( $\rho_{gy} = 0$ ). In section 5, we also look at the scenarios where the public consumption is counter-cyclical ( $\rho_{gy} = -0.7$ ) or procyclical ( $\rho_{gy} = 0.14$ ) by using some estimates of Fátas and Mihov (2010).<sup>18</sup> Finally, for tax rates, we used the implicit tax rates in the EA17 in 2009 for labour income ( $\tau_w = 0.33$ ), capital income ( $\tau_k = 0.25$ ) and consumption ( $\tau_c = 0.2$ ).<sup>19</sup>

#### 4. Impact of a household saving shock on output growth

To understand the impact of a saving shock on output growth, we simulate a 1% positive asymmetric shock on household savings in one country of the union (named domestic country hereafter). We chose an average value of 0.75 for the autoregressive coefficient ( $\rho_{\varepsilon}$ ) in order to account for a gradual process of increasing savings (for deleveraging or rebuilding wealth for instance) in some countries.

Figure 1 displays the effects of the shock on the main macroeconomic aggregates of the domestic country hit by the shock (straight line) and of the RoU (dotted line), under our baseline calibration. All variables are defined in deviation from the steady-state level. For a better understanding of the output dynamics in our model, we analyze the evolution of the main aggregate demand components in each economy. We start explaining the effects of the shock on the domestic country, and then, we will study the spillover effects on the RoU.

Since domestic households start to save more under the shock, this is detrimental to private consumption, which goes below its steady-state level. The surplus of saving in the domestic country insures the financing of more investment, simultaneously justified by the lower real interest rate in the economy after the shock. As for the fiscal policy, public spending is supposed to be acyclical in our baseline calibration, so its variation is very limited compared to the steady-state.<sup>20</sup> Its contribution to output growth is thus negligible. The dynamics of net exports in our model is closely linked to the evolution of the real effective exchange rate. A positive deviation of this variable from the steady-state would correspond to a real effective depreciation of the common currency for the domestic country, while a negative one translates a real effective appreciation. In our baseline simulation, the real effective appreciation of the source of the shock leads to a decline in net exports in the domestic country. The global negative impact of the saving shock on output growth in this country is thus due to lower consumption and lower net exports.

Moreover, as we will see below, net exports represent a key variable in our open economy model. Its evolution may significantly change the reaction of output to the saving shock. Thus, it is important at this stage to understand the dynamics of the real effective exchange rate. The latter variable has two components: *the intra-union real exchange rate* only depends on the expected inflation differential inside the union; *the extra-union real exchange rate* is influenced by the uncovered interest parity (UIP) defined in real terms (implicitly for the currency union as a whole). This second term induces

<sup>&</sup>lt;sup>18</sup> In Fátas and Mihov (2010), the estimated coefficient of output gap is 0.14 for procyclical public spending estimated for the EA and goes to 0.7 when considering the countercyclical policy conducted by Finland, for example.

<sup>&</sup>lt;sup>19</sup> Source: European Commission (2011), Taxation trends in the European Union.

<sup>&</sup>lt;sup>20</sup> Given the specification of the fiscal policy rule, public consumption has to be lowered (from its steady-state level) when the public debt ratio over GDP is above its steady-state level. Due to the negative impact of the shock on GDP, the public debt-GDP ratio increases.

a real appreciation in the two countries of the union. Indeed, a nominal appreciation of the common currency is required to restore the arbitrage condition after a decrease in the interest rate in the union. As for the decrease in the central bank interest rate, it is mainly due to the central bank reaction to the expected fall in the average output of the union (because of consumption). Since inflation is increasing after the shock in the domestic country, the real interest rate goes below its steady-state level. This explains in the same time the net lender position of domestic households with regards to the RoW, given the financial openness of the union towards the rest of the world and the increase in their savings after the shock.



Figure 1. Positive asymmetric shock on savings

Speaking about the spillover effects of the initial saving shock on the other country of the union (RoU), they are again mainly due to the central bank decision of reducing the nominal interest rate. The drop in net exports towards the RoW caused by the nominal appreciation of the common currency acts as a negative demand shock for the RoU and its inflation goes down. If consumption is quite unchanged in the RoU, the lower real interest rate stimulates investments in this country and the expected inflation is increasing for the next periods (more than in the domestic country). Subsequently, the RoU suffers from an additional loss of intra-union competitiveness, which explains the stronger appreciation of the real effective exchange rate in this country. Output decreases in the RoU even more than in the domestic country and the negative demand shock induces lower labour income for households. In order to finance their consumption, households from the RoU become net borrowers with regards to the RoW. Their borrowing position more than compensates the lending position of households from the domestic country with regards to the RoW.

## 5. Macroeconomic policy and households saving shocks transmission

In this section we look at how macroeconomic policy may influence the transmission of the saving shock. We first address the question about the role of the common monetary policy and then turn to the study of fiscal policy.

# 5.1 The monetary policy implications for the transmission of saving shocks

In order to study these implications, we conducted a robustness analysis of the previous results by changing the coefficients of the interest rate rule for the monetary policy. If changes in  $\beta_0$  or  $\beta_1$  have little influence on the simulation results and do not qualitatively change them, the coefficient  $\beta_2$  seems more important for our analysis. When the relative weight given by the central bank to the output stabilization objective compared to the inflation stabilization is sufficiently high ( $\beta_2$ =0.7), results are qualitatively similar to the ones exposed in the section 4 (Figure 1). We just note that a higher  $\beta_2$  coefficient corresponds in our simulations to a deeper fall in the output of the two member countries to the union. Intuitively, this is due to the stronger reaction of the central bank to the output drop after the shock. A stronger decrease in the central bank interest rate requires a higher appreciation of the common currency, with a negative impact on net exports and output in an open monetary union.

But results may qualitatively change if the common central bank is concerned by the inflation stabilization and pays very little attention to the output stabilization in the union. Figure 2 presents the macroeconomic effects of the previous saving shock when the relative weight given to the output stabilization by the central bank is very low, see  $\beta_2=0.1$ .

In that case, the saving shock first induces real effective exchange rate depreciation for the two countries of the union, with positive impact on their net exports and output. As long as the common monetary policy does not react to the initial fall in consumption (which should reduce output in the union and ask for lower interest rate), there is an increase in the central bank interest rate and a subsequent nominal depreciation of the common currency (given the uncovered interest rate parity). Subsequently, there is a positive effect on net exports towards the RoW. The increase in investments and net exports for the domestic country after the shock exceeds the decrease in consumption. Thanks to the net exports channel, the initial saving shock is transformed in a positive demand shock, being followed by an increase in output and inflation. This is the case in our simulations when  $\beta_2 < 0.65$ , but the benefit of the shock on output growth increases when  $\beta_2$  decreases.



Figure 2. Monetary policy and saving shocks transmission

The real effective depreciation of the common currency allowed by the nominal depreciation is not only beneficial to the domestic country, but also to the RoU. A first spillover of the saving shock hit by the domestic country for the RoU is additional net exports towards the RoW. But a second negative spillover is to be discussed. Because inflation is higher in the domestic country compared to the RoU and the common central bank stabilization objective concerns the average inflation in the union, the common nominal interest rate is over-adjusted from the point of view of the RoU. However, as long as the positive spillover on net exports exceeds the negative one on investment, even the RoU may benefits from output growth after the shock. Moreover, if the central bank is less concerned about output stabilization, then this positive effect on output growth is stronger.

In this context, it seems that an inflation targeting monetary strategy for the common central bank (that we will assimilate hereafter to our  $\beta_2=0.1$  case) is a more friendly regime for an open monetary union hit by saving shocks.

#### 5.2 The fiscal policy implications for the transmission of saving shocks

This section addresses the question of the fiscal policy that should be conducted in the country hit by a saving shock in order to limit the negative impact on output within the union. Beyond the a-cyclical fiscal policy considered in our baseline calibration, we assume hereafter that the domestic government may opt for counter-cyclical or pro-cyclical public spending policy. The degrees of counter-cyclicality ( $\rho_{gy} = -0.7$ ) or pro-cyclicality ( $\rho_{gy} = 0.14$ ) of public consumption are chosen in line with

Fatas and Mihov (2010). However, the case of a strong pro-cyclical public spending policy ( $\rho_{gy}$ = 1.3) gives interesting results that, we think, may be useful to European public authorities, given the current period of fiscal austerity in the euro area.<sup>21</sup> Figure 3 depicts the reaction of output to a saving shock, in the two countries of the union and for two scenarios of monetary policy. We first consider the baseline situation where the monetary policy simultaneously reacts to inflation and output deviations from the steady state and we turn, next, to the suitable inflation targeting monetary strategy described in section 4.



## Figure 3. Fiscal policy and the saving shocks transmission

We can notice that the conduct of a pro-cyclical fiscal policy in the domestic country is never suitable. The negative impact of the shock on output growth is amplified under such a policy in the baseline monetary scenario and the output grows less when the inflation target strategy is adopted by the central bank. Moreover, if the domestic government decides to implement a strong pro-cyclical fiscal policy, even the benefits of the inflation targeting monetary policy for output growth are suppressed and output drops under its steady-state level in the two countries of the union.

The fact that the unsuitable effects of the domestic pro-cyclical fiscal policy on output growth concern simultaneously the two countries of the union shows that the real effective exchange rate channel has again an important role in the transmission of shocks. In the baseline monetary scenario, for example, the decrease in public consumption during a period of economic slowdown reduces even more the output. This calls for even lower central bank interest rate and subsequently a stronger appreciation

<sup>&</sup>lt;sup>21</sup> It is to be noted here that the sensitivity of our results is very low in relation to the other coefficients of the fiscal policy rule, that's why we decided to not discuss them in the paper.

of the common currency, with even more negative impact on net exports in the two countries of the union. In the inflation targeting monetary strategy, the output drop is aggravated by a nominal and real effective appreciation of the common currency in the union. Given the loss of consumption after the shock (which is expected to reduce output), the pro-cyclical fiscal policy implies a strong reduction in public spending. The initial saving shock is thus transformed into a negative demand shock for the domestic country implying a decrease in output and inflation simultaneously. Inflation in the union is decreasing and the central bank reacts by reducing its interest rate. This in turn leads to a nominal and real appreciation of the common currency with regard to the RoW, with a reinforced negative impact on the output of all member countries. Thus, it is not advisable to implement a restrictive fiscal policy in a context of a decline in household consumption. Furthermore, such a restrictive pro-cyclical fiscal stance would not help economic recovery in both the domestic economy and the RoU. This has recently been the difficult position of some euro area countries.

If the pro-cyclical fiscal policy in the domestic country has no benefits for the union after a positive domestic saving shock, the counter-cyclical fiscal policy seems, on the contrary, to be suitable for output growth in all countries of the union. It reduces the real effective appreciation of the common currency in relation to the RoW under the baseline monetary scenario and can amplify its real effective depreciation under the inflation targeting strategy.

# 6. Robustness analysis

In order to assess our previous results, we propose in this section some robustness checks. The scenario we choose for the robustness analysis is our proxy of inflation targeting monetary strategy combined with a-cyclical fiscal policies conducted for the two countries of the union. We did the simulations of the saving shock again and we adjust the values of the following parameters for both the domestic country and the RoU: *i*) the degree of openness to trade with the RoW  $(a_2)$ ; *ii*) the inverse intertemporal elasticity of substitution  $(\sigma)$ ; and *iii*) the share of banking loans by national banks to each government  $(\zeta^i)$ . For each parameter, Figure 4 displays the effects on domestic output and on the output in the RoU.

First, if the monetary union is close to the RoW ( $a_2$  being equal to zero), the shock of domestic household savings has smaller positive effects on output. This positive effect comes from the investment increase which is more important than the decrease in consumption, given the lower real interest rate after the shock. However, the benefits to the growth is smaller than in the open-union model, because there is no more trade in goods with the RoW.

#### **Figure 4. Robustness checks**



Second, when the intertemporal elasticity of substitution is higher ( $\sigma$  below unity), households are more willing to substitute future consumption for current consumption. Face to the lower real interest rate, the domestic trajectory has a steeper slope in that case, just after the shock. The demand for the next period will be weaker and the expected inflation in domestic country even lower compared to the inflation in the RoU. Thus, domestic country gains intra-union competitiveness and the RoU loses it. That is why domestic output is higher when the intertemporal elasticity of substitution is higher and the shock adversely affects the economy in the RoU.

Third, we show that the assumption used in the model for the financing of public debt does not significantly affect the results. The effects of the shock are qualitatively similar when public debt is exclusively financed on the national market ( $\zeta^{i} = 1$ ) and when it is financed on all the markets of the union (with or without the consideration of a domestic bias). Quantitatively speaking, these effects are slightly different, being explained by adjustments in the relative intra-union competitiveness of the domestic country relative to the RoU. It thus seems that when the public debt is exclusively financed by domestic banks, the domestic country benefits from an improved intra-union competitiveness, because, after the shock, the expected drop in domestic inflation is deeper than that for the RoU.<sup>22</sup> The openness of the domestic public debt market to the RoU relatively reduces the intra-union competitiveness of the domestic country. That is why when we reduce the domestic bias in financing public debt we notice some benefits for the RoU output compared to the domestic output.

<sup>&</sup>lt;sup>22</sup> This is for example due to the fact that, after the shock, the lower public debt (due to higher taxes collected in the context of increasing output) requires a lower amount of deposits to finance it. There are subsequently relatively more funds allocated to the private sector which boost investment just after the shock. This creates a temporary excess of demand on the goods' market that explain the higher inflation in domestic country. Starting from the following periods, banks adjust the amount of deposits to the financial needs of the economy and the excess of demand is quickly reduced explaining the deeper decrease in domestic inflation.

# 7. Conclusion

Our work is about the consequences of a likely increase in household savings in some Euro area countries. We studied the effect of such a shock in an open-economy two-country monetary union, assuming that the saving shock occurs in one country of the monetary union. We found that output growth can be positive on some conditions, in particular the reaction of the common central bank and the subsequent effect of a change in the common interest rate on the exchange rate of the common currency.

More specifically, we showed that output would not decline in the domestic country and on the rest of the union (RoU) if net exports of both countries to the rest of the world (RoW) can expand following a real depreciation of the common currency. This can be observed if the central bank leads an inflation targeting policy. However, if fiscal policy (public consumption) were procyclical, then the effects of the shock on output in both countries would be negative.

The saving shock could also cause a contraction in economic activity for other reasons found in the literature: the source of the shock is household deleveraging with a severe initial debt overhang (Eggertsson and Krugman, 2012) or with subsequent falling house prices (Cuerpo *et al*, 2013), output growth in the RoW is not sufficient to enable net exports to increase both in the domestic country and in the RoU, and there is government deleveraging.

Our results point to the conclusion that an increase in government savings (fiscal consolidation) should not occur at the same time as an increase in household savings. This recommendation could well apply to some Euro area countries (for instance, Portugal).

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Description	Parameter	Value	References
Inverse intertemporal elasticity of substitution	$\sigma$	2	Literature on the EA
Inverse of the Frisch elasticity of labour supply	η	1	Literature on the EA
Subjective discount factor	p	0.99	Literature on the EA
Working time Habit persistence coefficient	N h	1/3	Literature on the EA
Share of imported goods from the rest of the union	n " <sup>i</sup>	0.05	
Share of imported goods from the rest of the union	$a_1$	0.21	EA average (AMECO)
Share of imported goods from the rest of the world	$a_2'$	0.11	EA average (AMECO)
Elasticity of substitution domestic/imported goods	heta	1.5	Coenen et al. (2008)
Elasticity of the risk premium with respect to NFA position	$\pmb{\psi}_b^i$	0.001	(2003)
Capital contribution to production	α	0.36	EA average (OECD)
Capital depreciation rate	$\delta$	0.03	Literature on the EA
Internal capital adjustment costs parameter	$\psi_{I}$	0.25	Literature on the EA
Fraction of retailers keeping their prices unchanged	$\phi_i$	0.8	Literature on the EA
Elasticity of the external finance premium (EFP) with respect to firm's leverage ratio	γ	0.005	<sup>a</sup> See Notes
Quarterly factor for the external finance premium for firms	$\Psi^i_E$	1.005	<sup>b</sup> See Notes
Firms' probability of leaving the economy	1-v	0.0272	Bernanke et al. (1999)
"Home bias" in the banking loans provided to governments	$\zeta^{i}$	0.7	EA average (ECB)
Elasticity of risk premium with respect to government debt	$\psi_l^i$	0.001	Coenen et al. (2008)
Steady-state ratios			
Consumption/GDP ratio	C/Y	0.6	EA average (AMECO)
Investment /GDP ratio	I/Y	0.2	EA average (AMECO)
Public expenditures/GDP ratio	G/Y	0.2	EA average (AMECO)
Transfers/GDP ratio	Tr/Y	0.13	EA average (AMECO)
Loans to Governments /GDP ratio	$l^i$	1	<sup>c</sup> See Notes
Public debt/GDP ratio	$DY^i = l^i$	1	Steady-state analytical solution
Macroeconomic policy			
Smoothing coefficient in the monetary policy rule	$oldsymbol{eta}_{0}$	0.8	Literature on the EA
Inflation stabilizing coefficient in the monetary policy rule	$oldsymbol{eta}_1$	2	Literature on the EA
Output stabilizing coefficient in the monetary policy rule	$eta_2$	0.1	Literature on the EA
Smoothing coefficient in the public expenditure rule	$ ho_{g}$	0.8	Coenen et al. (2008)
Output stabilizing coefficient in the public expenditure rule	$ ho_{_{gy}}$	1.3	<sup>d</sup> See Notes
Debt stabilizing coefficient in the public expenditure rule	$ ho_{_{gl}}$	0.01	Christoffel et al. (2011)
Tax rate on consumption	$ au_c$	0.20	EA average (EC data)
Tax rate on wages (labor income)	$ au_{_{W}}$	0.33	EA average (EC data)
Tax rate on capital income	$ au_k$	0.25	EA average (EC data)

# Appendix: Table 1. Calibration of the model

Notes:<sup>a</sup> Corresponding to a steady-state leverage NW/K=0.4 as in the literature on the EA. <sup>b</sup> Corresponding to a 2% average annual EFP for firms as in Bernanke et al. (1999). <sup>c</sup> The ratio of average total bank assets to GDP is close to 300% in Europe, with a share of banking loans to the private sector amounting to 67% of total banking loans (ECB database). Thus, the share of banking loans to the private sector is approximately 200% of GDP and the share of banking loans to governments is 100% of GDP. <sup>d</sup> In the literature on the cyclicality of the government budget balance, an estimated coefficient on the output gap above unity means that fiscal policy is highly counter-cyclical (as in the United States or the United Kingdom in the early 2000s).